ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

path to success

DISTANCE LEARNING PROGRAMME

(Academic Session : 2018 - 2019)

LEADER TEST SERIES / JOINT PACKAGE COURSE

TARGET : JEE (MAIN) 2019

Test Type : **ALL INDIA OPEN TEST** (MAJOR)   Test Pattern : JEE-Main

TEST # 03   **TEST DATE : 24 - 03 - 2019**

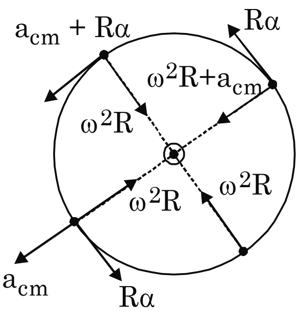
ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	4	4	1	4	2	2	1	3	4	1	1	2	1	2	2	2	4	4	1	1
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	2	2	2	3	3	1	1	1	3	1	2	2	3	3	4	2	3	3	2	3
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	4	2	1	3	1	1	1	1	4	2	2	3	4	4	4	1	2	1	1	2
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	3	4	4	1	2	3	1	2	4	2	1	4	2	1	3	2	4	2	4	2
Que.	81	82	83	84	85	86	87	88	89	90										
Ans.	2	1	3	2	4	3	3	1	4	4										

HINT – SHEET

1. Ans. (4)

Sol. After one revolution acceleration represent in the figure.



2. Ans. (4)

Sol. Initial velocity of point of contact of disc is in forward direction.

Hence friction backward direction and kinetic in nature.

Net force on the system is zero. So momentum is conserved.

3. Ans. (1)

Sol. $P_1 = \dfrac{K_1A(100-0)}{L} \Rightarrow Q = P_1t_1$

$P_2 = \dfrac{K_2A(100-0)}{L} \Rightarrow Q = P_2t_2$

$$P_3 = \frac{P_1P_2}{P_1+P_2} = \frac{\frac{Q}{t_1}\times\frac{Q}{t_2}}{\frac{Q}{t_1}+\frac{Q}{t_2}} = \frac{Q}{t_1+t_2}$$

$\Rightarrow Q = P_3(t_1+t_2)$

$\Rightarrow t_{10} = t_1 + t_2 = 100$ minutes

$$P_2 = P_1 + P_2 = \frac{Q}{t_1} + \frac{Q}{t_2} = \frac{Q(t_1+t_2)}{t_1 t_2}$$

$$\Rightarrow Q = P_4\left(\frac{t_1 t_2}{t_1+t_2}\right)$$

$$\Rightarrow t_{20} = \frac{t_1 t_2}{t_1+t_2} = \frac{20\times 80}{100} = 16\,\text{min}.$$

4. Ans. (4)

Sol. If final temp is T.

$$\frac{P_0V_0}{R4T_0}C_V(4T_0 - T) = \frac{P_0V_0}{RT_0}C_V(T - T_0)$$

$$\frac{4T_0 - T}{4} = T - T_0$$

$$4T_0 - T = 4T - 4T_0$$

$$8T_0 = 5T$$

$$T = \frac{8T_0}{5}$$

Final pressure

In left $P_f = \frac{T_f}{T_0}P_i = \frac{\frac{8}{5}T_0}{4T_0} = \frac{2}{5}P_0$

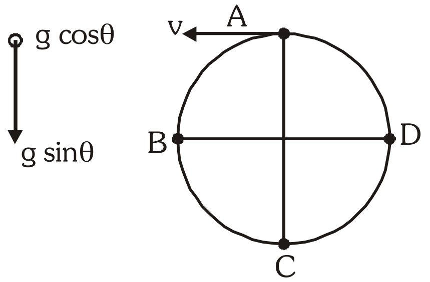
In right $P_f = \frac{\frac{8}{5}T_0}{T_0} = P_0 = \frac{8}{5}P_0$

Force $= \left(\frac{8P_0}{5} - \frac{2P_0}{5}\right)A = \frac{6P_0A}{5}$

5. Ans. (2)

Sol. $V_B = \sqrt{u^2 + 2g\ell\sin\theta}$

$$T_B = \frac{mv_B^2}{\ell}m\left[\frac{u^2}{\ell} + 2g\sin\theta\right]$$



$$V_C^2 = (u^2 + 4g\ell\sin\theta)$$

$$T_C - mg\sin\theta = \frac{mv_C^2}{\ell} \Rightarrow T_C = m\left[\frac{u^2}{\ell} + 5g\sin\theta\right]$$

6. Ans. (2)

Sol. In verticle direction.

$$v_y = \sqrt{2gH} = \sqrt{2\times 10\times 40} = 20\sqrt{2}\,m/s$$

$$T = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2\times 40}{10}} = 2\sqrt{2}\sec$$

$$\tan 37° = \frac{v_y}{v_x}$$

$$v_x = \frac{20\sqrt{2}\times 4}{3} = \frac{80\sqrt{2}}{3}$$

$$a_{\text{due to wind}} = \frac{2v_x}{T} = \frac{160\sqrt{2}}{3.2\sqrt{2}} = \frac{80}{3}m/s^2$$

7. Ans. (1)

Sol. $\frac{I}{\sqrt{2}} = mg$

$$\frac{I}{\sqrt{2}} = \mu N = \mu\left(mg + \frac{I}{\sqrt{2}}\right)$$

$$mg = \mu(2mg)$$

$$\mu = \frac{1}{2}$$

8. Ans. (3)

Sol. Because acceleration of base ball is constant and positive so force is constant and in direction of velocity and P = F.V Graph straight line.

