LEADER TEST SERIES / JOINT PACKAGE COURSE

TARGET : JEE (Main) 2019

Test Type : MAJOR   TEST # 06   Test Pattern : JEE (Main)

TEST DATE : 03 - 03 - 2019

ANSWER KEY

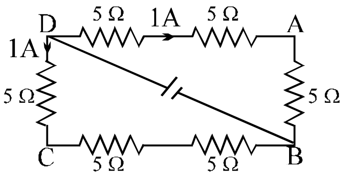
Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	3	2	1	3	3	1	3	3	1	2	3	4	3	1	2	3	3	1	3	1
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	2	2	3	2	4	1	3	3	2	1	3	4	1	2	3	3	4	2	3	3
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	1	3	2	2	1	2	4	2	4	1	2	4	3	1	4	2	3	3	3	4
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	4	4	2	4	2	3	3	3	3	2	3	2	1	4	1	4	2	2	3	2
Que.	81	82	83	84	85	86	87	88	89	90										
Ans.	2	2	2	4	4	3	4	1	2	4										

HINT – SHEET

1. Current in the circuit

$I = \frac{15}{15} \times 2$

I = 2 Amp



$V_D = +15$

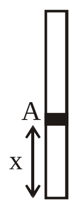
$V_D - 10 \times 1 = V_A$

$V_A = 15 - 10 = 5$ V

$V_C = 15 - 5 \times 1 = 10$ V

2. $T\sqrt{2} = \frac{m\omega^2 \ell}{\sqrt{2}}$

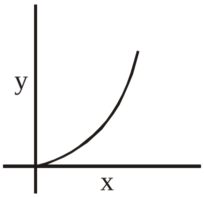
4.



Elastic energy density at

$A = \frac{1}{2}\frac{(\text{stress})^2}{Y} = \text{constant}$

$\frac{1}{2}\frac{\left(\frac{M}{L}\frac{x}{A}g\right)^2}{Y} = \text{constant}$



$Y \propto x^2$

8. $P_{total} = P_C\left(1+\frac{m^2}{2}\right)$

where "m" is modulation index. Putting m = 1

P_{total} = 1.5 kW

9. $\Delta p = F.\Delta t$ so

momentum of lighter cart (p_1) = momentum of heavier cart (p_2)

K.E. $_1$(lighter cart) = $\frac{p_1^2}{2m}$

K.E. $_2$ (heavier cart) = $\frac{p_2^2}{2(2m)}$

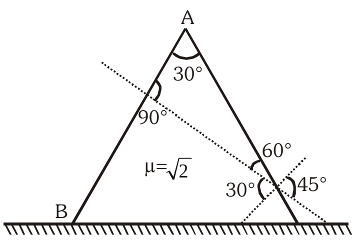
So. K.E.$_1$ > K.E.$_2$

10. Frequency of the wave doesn't change but speed α wavelength do

$v_{water} = \frac{v_{air}}{(4/3)}$ α $v_{glass} = \frac{v_{air}}{(8/5)}$

Dividing $v_g = \frac{5}{6} v_{water}$

11.



From the shown diagram μ sin 30° = 1 sinα

$\sqrt{2}\left(\frac{1}{2}\right) = \sin r$

r = 45°

Deviation angle $\delta = (i+e)-(r_1+r_2)$

$\delta = (0+45°)-30° \Rightarrow 15°$

12. $|V| = \left|-\frac{d\phi}{dt}\right| = \int \vec{E}.\vec{d}l$

$\Rightarrow \pi b^2 \frac{B}{\Delta t} = E \times 2\pi a$

∴ Force on circular wheel

$= E \times q = E \times 2\pi a \times \lambda = \frac{\pi b^2 B}{\Delta t} \times \lambda$

Torque about centre

$= a \times F = \frac{\pi b^2 B}{\Delta t} \times \lambda a = I_C \alpha$

$\therefore I_C \times \frac{\omega}{\Delta t} = \frac{\pi b^2 B \lambda a}{\Delta t}$

