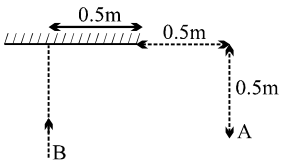
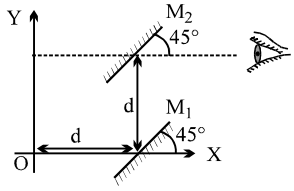
1. LAWS OF REFLECTION

Q.1 Eve, the 176 cm tall model is looking at herself in a mirror hanging on the wall. The mirror is vertical and its height is 75 cm. Eve stands straight, her eyes are at a height of 168 cm from the floor. How much length (in cm) of her image can she view if she can just see her shoes? [Ans. 0150.00]

Q.2 A man 'A' stands on outside of a plane mirror and a second man 'B' approaches the mirror along the line perpendicular to it which passes through its centre. At what distance from the mirror will 'B' be at the moment when 'A' and 'B' first see to each other in the mirror --------. [Ans.0.5 m]



Q.3 Find the coordinates of the image formed that of an object placed at origin, which the eye will observe in mirror M_2. [Ans.(– d, d)]



2. SPHERICAL MIRRORS

Q.4 Two rays are incident on a spherical mirror of radius of R = 5 cm parallel to its optical axis at the distance $h_1 = 0.5$ cm and $h_2 = 3$ cm. Determine the distance Δx (in cm) between the points at which these rays intersect the optical axis after being reflected at the mirror. [Ans.5/8= 0.625 cm]

Q.5 A bright source S is kept on the principal axis of a concave mirror of focal length f = 20 cm at a distance d = 30 cm from its pole. Find the distance D (in cm) in front of the concave mirror where a plane mirror should be placed so that the ray after being reflected first at the concave mirror and then by the plane mirror return to the point S. [Ans.45 cm from concave mirror]

Q.6 Two rays are incident on a spherical mirror of radius of R = 5 cm parallel to its optical axis at the distance $h_1 = 0.5$ cm and $h_2 = 3$ cm. Determine the distance Δx between the points at which these rays intersect the optical axis after being reflected at the mirror. [Ans. 5/8= 0.625 cm]

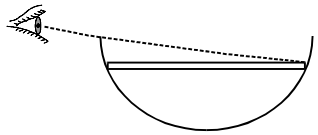
Q.7 A thief is running away in a car with velocity of 20 m/s. A police jeep is following him, which is sighted by thief in his rear view mirror which is a convex mirror of focal length 10 m. He observes that the image of jeep is moving towards him with a velocity of 1 cm/s. If the magnification of the mirror for the jeep at that time is 1/10. Find
(a) actual speed of jeep (b) rate at which magnification is changing.
Assume that police jeep is on axis of the mirror. [Ans. (a) 21 m/s, (b) 1×10^{-3} /sec]

Q.8 A luminous point P is inside a circle. A ray enters from P and after two reflections by the circle, return to P. If θ be the angle of incidence, a the distance of P from the centre of the circle and b the distance of the centre from the point where the ray in its course crosses the diameter through P, prove that $\tan\theta = \sqrt{\frac{a-b}{a+b}}$.

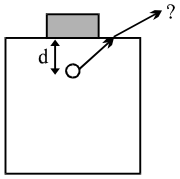
3. REFRACTION - PLANE SURFACE

Q.9 A surveyor on one bank of canal observed the image of the 4 inch and 17 ft marks on a vertical staff, which is partially immersed in the water and held against the bank directly opposite to him, coincides. If the 17ft mark and the surveyor's eye are both 6ft above the water level, estimate the width of the canal, assuming that the refractive index of the water is 4/3. [Ans.16 feet]

Q.10 A circular disc of diameter d lies horizontally inside a metallic hemispherical bowl radius a. The disc is just visible to an eye looking over the edge. The bowl is now filled with a liquid of refractive index μ. Now, the whole of the disc is just visible to the eye in the same position. Show that $d = 2a\frac{(\mu^2 - 1)}{(\mu^2 + 1)}$.