9. Ans. (4)

Sol. $I_{AB} = \int y^2 dm = \frac{\lambda_0}{\ell^3}\int_{-x}^{\ell-x}(y+x)^3 y^2 dy$

$$\int (y+x)^3 y^2 dy = y^2\frac{(y+x)^4}{4} - \frac{1}{2}y(y+x)^4 dy$$

$$= \frac{y^2(y+x)^2}{4} - \frac{y}{10}(y+x)^5 + \frac{1}{10}(y+x)^5 dy$$

$$= \frac{y^2(y+x)^2}{4} - \frac{y(y+x)^5}{10} + \frac{(y+x)^2}{60}$$

$$\int_{-x_0}^{\ell-x} y^2(y+x)^3 dx = \frac{(\ell-x)^2(\ell)^4}{4} - \frac{(\ell-x)\ell^5}{10} + \frac{\ell^6}{60}$$

$$I_{AB} = \lambda_0\left[\frac{(\ell-x)^2}{4}\ell - \frac{1}{10}(\ell-x)\ell^2 + \frac{1}{60}\ell^3\right]$$

$$= \lambda_0\ell\left[\frac{(\ell-x)^2}{4}\ell - \frac{1}{10}(\ell-x) + \frac{1}{60}\ell^2\right]$$

$$\frac{dI_{AB}}{dx} = 0 \Rightarrow -\frac{(\ell - x)}{2} + \frac{\ell}{10} = 0$$

$$\Rightarrow 5x = 4\ell \quad \Rightarrow x = \frac{4\ell}{5}$$

10. Ans. (1)

Sol. $\tau = MB = C\theta$

$$\Rightarrow \; N\,i\,AB = C\,\frac{\pi}{2} \Rightarrow C = \frac{2N\,i\,AB}{\pi} \quad ....(1)$$

When Q is passed,

$$\int \tau\, dt = \int Ni\,AB\,dt$$

$$I\omega = N\,A\,B\,Q \quad ....(2)$$

Also, maximum deflection happens when entire $\frac{1}{2}I\omega^2$ converts to $\frac{1}{2}C\theta_{max}^2$

So $\theta_{max} = \sqrt{\frac{I\omega^2}{C}} = \frac{1}{\sqrt{IC}}I\omega = \frac{NABQ}{\sqrt{IC}}$

(from (1))

11. Ans. (1)

Sol. $\frac{1}{2}LI^2 = \frac{L\varepsilon^2}{8R^2}$

$$I = \frac{\varepsilon}{2R}$$

$$IR = \frac{\varepsilon}{2}$$

So $\frac{LdI}{dt} = \frac{\varepsilon}{2}$

$$\frac{dI}{dt} = \frac{\varepsilon}{2L}$$

12. Ans. (2)

Sol. Charge on outer surface of shell is

$$Q_0 + Q_0 - 2Q_0 = 0$$

So potential at surface $= \frac{K3Q_0}{2R}$

13. Ans. (1)

Sol. $F = q_0E$

and $E_1 > E_2 > E_3 > E_4$.

14. Ans. (2)

Sol. $f = 0.1 \times 4 \times \frac{10}{h_1} + 0.4 \times 4 \times \frac{10}{h_2}$

$$= \frac{0.1 \times 4 \times 10}{3 - h_2} + \frac{0.4 \times 4 \times 10}{h_2}$$

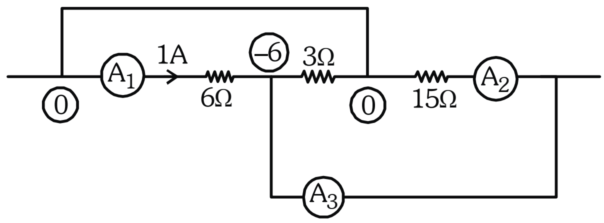
for $f_{min.}$ $\frac{df}{dh_2} = 0$

$h_2 = 2m$

$F_{min.} = 12\ N$

15. Ans. (2)

Sol.



current in $3\Omega = 2A$

current in $A_3 = 3A$

16. Ans. (2)

Sol. $I = \chi H$ and $\chi = \frac{C}{T}$ (according to curie law)

Magnetization (I)