$\therefore \omega = \frac{\pi b^2 B \lambda a}{I}$

13. $E = E_0 e^{-\gamma t}$

14. Maximum current thorugh capacitor = $\frac{V_0}{z}$

$i = \frac{V_0}{z}$

$Q\omega = \frac{V_0}{z}$

$Q = \frac{V_0}{\omega z}$

15. $\frac{1}{mS_{gas}} = \frac{30}{10} = 3$

$\frac{1}{mS_{liquid}} = \frac{20}{20} = 1$

$\frac{1}{mS_{solid}} = \frac{40}{20} = 2$

$\frac{1}{3m} : \frac{1}{m} : \frac{1}{2m}$

2 : 6 : 3

16. $\frac{1}{2}mV^2 = \frac{3GMm}{2R} \Rightarrow V = \sqrt{\frac{3GM}{R}}$

17. Area = $m[v_f - v_i]$

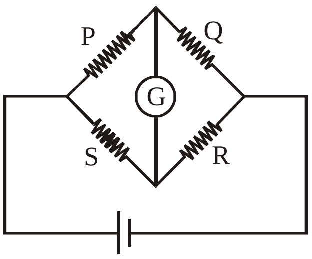
$20 - 10 = 2[v_f - 0]$, $v_f = 5$

$W = \frac{1}{2}m[v_0^2 - v_1^2]$

18. $5(2\mu F)$ in series with $\left(\frac{2\mu F}{2}\right)$, $10\mu F$ in series with $1\mu F$, $C_{eq} = \frac{10\times 1}{10+1} = \frac{10}{11}\mu F$

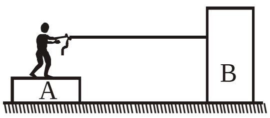
19. $R\downarrow \Rightarrow i\uparrow \Rightarrow v_1\downarrow$

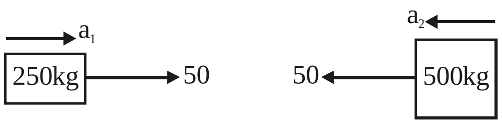
20.



$$\frac{P_P + Q}{P_R + S} = \frac{R+S}{P+Q} = \frac{R}{Q}\left(\frac{1+\frac{S}{R}}{1+\frac{P}{Q}}\right) = \frac{R}{Q}$$

21.





$a_1 = \frac{1}{5}$ m/s^2 $a_2 = \frac{1}{10}$ m/s^2

$v_1 = 5 \times \frac{1}{5} = 1$ m/s $\quad v_2 = 5 \times \frac{1}{10} = \frac{1}{2}$ m/s

$v_{12} = 1.5$ m/s

22. $t = \frac{\ell}{v}$, $v = \sqrt{\frac{T}{\mu}}$

23. $f' = \left(\frac{V \pm V_0}{V \pm V_S}\right)f$

$f_1 = \left(\frac{V \pm V_0}{V}\right)f$

$f_2 = \left(\frac{V}{V \pm V_S}\right)S$

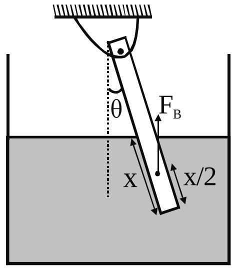
$\lambda_2 = \left(\frac{V \pm V_S}{f}\right)$

V_0 = velocity of listener; V_S = Velocity of source.