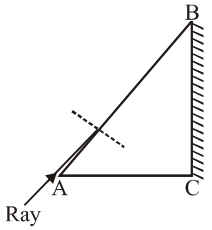
Q.11 A large Lucite cube (n = 1.5) has a small air bubble (a defect in the casting process) below one surface. When a rupee coin (diameter 2 cm) is placed directly over the bubble on the outside of the cube, the bubble cannot be seen by looking down into the cube at any angle. However, when a 50 paise coin (diameter 1.5 cm) is placed directly over it, the bubble can be seen by looking down into the cube. What is the range of the possible depths d of the air bubble beneath the surface?



[Ans. $d_{max} = \frac{\sqrt{5}}{2}$, $d_{min} = \frac{3\sqrt{5}}{8}$]

4. REFRACTION THROUGH PRISM

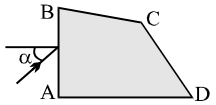
Q.12 The **diagram** shows a right angled prism, the other surface of which is made perfectaly reflecting. The angle ABC = 30° and the refractive index of the prism is $2/\sqrt{3}$. The light ray is incident from air on the inclined face at grazing incidence. What will be the overall deviation of the light ray (in degrees) after it comes back in air ?



[Ans. 0090]

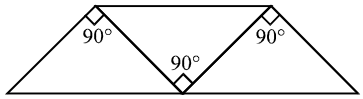
Q.13 A ray of light passes through a prism in a principle plane the deviation being equal to angle of incidence which is equal to 2α. It is given that α is the angle of prism. Show that $\cos^2\alpha = \frac{(\mu^2 - 1)}{8}$, where μ is the refractive index of the material of prism.

Q.14 The faces of prism ABCD made of glass of refractive index μ from dihedral angles ∠A = 90°, ∠B = 75°, ∠C = 135° and ∠D = 60°. A beam of light falls on face AB and after complete internal reflection from face BC, escapes through face AD. Find the range of μ and angle of incidence α of the beam, if a beam that has passed through the prism in this manner, is perpendicular to the incident beam.

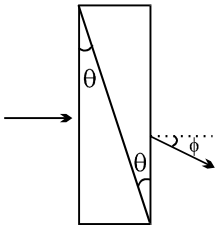


[Ans. $2 > \mu > \sqrt{2}$, $\alpha = \sin^{-1}\left(\frac{\mu}{2}\right)$]

Q.15 Three right angled prisms of refractive indices μ_1, μ_2 and μ_3 are joined together so that the faces of the middle prism in are in contact each with one of the outside prisms. If the ray passes through the composite block undeviated, show that $\mu_1^2 + \mu_3^2 - \mu_2^2 = 1$.

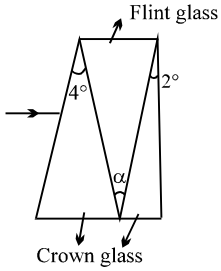


Q.16 Two identical prisms with slightly different indices are located as shown. Angle θ is small. When a laser beam strikes one of the prisms perpendicular to the surface, the refracted ray is deviated by a small angle ϕ. Find the difference between the indices of refraction of the prism in terms of θ and ϕ.



[Ans. $\frac{\phi}{\theta}$]

Q.17 The refractive indices of the crown glass for violet and red lights are 1.51 and 1.49 respectively and those of the flint glass are 1.77 and 1.73 respectively. A narrow beam of white light is incident at a small angle of incident on the shown combination of thin prisms. Find the value of α for which the mean deviation of the incident beam is zero. Also calculate the net dispersion produced by the combination.



[Ans. 4°, 0.04°]

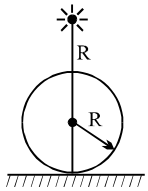
Q.18 The dispersive power of crown and flint glasses are 0.03 and 0.05 respectively. The refractive indices for yeloow light for these glasses are 1.517 and 1.621. It is designed to form an achromatic combination of prisms of crown and flint glasses which can produce a deviation of 1° for yellow ray. Find the refracting angles of the two prism needed. [Ans. 4.84°, 2.42°]

5. *REFRACTION AT SPHERICAL SURFACE*

Q.19 A spider & a fly are on the surface of a glass sphere of radius R over. What area the fly be for the spider to be able to see it ? Assume that the radius of the sphere is much larger than the sizes of the spider & the fly. Take the refractive index of glass to be $\mu_g = 2$. [Ans.πR^2]

Q.20 A transparent sphere with a refraction index of n relative to air has a radius r. An object is placed at a distance 4r from the centre of the sphere. Its final image is obtained at a distance _____ from the centre of the sphere. [Ans : $\frac{4nr}{7n-8}$]

Q.21 An opaque sphere of radius R lies on a horizontal plane. On the perpendicular through the point of contact there is a point source of light a distance R above the sphere.