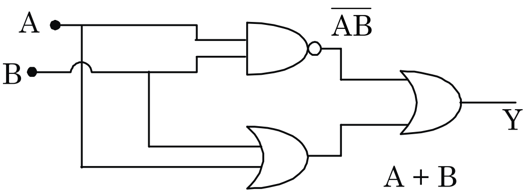
$$\propto \frac{\text{Magnetic Induction (B)}}{\text{Absolute temperature (T)}}$$

$$\frac{I_2}{I_1} = \frac{B_2T_1}{B_1T_2}$$

17. Ans. (4)

Sol. $Y = A + B + \overline{AB}$

$$\Rightarrow \; Y = A + B + \overline{A} + \overline{B} = 1$$



18. Ans. (4)

Sol. $\omega' = \sqrt{\frac{K}{m} - \frac{b^2}{4m^2}}$ and $\omega' = \frac{2\pi}{T}$

19. Ans. (1)

Sol. $\frac{dF}{dL} = IB = \text{slope(s)}$

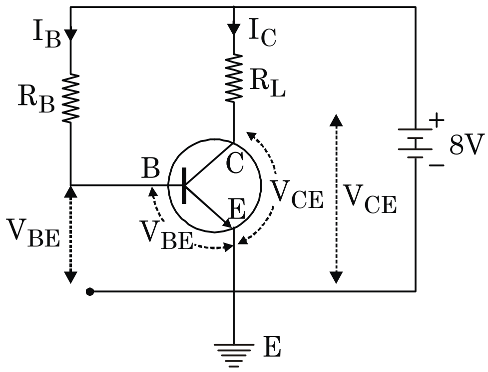
$$B = \frac{S}{I}$$

$$\frac{\Delta B}{B} = \frac{\Delta S}{S} + \frac{\Delta I}{I} = \left(\frac{1}{10} + \frac{1}{15}\right) = \frac{1}{6}$$

$$\%\ \text{Error} = \frac{1}{6} \times 100 = \frac{50}{3}\%$$

20. Ans. (1)

Sol. See figure. Potential difference across R_L



$= 8V - V_{CE}$

$= 8V - 4V = 4V$

Now $I_C R_L = 4V$

$$R_L = \frac{4}{4\times10^{-3}} = 10^3\Omega = 1k\Omega$$

Further, for base-emitter equation,

$V_{CC} = I_B R_B + V_{BE}$

or $I_B R_B$ = Potential difference across R_B

$= V_{CC} - V_{BE} = 8 - 0.6 = 7.4$ V

Again, $I_B = \frac{I_C}{\beta} = \frac{4\times10^{-3}}{100} = 4\times10^{-5}A$

$$R_B = \frac{7.4}{4\times10^{-5}} = 1.85\times10^5\Omega = 185k\Omega$$

21. Ans. (2)

Sol. If attenuation of a signal is expressed in dB.

$10 \log_{10} (I/I_0) = -\alpha x$

(α is the attenuation in dB/km)

or $\frac{I}{I_0} = \frac{I}{2}$

Thus, $10\log_{10}\left(\frac{1}{2}\right) = -\alpha x$

or $10\log_{10} 2 = 50\alpha$

or $\alpha = \frac{\log_{10} 2}{5} = \frac{0.3010}{3} = 0.0602\,dB/km$

22. Ans. (2)

Sol. For open pipe $n_0 = \frac{v}{2\ell} = \frac{360}{2\times15} = 120\,Hz$

Let velocity of sound in gas at 30°C = v'

$n_1 = \frac{v'}{4\ell} = \frac{v'}{4\times1.5} = \frac{v'}{6}$

$n_1 = n_0 \Rightarrow$ v' = 720 m/sec.

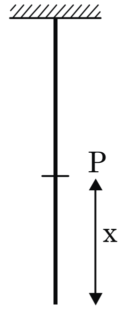
Also

$$v \propto \sqrt{T} \Rightarrow v = 720\times\sqrt{\frac{273}{303}} = 683\,m/sec.$$

23. Ans. (2)

Sol. Tension at P = $\frac{m}{L}gx$

$v_\omega = \sqrt{gx}$



$$a_\omega = v_\omega \frac{dv_\omega}{dx} = g/2$$

$\therefore$ (2)

24. Ans. (3)

Sol. $\eta = 1 - \frac{T_2}{T_1}$

$0.8 = 1 - \frac{T_2}{T_1}$

$\frac{T_2}{T_1} = 0.2$

$T_2 V_2^{\gamma-1} = T_1 V_1^{\gamma-1}$

$\left(\frac{V_2}{V_1}\right)^{\gamma-1} = \frac{T_1}{T_2} = 5$

$\frac{V_1}{V_2} = \left(\frac{1}{5}\right)^{\frac{5}{2}}$

25. Ans. (3)

Sol. Path difference at point P = $\frac{xd}{D}$

$\therefore$ phase deff. at P = $\frac{2\pi}{x}\frac{xd}{D} = \frac{2\pi x}{\beta}$