But if $\vec{V}_S = \vec{V}_0$ then $f' = f$

Thus statement-3 is not always correct

24.



where $F_B = \frac{9\rho}{5}(xA)g$

Balancing torque about hinge

$$(\rho\ell A)g \times \left(\frac{\ell}{2}\sin\theta\right)$$

$$= \left[\frac{9\rho}{5}\times(xA)\right]g \times \left[\left(\ell - \frac{x}{2}\right)\sin\theta\right]$$

[where θ is very small]

25. $C = \frac{Q}{n\Delta T}\ \frac{\Delta U + W}{n\Delta T} = \frac{nC_V\Delta t + \int_{V_i}^{V_f} PdV}{n\Delta T}$

$$= C_V + \frac{\alpha}{n\Delta t}.\int_{V_i}^{V_f} VdV$$

$$= \frac{R}{\gamma - 1} + \frac{1}{2}\frac{\alpha V_f^2 - \alpha V_i^2}{n\Delta T}$$

$$= \frac{R}{\gamma - 1} + \frac{1}{2}\frac{P_f V_f - P_i V_i}{n\Delta T}$$

$$= R\left[\frac{1}{\gamma-1} + \frac{1}{2}\right] = \frac{R}{2}\left(\frac{\gamma+1}{\gamma-1}\right)$$

26. $\frac{1}{-(N.P.)} - \frac{1}{(-D)} = \frac{1}{f}$

$-1 + \frac{100}{25}$

$-1 + 4$

$3 = \frac{1}{f}$

27. $(\mu_1 - 1)A_1 - (\mu_2 - 1)A_2 = 0$

$\delta_1 + \delta_2 = 0$

28. $\frac{1}{\lambda} \propto (Z - b)2$

$$\frac{\lambda_2}{\lambda_1} = \frac{(Z_1 - b)^2}{(Z_2 - b)^2} = \left(\frac{28}{14}\right)^2 = 4$$

$$\boxed{\lambda_2 = 4\lambda_1}$$

29. $\frac{A_0}{\sqrt{3}} = A_0 e^{-\lambda t}$ t = 1

N = $A_0 e^{-\lambda 4}$

N = $A_0 \left(\frac{1}{\sqrt{3}}\right)^4 = \frac{A_0}{9}$

30. $[\text{Area}] = \left(\frac{[V]^2}{[A]}\right)^2 = V^4A^{-2}$

$[Y] = \frac{[F]/[A]}{[\Delta^2]/[\ell]} = FV^{-4}A^2$

31. $KClO_3 \rightarrow KCl + \frac{3}{2}O_2$

m

mass

decomposed= $\frac{50m}{100}$ $\frac{3}{2} \times \frac{50m}{100 \times 122.5}$

moles = $\frac{50m}{100 \times 122.5} = \frac{67.2}{22.4}$, m = 490 gm

33. $\frac{r_{He}}{r_{CH_4}} = \frac{P^0_{He}}{P^0_{CH_4}}\sqrt{\frac{M_{CH_4}}{M_{He}}} = \frac{4}{1}\sqrt{\frac{16}{4}} = 8:1$

34. $1.5 \times 10^{-3} \times (10\text{-}2x) = 0.03 \times 5 \times \frac{40}{1000}$

x = 3

35. $H^+(aq) + OH^-(aq) \longrightarrow H_2O(\ell)$

$\Delta H^\circ = -57.32$ kJ/mol

$\Delta H_r = \Delta H^\circ_f\,(H_2O) - [\Delta H^\circ_f\, H^+(aq) + \Delta H^\circ_f\,(OH^-)\,(aq)]$

36. W = – PΔV = –25 × 8

= – 200 bar- L

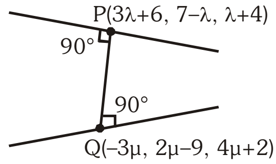
= – 20 kJ

ΔU = Q + W

= 80 – 20 = 60 kJ

40. $\frac{-dN}{dt} = \lambda N$

61.