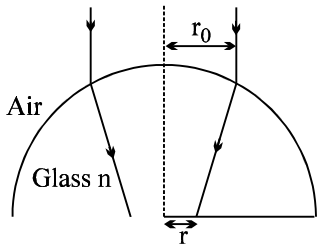
(a) Show that the area of the shadow on the plane is $3\pi R^2$.

(b) A transparent liquid of refractive index $\sqrt{3}$ is filled above the plane such that the sphere is just covered with the liquid. Show that the area of shadow now becomes $2\pi R^2$.

Q.22 A ray of light refracted through a sphere, whose material has refractive index μ in such a way that it passes through the extremities of two radii which make an angle θ with each other. Prove that if α is the deviation of the ray caused by its passage through the sphere

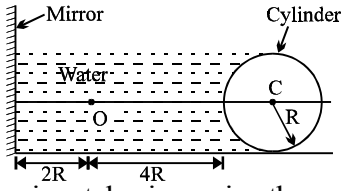
$$\cos\frac{1}{2}(\theta-\alpha)=\mu\cos\frac{\theta}{2}$$

Q.23 A beam of light is incident vertically on a glass hemisphere of radius R lying with its plane side on a table . The axis of the beam coincides with the vertical axis passing through the centre of the base of the hemisphere and the radius r_0 of the cross section of the beam is smaller than R. Find the radius of the luminous spot formed on the table.



[Ans. $r=\dfrac{r_0}{\sqrt{\{1-(r_0/R)^2\}\{n^2-(r_0/R)^2\}+(r_0/R)^2}}=\dfrac{r_0}{n}$, if $r_0<R$

Q.25 The figure shows a very long circular cylinder of radius R made of glass $\mu_g=3/2$ fixed to the horizontal surface. The space between the cylinder and a vertical wall (plane mirror) is filled with water of refractive index $\mu_w=4/3$, as shown in figure.



Find the positions of the final image formed of an object O placed on the horizontal axis passing through the centre of mass C of the cylinder. as shown in figure. Express you answer with respect to centre C of the cylinder. Consider only paraxial rays lying in the vertical plane containing the point O and C.

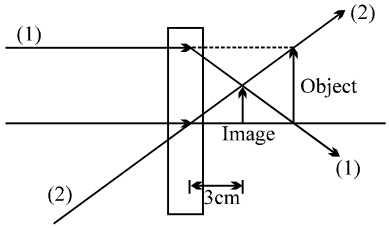
[Ans : $\dfrac{15R}{4}$, 3R]

6. *REFRACTION IN LENS*

Q.26 The rectangular box is the place of a lens. By looking at the ray diagram, answer the following questions:

(i) What is the focal length of the lens?

(ii) The magnification factor is equal to



[Ans. (i) 6 cm, (ii) $\dfrac{1}{2}$]

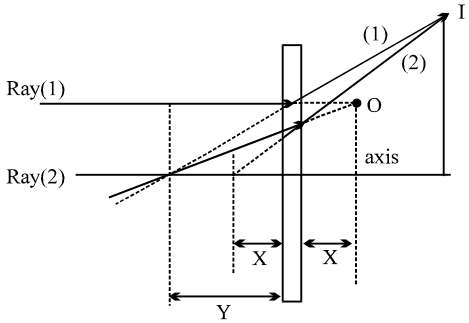
Q.27 A transparent sphere with a refraction index of n relative to air has a radius r. An object is placed at a distance 4r from the centre of the sphere. Its final image is obtained at a distance _____ from the centre of the sphere.