$\therefore$ Intensity at P = I + I + 2I cos $\frac{2\pi x}{\beta}$

$= 2I\left[1+\cos\frac{2\pi x}{\beta}\right]$ $[\because I_0 = 4I]$

$= 4I\cos^2\frac{\pi x}{\beta} = I_0\cos^2\frac{\pi x}{\beta}$

26. Ans. (1)

Sol. emf $= \frac{\Delta\phi}{\Delta t} = \frac{\pi(0.1)^2\times B}{0.314} = 0.1\,v$

27. Ans. (1)

Sol. Positive half cycle

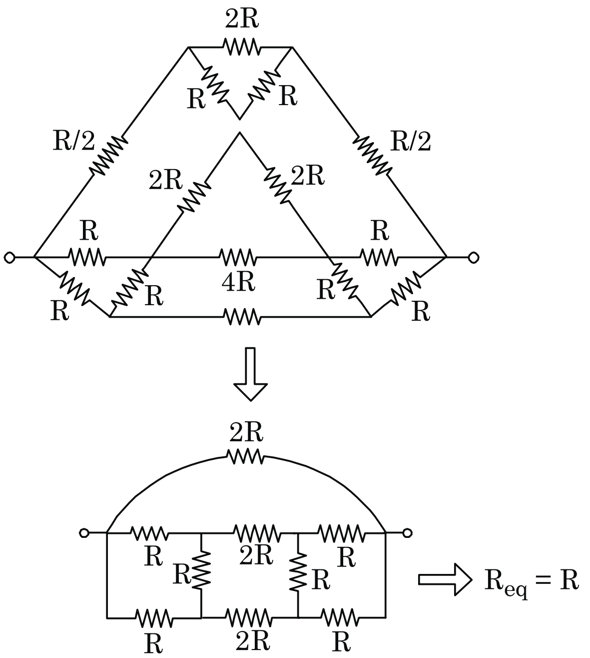
$V_{output} = 10V$

For negative half cycle

potential difference across output = –25V

28. Ans. (1)

Sol.



29. Ans. (3)

Sol. Capacitors already in steady state.

30. Ans. (1)

Sol. $D = 4f \Rightarrow f = D/4 = 0.50$

$\therefore \frac{1}{50} = (1.5 - 1)\left(\frac{1}{R}\right) \Rightarrow R = 25cm$

$\therefore a = \sqrt{8Rt} \Rightarrow \sqrt{8 \times 25 \times 0.5} = 10cm$

31. Ans. (2)

Sol. During adsorption the –ve value of ΔH goes on decreasing because as adsorption proceeds number of adsorption sites decreases and hence exothermicity decreases.

32. Ans. (2)

Sol. (n – 2)

6 – 2 = 4

33. Ans. (3)

Sol. At critical point, $Z = \frac{P_C.V_C}{R.T_C} = \frac{\frac{a}{27b^2} \times 3b}{R \times \frac{8a}{27Rb}} = \frac{3}{8}$

(Same for all vander waal's gases)

But, at critical temperature, $Z = \frac{P.V_m}{R.T_C}$

(depends on P)

34. Ans. (3)

Sol. $pH = pK_a + \log\frac{[Salt]}{[Acid]}$

$\Rightarrow \quad 4.5 = 4.2 + \log \frac{300 - x}{x}$

$\Rightarrow \quad 0.3 = \log \frac{300 - x}{x}$

$\Rightarrow \quad \log 2 = \log \frac{300 - x}{x}$

$\Rightarrow \quad 2 = \frac{300 - x}{x}$

$\Rightarrow \quad 2x = 300 - x$

$\Rightarrow \quad 3x = 300 \Rightarrow x = 100$ ml

35. Ans. (4)

Sol. Cellulose is natural polymer of glucose.

Nylon, Teflon and PVC are synthetic polymers.

36. Ans. (2)

Sol. Phenelzine is anti–depressant.

37. Ans. (3)

Sol. Maltose + $H_2O \xrightarrow{H^+}$ 2 (glucose)

(1) Sucrose + $H_2O \xrightarrow{H^+}$ glucose + fructose

(2) Lactose + $H_2O \xrightarrow{H^+}$ glucose + galactose

(4) Raffinose + $H_2O \xrightarrow{H^+}$ glucose + fructose + galactose

38. Ans. (3)

Sol. Glycine is amino acid. It cannot be present in nucleotide which contain deoxyribose or ribose as sugar nd heterocyclic bases like guanine, cytosine, etc.

39. Ans. (2)

Sol. The elctronic configuration of

$N_2 = KK\sigma 2s^2\, \sigma^* 2s^2\, \pi 2p_x^2 = \pi 2p_y^2\, \sigma 2p_z^2$

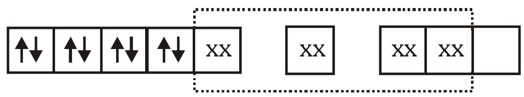
$N_2^+ = KK\sigma 2s^2\, \sigma^* 2s^2\, \pi 2p_x^2 = \pi 2p_y^2\, \sigma 2p_z^1$

40. Ans. (3)

Sol. $PtCl_4]^{2-}$ In case of Pt(78), Cl will cause pairing of electrons.