$\overrightarrow{PQ}.(3\hat{i} - \hat{j} + \hat{k}) = 0$ &

$\overrightarrow{PQ}.(-3\hat{i} + 2\hat{j} + 4\hat{k}) = 0$

$\overrightarrow{PQ} = (3\lambda + 3\mu + 6)\hat{i} + (16 - \lambda - 2\mu)\hat{j} + (2 + \lambda - 4\mu)\hat{k}$

$\Rightarrow 7\mu + 11\lambda = -4$ & $29\mu + 7\lambda = 22$

$\Rightarrow \lambda = -1$ & $\mu = 1$

P & Q are (3, 8, 3) & (–3, –7, 6) respectively

Hence equation of PQ is $\frac{x-3}{2} = \frac{y-8}{5} = \frac{z-3}{-1}$

62. Two plane figures are triangle and trapezium

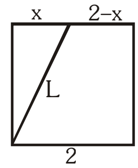
Let side of triangle be x and its perimeter be P_1

$\Rightarrow P_1 = x + 2 + L$, where L is length of cut.

Now, perimeter of trapezium = P_2

= 2 + 2 + (2 – x) + L = 6 – x + L

⇒ sum of the perimeters = 8 + 2L



which is maximum, when L is maximum

⇒ cut must be along the diagonal

$\Rightarrow L^2 = 4 + 4 \Rightarrow L = 2\sqrt{2}$

$\Rightarrow P = 8 + 4\sqrt{2}$

63. $x^2 + 2 = 2|x| - \cos\pi x$

$\Rightarrow x^2 - 2|x| + 2 = -\cos\pi x$

$\Rightarrow (|x|-1)^2 + 1 = -\cos\pi x$

$\Rightarrow x = \pm 1$

$\therefore$ Area = $\int_{-1}^{1}(x^2 + 2 - 2|x| + \cos\pi x)dx$

$= 2\left[\frac{x^3}{3} + 2x - x^2 + \frac{\sin\pi x}{\pi}\right]_0^1 = 2\left(\frac{4}{3}\right) = \frac{8}{3}$

64. In ΔAOB, AB = 2tan60° = $2\sqrt{3}$

⇒ Area of ΔAOB = $\frac{1}{2} \times 2 \times 2\sqrt{3} = 2\sqrt{3}$

Area of sector OAC = $\frac{60}{360}\pi.(2)^2 = \frac{2\pi}{3}$

⇒ Ratio = $\frac{2\sqrt{3} - \frac{2\pi}{3}}{\frac{2\pi}{3}} = \frac{3\sqrt{3}}{\pi} - 1$

65. Clearly $9 - x \ge 0,\ x - 1 \ge 0$ & $9 - x - (x - 1) \ge 0$

$\Rightarrow 1 \le x \le 5,\ x \in I$

$\therefore m = 5$

Range of $f(x) = \{^8C_0, {^7C_1}, {^6C_2}, {^5C_3}, {^4C_4}\}$

or $\{1, 7, 15, 10\}$

$\therefore n = 4$

66. $x^3 + y^2 = 5$ & $x^3y^2 = -6$

$\Rightarrow t^2 - 5t - 6 = 0$ has roots given by x^3 & y^2.

or $t = 6, -1$

$\Rightarrow x^3 = -1$ & $y^2 = 6$

$\therefore \arg(x + iy) = \tan^{-1}(\pm\sqrt{6})$ or $\tan^2\theta = 6$

67. $$f(x) = \begin{cases} \dfrac{4x}{5-6x}, & x < 0 \\ \dfrac{4x}{5+6x}, & x \ge 0 \end{cases}$$

$$f'(x) = \begin{cases} \dfrac{20}{(6x-5)^2}, & x < 0 \\ \dfrac{20}{(6x+5)^2}, & x > 0 \end{cases}$$