[Ans : $\dfrac{4nr}{7n-8}$]

Q.28 The rectangular box shown is the place of lens. By looking at the ray diagram, answer the following questions:

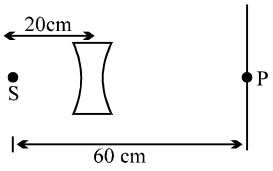
(i) If X is 5 cm then what is the focal length of the lens?

(ii) If the point O is 1 cm above the axis then what is the position of the image? Consider the optical center of the lens to be the origin.



[Ans. 10cm, 10,2]

Q.29 A point source of light is placed 60cm away from screen. Intensity detected at point P is I. Now a diverging lens of focal length 20 cm is placed 20 cm away from S between S and P. The lens transmits 75% of light incident on it. Find the new value of intensity at P.



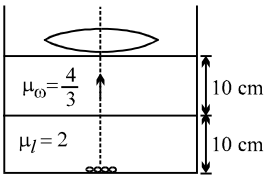
[Ans. 0.27 I]

Q.30 The dispersive power of the material of a lens is 0.4 and the focal length of the lens is 10 cm. Find the difference in the focal lengths of the lens for violet and red colour. [Ans. $\frac{25}{6}$ cm]

Q.31 An observer holds in front of himself a thin symmetrical converging lens ; a is the numerical value of the radius of each face. He sees two images of his eyes, one erect and the other inverted. Explain the formation of these images and assuming the refractive index of glass to be 1.50, prove that he will see two erect images if the distance of the lens is less than 0.25a.

Q.32 A thin converging lens is arranged between a small illuminated object & a screen so that an image of the object of linear magnification 3 is formed on a screen . The object and the screen are then 64 cm apart. A thin biconcave lens is then placed between the converging lens & the screen so that the lenses are coaxial & 6 cm apart . To restore a sharply focussed image on the image screen the object was moved away from the converging lens through a distance of 14 cm . The biconcave lens has a surface of radii of curvature 14 cm & 21 cm. Calculate the focal length of the biconcave lens. Also find the R. I. of the biconcave lens. [Ans. $f = -21$ cm, $\mu = 1.4$]

Q.33 An insect is placed at the base (at centre) of a cylindrical vessel and oil of refractive index $\mu_l = 2$ is filled upto height 10 cm. Then water is poured in vessel upto height 20 cm (length of water column 10 cm). If a convex lens of focal length 50 cm is placed just above the surface of water. Then find location of final image of the insect by this system. If the insect starts moving up with speed 1.8 cm/s, then what will be the initial speed of the final image of the insect.

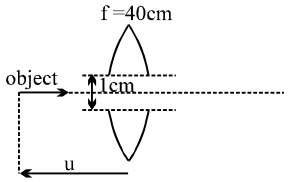


[Ans.1.6 cm / sec up, $\frac{50}{3}$ cm below water surface]

Q.34 Two thin similar watch glass pieces are joined together, front to front, with rear portion silvered and the combination of glass pieces is placed at a distance $a = 60$ cm from a screen. A small object is placed normal to the optical axis of the combination such that its two times magnified image is formed on the screen. If air between the glass pieces is replaced by water ($\mu = 4/3$), calculate the distance through which the object must be displaced so that a sharp image is again formed on the screen.

[Ans. 15 cm towards the combination]

Q.35 In the figure shown, find the relative speed of approach/separation of the two final images formed after the light rays pass through the lens, at the moment when u = 30 cm. The speed object = 4 cm/s. The two lens halves are placed symmetrically w.r.t. the moving object.



[Ans. 8/5 cm/s]

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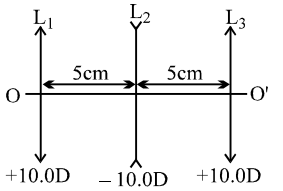
Q.36 The figure illustrates an aligned system consisting of three thin lenses. The system is located in air. Determine:

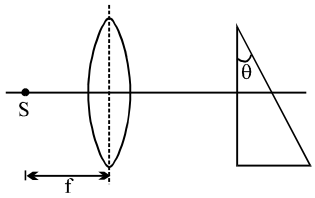


(a) the position (relative to right most lens) of the point of convergence of a parallel ray incoming from the left after passing through the system ;

(b) The distance between the first lens and a point lying on the axis to the left of the system, at which that point and its image are located symmetrically with respect to the lens system?