Pt (78) : [Xe] $6s^1$ $5d^9$ Pt^{2+} : [Xe] $6s^0$ $5d^8$

$PtCl_4]^{2-}$



dsp^2 square planar

In other Cl being weak filed ligand cannot cause pairing of electrons, therefore, they undergo sp^3 hybridisation and have tetrahedral shape.

41. Ans. (4)

Sol. (1) $PbO + C \longrightarrow Pb + CO$

(2) $SnO_2 + 2C \longrightarrow Sn + 2CO$

(3) $ZnO + C \longrightarrow Zn + CO$

(4) Temp. requirement is very high.

42. Ans. (2)

Sol. $4KO_2 + 2CO_2 \longrightarrow 2K_2CO_3 + 3O_2$

43. Ans. (1)

Sol. $\Delta S = nC_P \ell n\frac{T_2}{T_1} - nR\,\ell n\frac{P_2}{P_1}$

$= 3 \times 7 \times 2.3 \log \frac{1000}{300} - 3\times 2\times 2.3\log\frac{1.0}{0.1}$

= 11.316 cal/degree

44. Ans. (3)

Sol. In HCP, the number of octahedral void is equal to the number of atoms. Hence, the formula is $AC_{2/3}$ i.e. A_3C_2.

45. Ans. (1)

Sol. $k_1 = k_2$

$10^{15} e^{-25000/8.314T} = 10^{14} e^{-15000/8.314T}$

$\frac{10^{15}}{10^{14}} = e^{10000/8.314T}$

$2.303\log_{10} 10 = \frac{10000}{8.314\,T}$;

$T = \frac{10000}{8.314 \times 2.303} = 522\,K$

Hence the correct answer is (1).

46. Ans. (1)

Sol. Acetone and chloroform forms hydrogen bonding so volume decreases.

47. Ans. (1)

Sol. Even though there is an inversion of bond configuration in the above reaction, the R/S configuration remains the same because of the change in priority order.

48. Ans. (1)

Sol. Theory based.

49. Ans. (4)

50. Ans. (2)

Sol. Order of basic strength is

$\underbrace{CH_3\text{–}CH_2\text{–}NH_2}_{+I} > \underbrace{Ph}_{-I}\text{–}CH_2\text{–}NH_2 > Ph\text{–}\ddot{N}H_2$ (delocalise l.p.)

51. Ans. (2)

Sol. A halate will be formed from halogen and the greenish yellow gas is Cl_2. The halate which is used in fire works and safety matches is $KClO_3$. The reaction is

$\underset{\text{greenish yellowgas}}{3Cl_2} + 6KOH \longrightarrow \underset{\text{halate}}{KClO_3} + 5KCl + 3H_2O$

52. Ans. (3)

Sol. C(6) $1s^2\, 2s^2\, 2p_x^1 2p_y^1$

N(7) $1s^2 2s^2 2p_x^1 2p_y^1 2p_z^1$

O(8) $1s^2\, 2s^2\, 2p_x^2\, 2p_y^1 2p_z^1$

F(9) $1s^2\, 2s^2\, 2p_x^2 2p_y^2 2p_z^1$

O has highest 2^{nd} ionisation potential i.e ionisation energy because after losing one electron, it acquires noble gas configuration.

F is smaller than N, therefore, it has higher second ionisation potential. C has least due to bigger size.

53. Ans. (4)

Sol. $2KMnO_4 \xrightarrow{\text{heat}} K_2MnO_4 + MnO_2 + O_2$

54 Ans. (4)

Sol. F^- concentration above 2ppm causes various health related problems

$[SO_4^{2-}] > 500\,ppm$ is harmful.

$[NO_3^-] > 50\,ppm$ is harmful.

55. Ans. (4)

Sol. m = Z × Q

$7.42 = \frac{118}{n \times 96500} \times 24125$

$n = \frac{118}{7.42 \times 4} \approx 4$

56. Ans. (1)

Sol. Total volume = 500 + 1500 = 2000 ml = 2L

Moles of $[Na^+]$ ions = 0.2 × 0.5 = 0.1

Concentration of $[Na^+]$ ions $= \frac{0.1}{2} = 0.05M$

Moles of $[Mg^{2+}]$ ions = 0.4 × 1.5 = 0.6

Concentration of $[Mg^{2+}]$ ions

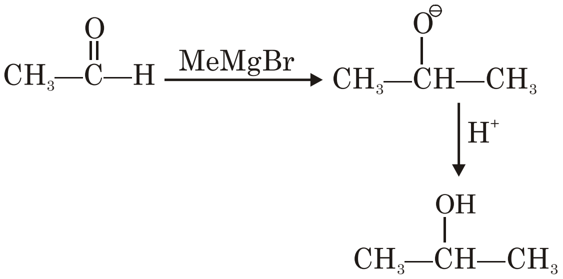
$= \frac{0.6}{2} = 0.3M = 7.2 gm/L$

Moles of $[Cl^-]$ ions = 0.2 × 0.5 + 2 × 0.4 × 1.5 = 1.3

Concentration of $[Cl^-]$ ions $= \frac{1.3}{2} = 0.65$ M

57. Ans. (2)

Sol.