$f'(0^+) = f'(0^-) = 4/5$

$\therefore f$ is differentiable in $(-\infty, \infty)$

68. The curve cuts x-axis at (1, 0)

$$y' = \left(\sqrt{5-x^6}\right).\ 3x^2 - \left(\sqrt{5-x^4}\right)2x$$

$$y'\big|_{x=1} = \left(\sqrt{5-1}\right)3 - \left(\sqrt{5-1}\right)2$$

$\tan\theta = 6 - 4$

$$\theta = \tan^{-1}2 = \cot^{-1}\frac{1}{2}$$

69. $2xy\,dy = (x^2 + 1)\,dx + y^2\,dx$

$$\Rightarrow \frac{2xydy - y^2dx}{x^2} = \left(1 + \frac{1}{x^2}\right)dx$$

$$\Rightarrow d\left(\frac{y^2}{x}\right) = d\left(x - \frac{1}{x}\right)$$

$$\Rightarrow \frac{y^2}{x} = x - \frac{1}{x} + c \quad \Rightarrow \quad y^2 = x^2 - 1 + cx$$

$$\Rightarrow \left(x + \frac{c}{2}\right)^2 - y^2 = 1 + \frac{c^2}{4}$$

which represents a family of rectangular hyperbolas with centre on x-axis.

70. $$\frac{dy}{dx} = 1 + e^x$$

$$\Rightarrow \frac{dx}{dy} = \frac{1}{1+e^x}$$

$$\frac{d^2x}{dy^2} = \frac{d}{dx}\left(\frac{1}{1+e^x}\right)\frac{dx}{dy} = \frac{-e^x}{(1+e^x)^2}\cdot\frac{1}{(1+e^x)}$$

$$\therefore \left.\frac{d^2x}{dy^2}\right|_{x=1} = -\frac{e}{(1+e)^3}$$

71. AB + BC + CA = constant

BC = constant then AB + AC = constant

A is lying on the ellipse.

72. $$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = 2A \Rightarrow A^3 = 2A^2$$

$= 2^2A$

Similarly $A^4 = 2^3$ A and so on

So $A^n = 2^{n-1}A$

$$\Rightarrow A^n = 2^{n-1}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A^n - I = \begin{bmatrix} 2^{n-1} - 1 & 2^{n-1} \\ 2^{n-1} & 2^{n-1} - 1 \end{bmatrix}$$

$|A^n - I| = (2^{n-1} - 1)^2 - (2^{n-1})^2$

$= 1 - 2^n$

$\Rightarrow \lambda = 2$

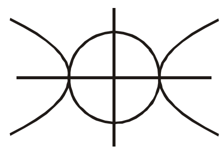
73. For f to be one–one vertex must lie on or to the right of y–axis.

$\therefore -m \ge 0 \Rightarrow m \le 0$

for m = 0, $f(x) = \begin{cases} x^2 - 1, & x \le 0 \\ -1, & x > 0 \end{cases}$ which is not one–one

$\therefore m \in (-\infty, 0)$

74.



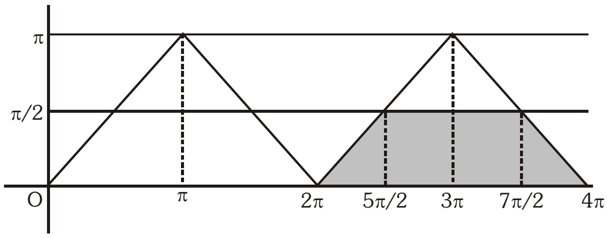
Equation of chord of the given circle whose middle point is (h, k) is $xh + yk = h^2 + k^2$
and equation of tangent line upon hyperbola is $y - mx = \sqrt{a^2m^2 - b^2}$
As both lines are identical

$\Rightarrow -\frac{m}{h} = \frac{1}{k} = \frac{\sqrt{a^2m^2 - b^2}}{h^2 + k^2}$

Eliminating m
$\Rightarrow (h^2 + k^2)^2 = (a^2h^2 - b^2k^2)$
hence the locus is $(x^2 + y^2)^2 = (a^2x^2 - b^2y^2)$

75. $y = \cos^{-1}(\cos x) = \begin{cases} x - 2\pi & x \in [2\pi, 3\pi] \\ 4\pi - x & x \in [3\pi, 4\pi] \end{cases}$