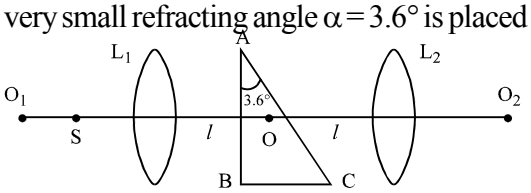
[Ans (a)3.3 cm, (b) $l = (50/3)$ cm]

Q.36 A glass wedge with a small angle of refraction θ is placed at a certain distance from a convergent lens with a focal length f, one surface of the wedge being perpendicular to the optical axis of the lens. A point sources S of light is on the other side of the lens at its focus. The rays reflected from the wedge (not from base) produce, after refraction in the lens, two images of the source displaced with respect to each other by d. Find the refractive index of the wedge glass.



[Ans. $d/2f\theta$]

Q.37 A thin right angled glass prism of cross-section ABC with a very small refracting angle $\alpha = 3.6°$ is placed between the convex lenses along a common axis O_1O_2 (as shown in figure)

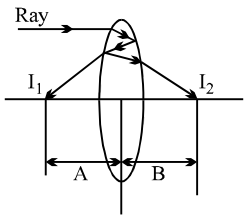


The lenses are placed at equal distances $l = 30$ cm from the prism. The refractive index of the prism glass is $\mu = 1.5$. The surface AB does not reflect any light energy while the surface AC is partially reflecting and partially refracting. A point source of light S is placed at a distance of 20 cm to the left of the lens L_1 as shown. Considering the rays that are closed to the axis O_1O_2, find the possible position of the images(s) formed by this system. Show the image on a proper ray diagram. The focal lengths of the lenses L_1 and L_2 are 20 cm each. The face AB of the prism is normal to the axis O_1O_2.

[Ans. $y_1 = f\delta_1 = 20 \times \frac{\pi}{100}$ cm $= \pi/5$ cm ; $y_2 = f\delta_2 = 20 \times \frac{3\pi}{50}$ cm $= 1.2\pi$ cm]

7. LENS MIRROR COMBINATIONS

Q.38 A light ray enters into a double convex lens parallel to the axis. The image I_1 is obtained after single internal reflection and the image I_2 is obtained after two internal reflections. If the lens is equiconvex then find out the refractive index of the lens. Where $k = B/A$.



[Ans. $\frac{K-1}{3K-2}$]

Q.39 A concave mirror has the form of a hemisphere with a radius of $R = 60$ cm. A thin layer of an unknown transparent liquid is poured into the mirror. The mirror-liquid system forms one real image and another real image is formed by mirror alone, with the source in a certain position. One of them coincides with the source and the other is at a distance of $l = 30$ cm from source. Find the possible value(s) refractive index μ of the liquid. [Ans. 1.5 or $(\sqrt{5}-1)$]

Q.40 A parallel beam of light of diameter 4 cm falls on a thin convex lens of focal length 40 cm and then on a thin convex lens of focal length 10 cm as shown in figure (A). In figure (B) the second lens is an equiconcave lens of focal length 10 cm and made of material of refractive index 1.5. In both the case, the second lens has an aperture equal to 1 cm.

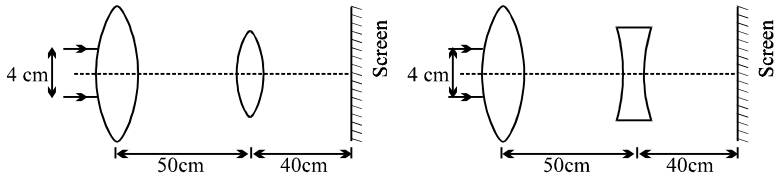


Figure (A) Figure (B)

(a) Compare the area illuminated by the beam of light on the screen, which passes through the second lens in the two cases. Now liquid of refractive index μ is filled to the right of the second lens in case B such that the area illuminated in both the cases is the same.