58. Ans. (1)

Sol. Anti markownikoff addition.

59. Ans. (1)

60. Ans. (2)

Sol. $Na_2S_2O_3 + 2AgNO_3 \longrightarrow \underset{\text{(white ppt)}}{Ag_2S_2O_3} + 2NaNO_3$

$$Ag_2S_2O_3 \xrightarrow{\text{standing}\ H_2O} \underset{\text{(Black ppt)}}{Ag_2S + H_2SO_4}$$

61. Ans. (3)

Sol. Null relation is symmetric, transitive but not reflexive.

62. Ans. (4)

Sol. $\sim(\sim s \vee (\sim r \wedge s))$

$\equiv\ s \wedge \sim(\sim r \wedge s)$

$\equiv\ s \wedge (r \vee \sim s)$

$\equiv\ (s \wedge r) \vee (s \wedge \sim s)$

$\equiv\ s \wedge r$

63. Ans. (4)

Sol. $\sigma^2 = \dfrac{\sum (x_i - \bar{x})^2}{N}$

required variance $(\sigma)^2 = 2^2 \times$ Variance of (1, 2, 3, ... 50)

$= 4 \times \dfrac{n^2 - 1}{12}$

$= 4 \times \dfrac{2500 - 1}{12} = 833$

64. Ans. (1)

Sol. $\begin{vmatrix} 2 & 3 & a \\ b & 5 & -1 \\ 1 & 1 & 3 \end{vmatrix} = ab - 5a - 9b + 29 = 0$ and $2b - a = 5$

solving $\Rightarrow$ a = 1 and b = 3 or a = 13, b = 9

65. Ans. (2)

Sol. The line is parallel to the vector $\vec{b} = 3i + 2j + 4k$ and normal to the plane is the vector $\vec{h} = 2i + j - 3k$.

$\Rightarrow \sin\theta = \dfrac{\vec{b}\cdot\vec{h}}{|\vec{b}||\vec{h}|} = \dfrac{6 + 2 - 12}{\sqrt{29}\sqrt{14}} = \dfrac{-4}{\sqrt{406}}$

$\Rightarrow 8\,\text{cosec}^2\theta = 203 \Rightarrow 64\,\text{cosec}^2\theta = 1624$

66. Ans. (3)

Sol. $\dfrac{dy}{dx} = \dfrac{(1+x)y}{(y-1)x}$ can be written as

$\dfrac{y-1}{y}dy = \dfrac{(1+x)}{x}dx$

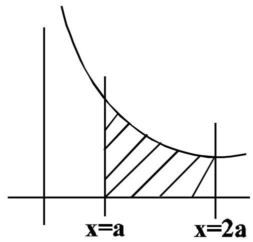
$\Rightarrow \left(1 - \dfrac{1}{y}\right)dy = \left(1 + \dfrac{1}{x}\right)dx$

$\Rightarrow (y - \log y) = (x + \log x) + c$

$\Rightarrow x - y + \log xy = c$

67. Ans. (1)

Sol. $A = \displaystyle\int_a^{2a}\left(\frac{x}{6} + \frac{1}{x^2}\right)dx$



$= \left(\dfrac{a^2}{3} - \dfrac{1}{2a}\right) - \left(\dfrac{a^2}{12} - \dfrac{1}{a}\right)$

$f(a) = \dfrac{a^2}{4} + \dfrac{1}{2a}$

now $f'(a) = \dfrac{a}{2} - \dfrac{1}{2a^2} = 0 \Rightarrow a^3 = 1$

$\Rightarrow a = 1$

68. Ans. (2)

Sol. $u = \displaystyle\int_0^1 \frac{\ell n(x+1)}{x^2+1}dx$ put $x = \tan\theta$

$= \displaystyle\int_0^{\pi/4} \ell n(1 + \tan\theta)\,d\theta$

$= \displaystyle\int_0^{\pi/4} \ell n\left(1 + \frac{1 - \tan\theta}{1 + \tan\theta}\right)d\theta$

$= \displaystyle\int_0^{\pi/4} \ell n\frac{2}{1 + \tan\theta}\,d\theta = \frac{\pi}{4}\,\ell n\,2 - u$

$\therefore u = \dfrac{\pi}{8}\,\ell n\,2 \Rightarrow 4u = \dfrac{\pi}{2}\,\ell n\,2$ ....(1)

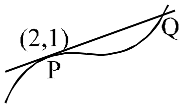
again $v = \displaystyle\int_0^{\pi/2} \ell n(\sin 2x)\,dx$ (put 2x = t) ;

$$v = \frac{1}{2}\int_0^{\pi} \ell n(\sin t)\, dt = \int_0^{\pi/2} \ell n(\sin t)\, dx$$

$$v = -\frac{\pi}{2}\ \ell n\, 2 \qquad ....(2)$$

(1) + (2) $\Rightarrow 4u + v = 0 \Rightarrow$ (C)

69. Ans. (4)

Sol. 

eliminating t gives $y^2(x - 1) = 1$.