$y = \tan^{-1}x + \tan^{-1}\frac{1}{x} = \frac{\pi}{2}, \quad x > 0$



Area of the shaded region

$= \frac{1}{2}(\pi + 2\pi)\frac{\pi}{2} = \frac{3\pi^2}{4}$

76. L.H.L $= \lim_{x \to 1^-} \frac{e^{\frac{1}{x-1}} - 2}{e^{\frac{1}{x-1}} + 2} = \lim_{h \to 0^-} \frac{e^{-\frac{1}{h}} - 2}{e^{-\frac{1}{h}} + 2} = -1$

R.H.L $= \lim_{x \to 1^+} \frac{e^{\frac{1}{x-1}} - 2}{e^{\frac{1}{x-1}} + 2} = \lim_{h \to 0^+} \frac{e^{\frac{1}{h}} - 2}{e^{\frac{1}{h}} + 2}$

$= \lim_{h \to 0^+} \frac{1 - 2e^{-\frac{1}{h}}}{1 + 2e^{-\frac{1}{h}}} = 1$

as L.H.L ≠ R.H.L.
limit does not exist.

77. If we calculate $A^2 = \begin{bmatrix} 1 & 0 \\ 2\left(\frac{1}{2}\right) & 1 \end{bmatrix}$,

$A^3 = \begin{bmatrix} 1 & 0 \\ 3\left(\frac{1}{2}\right) & 1 \end{bmatrix}$,.........., $A^{50} = \begin{bmatrix} 1 & 0 \\ 50\left(\frac{1}{2}\right) & 1 \end{bmatrix}$

78.

1 3 2 4	2 1 4 3	3 1 4 2	4 1 3 2
1 4 3 2	2 4 1 3	3 2 1 4	4 1 1 3
	2 4 3 1	3 2 4 1	4 3 2 1

Total 11 numbers

79. Coordinates of any points lying on the line $y = \sqrt{3}x$ will be $\left(\frac{r}{2}, \frac{r\sqrt{3}}{2}\right)$

If the given line intersects the curve $x^4 + ax^2y + bxy + cx + dy + 6 = 0$, then

$\frac{r^4}{16} + a\frac{r^3\sqrt{3}}{8} + \frac{br^2\sqrt{3}}{4} + \frac{cr}{2} + \frac{dr\sqrt{3}}{2} + 6 = 0$

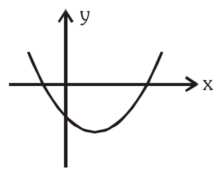
$r^4 + r^3\, 2a\sqrt{3} + r^2 4b\sqrt{3} + r8(c + d\sqrt{3}) + 96 = 0$
$\therefore\ r_1r_2r_3r_4 = 96$
hence OA . OB . OC . OD = 96

80. P(all tails or all heads in one trial) $= \frac{1}{4}$

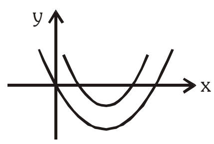
$\therefore$ P(this happens second time in 3^{rd} trial)

$= {}^2C_1 \times \frac{1}{4} \times \frac{3}{4} \times \frac{1}{4} = \frac{3}{32}$

81. $f(x) = x^2 - (a + 2)x - (a + 3)$
for $f(x)$ to be negative for atleast one positive x, following cases may be there -
Case-I $\quad f(0) < 0$



$-(a + 3) < 0 \Rightarrow a + 3 > 0$
$\Rightarrow a > -3$
Case-II (1) D > 0
$(a + 2)^2 + 4(a + 3) > 0$
$a^2 + 8a + 16 > 0$
$\Rightarrow (a + 4)^2 > 0$
$\Rightarrow a \in R - \{-4\}$



(2) $f(0) \geq 0$
$-(a + 3) \geq 0$
$\Rightarrow a + 3 \leq 0$
$\Rightarrow a \leq -3$