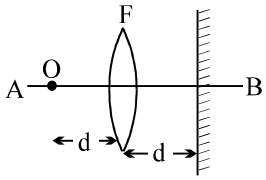
(b) Determine the refractive index of the liquid. [Ans : (a) 81, (b) 3]

Q.41 One side of radius of curvature $r_1 = 120$ cm of a convex lens of material of refractive index $\mu=3/2$ and focal length $f_1 = 20$ cm is silvered. It is placed on a horizontal surface with silvered surface in contact with it. Another convex lens of focal length $f_2 = 20$ cm is fixed coaxially d = 10 cm above. A luminous point object O on the axis gives rise to an image coincident on it. Find its height above the upper lens.

[Ans. 100/19 cm]

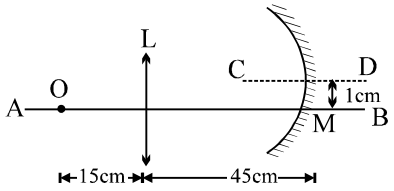
Q.42 Two identical convex lenses L_1 and L_2 are placed at a distance of 20 cm from each other on the common principal axis. The focal length of each lens is 15 cm and the lens L_2 is to the right of lens A. A point object is placed at a distance of 20 cm on the left of lens L_1, on the common axis of two lenses. Find, where a convex mirror of radius of curvature 5 cm should be placed so that the final image coincides with the object? [Ans. 5.9 cm,10.9 cm]

Q.43 In the figure shown 'O' is point object. AB is principal axis of the converging lens of focal length F. Find the distance of the final image from the lens.



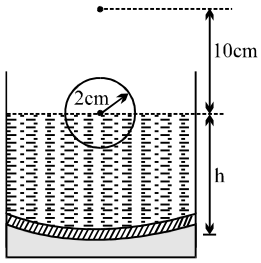
$$[\text{Ans. } l = \left[\frac{(3f-2d)fd}{4fd-2d^2-f^2}\right]]$$

Q.44 In the figure shown L is a converging lens of focal length 10cm and M is a concave mirror of radius of curvature 20cm. A point object O is placed in front of the lens at a distance 15cm. AB and CD are optical axes of the lens and mirror respectively. Find the distance of the final image formed by this system from the optical centre of the lens. The distance between CD & AB is 1 cm.



[Ans. $6\sqrt{26}$ cm]

Q.45 A transparent solid sphere of radius 2 cm and density ρ floats in a transparent liquid of density 2ρ kept in a beaker. The bottom of the beaker is spherical in shape with radius of curvature 8 cm and is silvered to make it concave mirror as shown in the figure. When an object is placed at a distance of 10 cm directly above the centre of the sphere its final image coincides with it. Find h (as shown in the figure), the height of the liquid surface in the beaker from the apex of the bottom. Consider the paraxial rays only. The refractive index of the sphere is 3/2 and that of the liquid 4/3.

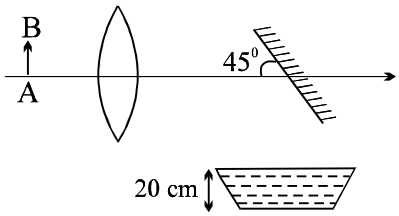


[Ans.194/13 cm]

Q.46 A thin bi-convex lens of refractive index 3/2 and radius of curvature 50 cm is placed on a reflecting convex surface of radius of curvature 100 cm. A point object is placed on the principle axis of the system such that its final image coincides with itself. Now few drops of a transparent liquid is placed between the mirror and lens such that final image of the object is at infinity. Find the refractive index of the liquid used. And also find the position of the object.

[Ans. $\mu = 7/6$, 100 cm from the lens]

Q.47 An object AB is at a distance of 18 cm from a lens with focal length 15 cm. A flat mirror turned through 45^0 with respect to optical axis of the lens is placed at a distance 0.5 m behind the lens as shown in figure. A sharp image of the object is formed at the bottom of the vessel filled with water upto a height of 20 cm. Find the distance of the image from the optical axis of the system.(use $\mu_{water} = 4/3$)



[Ans.45 cm]