Equation of tangent at P(2, 1) is

$x + 2y = 4$.

Solving with curve $x = 5$ & $y = -1/2$

$\Rightarrow Q\left(5, \frac{1}{2}\right)$

70. Ans. (2)

Sol. $f\,'(x) = 2x^3 - 3x + 1$ this is always positive in (1, 3)

$\therefore$ increasing in [1, 3]

$\therefore$ f (3) will be the greatest value]

71. Ans. (1)

Sol. $y = x^2 + \frac{1}{y}$

or $y^2 = x^2y + 1$

or $2y\frac{dy}{dx} = y.2x + x^2\frac{dy}{dx}$

or $\frac{dy}{dx} = \frac{2xy}{2y - x^2}$

72. Ans. (4)

Sol. Function satisfying the given rule and for which $f'(1) = 3$ will be $f(x) = x\ell nx + 2x$.

No point of inflection.

No asymptote.

73. Ans. (2)

Sol. Let $f(x) = a(x - 3)^3 + b(x - 3)^2 + c(x - 3) + d$

Then,

$f(3)=1 = d \quad \Rightarrow d = 1$

$f'(3) = -1 = c \quad \Rightarrow c = -1$

$f''(3) = 0 = 2b \quad \Rightarrow b = 0$

$f'''(3)=12 = 6a \Rightarrow a = 2.$

$\therefore \quad f(x) = 2(x - 3)^3 - x + 4$

$\Rightarrow \quad f'(x) = 6(x - 3)^2 - 1$

$\therefore \quad f'(1) = 6(4) - 1 = 23.$

74. Ans. (1)

Sol. As we are interested in coefficient of t^{50}, we shall ignore all the term with exponent more than 50. Thus we can write as

$(1 + {}^{25}C_1t^2 + ... + {}^{25}C_{25}t^{50}) \times$
$(1+t^{25}+t^{40}+t^{45}+t^{47})$

As all the terms in the first have even exponent we can ignore t^{25}, t^{45} and t^{47} too thus coefficient of t^{50} is

$= {}^{25}C_{25} + {}^{25}C_5 = 1 + {}^{25}C_5$

75. Ans. (3)

Sol. $x - 2y + 5z = 3$

$2x - y + z = 1$

$11x - 7y + pz = q$

$$\Delta = 0 \Rightarrow \begin{vmatrix} 1 & -2 & 5 \\ 2 & -1 & 1 \\ 11 & -7 & p \end{vmatrix} = 0$$

$(-p + 7) + 2(2p - 11) + 5(-14 + 11) = 0$

$\Rightarrow \quad p = 10$

$$\Delta_x = 0 \Rightarrow \begin{vmatrix} 3 & 2 & 5 \\ 1 & 1 & 1 \\ q & 7 & 10 \end{vmatrix} = 0$$

$\Rightarrow \quad 3(10 - 7) - 2(10 - q) + 5(7 - q) = 0$

$\Rightarrow \quad q = 8$

$p - q = 2$

76. Ans. (2)

Sol.
$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \text{Rational \& Positive}$$

77. Ans. (4)

Sol. Applying Bayels theorem required probability is

$$=\frac{\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)^2}{\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)^2+\left(\frac{1}{2}\right)\left(\frac{3}{4}\right)^2}=\frac{1}{1+9}=\frac{1}{10}$$

78. Ans. (2)

For 2 selected elements there is only one options and for rest there will be 3 options $= {}^nC_2\, 3^{n-2}$

79. Ans. (4)

Sol. $\alpha+\beta=\frac{3}{5}$

$\alpha\beta=-\frac{1}{5}$

$$\left[(\alpha+\beta)x-\left(\frac{\alpha^2+\beta^2}{2}\right)x^2+\left(\frac{\alpha^3+\beta^3}{3}\right)x^3..........\right]$$

$$=\left(\alpha x-\frac{(\alpha x)^2}{2}+\frac{(\alpha x)^3}{3}.......\right)+\left(\beta x-\frac{(\beta x)^2}{2}+\frac{(\beta x)^3}{3}......\right)$$