(3) $-\frac{b}{2a} > 0 \Rightarrow a > -2$

intersection of (1), (2) & (3) is $a \in \phi$.
$\therefore\ a \in (-3, \infty)$

82. $a^{\left(\frac{1}{a}+\frac{1}{2a}+\frac{1}{4a}+......\infty\right)} . 2^{\frac{1}{2a}+\frac{1}{4a}+\frac{1}{8a}+......\infty} = \frac{8}{27}$

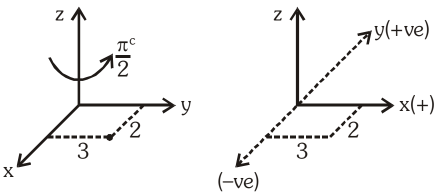
now $\frac{1}{a}\left(1+\frac{1}{2}+\frac{1}{2^2}+........\infty\right) = \frac{2}{a}$ and

$\frac{1}{2a}+\frac{2}{4a}+\frac{3}{8a}+........\infty = \frac{2}{a}$ (use AGP)

$\therefore\ a^{\frac{1}{a}}.2^{\frac{1}{a}} = \frac{8}{27} = \left(\frac{1}{3}\right)^3 . 2^3 \Rightarrow a = \frac{1}{3}$

83. tan20 . tan80 . tan40 = tan3(20) = $\sqrt{3}$ as $\tan\theta\tan(60-\theta)\tan(60+\theta) = \tan3\theta$

84.



There will be no change in z-component

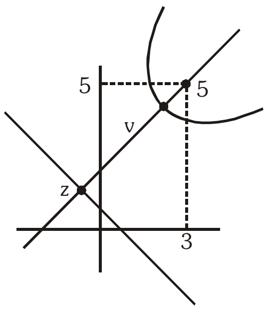
$\therefore$ New component are (3, –2, 7).

85. a = A + (p – 1)D, b = A + (q – 1)D & c = A + (r–1)D

$\vec{x}\,.\,\vec{y}$ = (q – r)a + (r – p)b + (p – q)c ...... (i)

put value of a, b, c in (i) we get $\vec{x},.\vec{y} = 0$

86.



axis x – y = l

x – y = –2

now foot of directrix z

$\left.\begin{matrix} x-y+2=0 \\ x+y-4=0 \end{matrix}\right\} x=1,\ y=3$

v is mid point of x & z

so v(2, 4)

87. Length of tangent = length of subnormal

$\Rightarrow \frac{dy}{dx} = \pm1$

If $\frac{dy}{dx} = 1$, then equation of tangent at (3, 4) is

y – 4 = x – 3 ⇒ y = x + 1 which cuts coordinate axes at (0, 1) & (–1, 0)

If $\frac{dy}{dx} = -1$, then equation of tangent at (3, 4) is

y – 4 = –(x –3) or x + y = 7

which cuts positive coordinate axes at A(7, 0) and B(0, 7)

$\therefore$ Area of ΔOAB = $\frac{1}{2}.7.7 = \frac{49}{2}$

88. $I = \int \frac{\sec x.\operatorname{cosec} x}{2\cot x - \sec x \operatorname{cosec} x}dx = \int \frac{dx}{2\cos^2 x - 1}$

$= \int (\sec 2x)dx$

$I = \frac{\ell n|\sec 2x + \tan 2x|}{2} + c$

89.

p	q	p ∧ q	p ∨ q	(p ∧ q→p ∨ q)	
T	T	T	T	T	
T	F	F	T	T	i.e. tautology
F	T	F	T	T	
F	F	F	F	T	

90. $n_1 = 10,\ n_2 = 10$

average $m_1 = 60,\ m_2 = 40$

$\sigma_1 = 4,\ \sigma_2 = 6$

Standard deviation of combined series

$$\sigma = \sqrt{\frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1+n_2} + \frac{n_1 n_2 (m_1 - m_2)^2}{(n_1+n_2)^2}}$$

$$= \sqrt{\frac{10\times16+10\times36}{10+10} + \frac{10\times10(60-40)^2}{(10+10)^2}}$$

$$= \sqrt{8+18+100} = \sqrt{126} = 11.2$$