$= \ell n(1-\alpha x)+\ell n(1-\beta x)$

$= \ell n((1-\alpha x)(1-\beta x))$

$= \ell n(1-x(\alpha+\beta)+\alpha\beta x^2)$

$=\ell n\left(1-x\frac{3}{5}-\frac{1}{5}x^2\right)$

80. Ans. (2)

Sol. $A=G+\frac{3}{2},\ G=H+\frac{6}{5}$

Using $G^2=AH$ we get $G=6$, $A=\frac{15}{2}$ and $H=\frac{24}{5}$

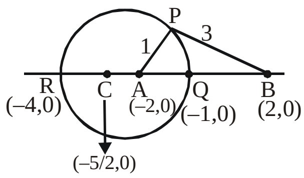
Hence, $a+b=15$ and $ab=36$

$a=12, b=3$ or $a=3, b=12$

$|a^2-b^2|=135.$

81. Ans. (2)

Sol. $z_1+z_3=z_2+z_4 \Rightarrow$ PQRS is a parallelogram and set A contain points on the following circle



parallelogram inscribed in a circle is rectangle and area of rectangle will be maximum when it is a square so area $=\frac{1}{2}(3)^2=\frac{9}{2}$

82. Ans. (1)

Sol. Slope of tangent $=\frac{2}{5}$

Now $c^2=a^2m^2-b^2$

$=9\left(\frac{2}{5}\right)^2-4<0$

No such tangent exist

83. Ans. (3)

Sol. $$\frac{\left(\frac{x-y+1}{\sqrt{2}}\right)^2}{10}+\frac{\left(\frac{x+y-3}{\sqrt{2}}\right)^2}{5/2}=1$$

Here $a^2=10$ and $b^2=5/2$ and centre is (1,2)

$\because$ Locus of feet of perpendicular lie on auxiliary circle of ellipse

$\therefore$ Equation of circle is

$(x-1)^2+(y-2)^2=10$

$x^2+y^2-2x-4y-5=0$

84. Ans. (2)

Sol. Let mid point be (h, k).

$\therefore$ Equation of chord is $T=S_1$

$yy_1-2a(x+x_1)=y_1^2-4ax_1$

Since, it passes through origin

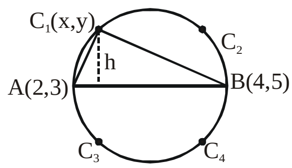
$\therefore\ -2ax_1=y_1^2-4ax_1$

$\Rightarrow\ y_1^2=2ax_1$

$\therefore$ Locus is $y^2=2ax$

85. Ans. (4)

Sol. $AB=2\sqrt{2}$



area $=\frac{1}{2}(AB)h$

$\frac{2\sqrt{2}}{2}.h=\sqrt{2}$

$h=1$

$\therefore$ there are four positions

86. Ans. (3)

Sol. Given lines will be concurrent if

$$\begin{vmatrix}1 & 2a & a\\ 1 & 3b & b\\ 1 & 4c & c\end{vmatrix}=0 \Rightarrow -bc+2ac-ab=0$$

$\Rightarrow b=\frac{2ac}{a+c} \Rightarrow$ a, b, c are in H.P.

87. Ans. (3)

Sol. $\cot^{-1}\frac{n}{\pi} > \frac{\pi}{6}$

$\Rightarrow \frac{n}{\pi} < \cot\frac{\pi}{6}$

[as $\cot^{-1}$ x is a decreasing function]

$\Rightarrow \quad \frac{n}{\pi} < \sqrt{3}$

$\Rightarrow \quad n < \sqrt{3}\,\pi$

$\Rightarrow \quad n < 5.46$

$\Rightarrow$ maximum value of n is 5

88. Ans. (1)

Sol. $\sin\theta + \sqrt{3}\cos\theta = -2 - (x-3)^2$

minimum vlaue of $\sin\theta + \sqrt{3}\cos\theta$ is – 2

Hence solution of equation is possible only if x = 3

89. Ans. (4)

Sol. We know that,

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \Rightarrow \sum\cos^2\alpha = 1$... (i)

Given that

$$2\left(\frac{\tan^2\alpha}{1+\tan^2\alpha} + \frac{\tan^2\beta}{1+\tan^2\beta} + \frac{\tan^2\gamma}{1+\tan^2\gamma}\right) = 3\sec^2\frac{\theta}{2}$$

$$\Rightarrow 2(\sin^2\alpha + \sin^2\beta + \sin^2\gamma) = 3\sec^2\frac{\theta}{2}$$

$$\Rightarrow 4\cos^2\frac{\theta}{2} = 3 \qquad \text{[From (i)]}$$

$$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

90. Ans. (4)

Sol. $\frac{dy}{dx} = 2 - \frac{1}{1+x^2} - \frac{1}{\sqrt{1+x^2}} \geq 0 \quad \forall\ x \in R$

Since $\frac{1}{(1+x^2)}$ and $\frac{1}{\sqrt{1+x^2}}$ are less than or equal to 1 for all x. So f(x) increases on $(-\infty, \infty)$