LEADER TEST SERIES / JOINT PACKAGE COURSE

TARGET : JEE (Main) 2019

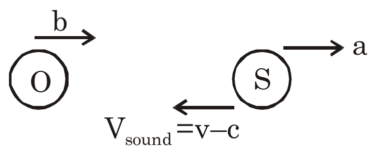
Test Type : MAJOR TEST # 07 Test Pattern : JEE (Main)

TEST DATE : 17 - 03 - 2019

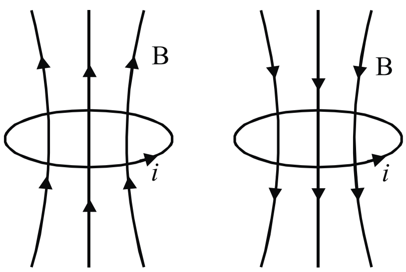
ANSWER KEY

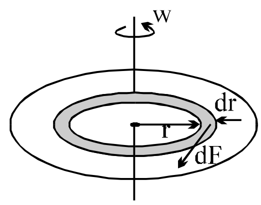
Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	2	2	3	2	2	2	4	3	1	3	3	2	4	2	3	4	4	2	1	4
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	4	2	1	1	4	3	3	1	3	4	3	2	4	4	4	3	2	2	4	4
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	2	4	3	2	1	3	3	3	3	3	1	4	2	1	3	4	3	3	4	2
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	3	3	1	2	1	3	3	4	1	2	3	1	3	3	4	3	2	2	3	1
Que.	81	82	83	84	85	86	87	88	89	90										
Ans.	1	1	3	2	3	3	3	3	1	3										

HINT – SHEET

1. 

$$f = \frac{v - c + b}{v - c + a} \times n$$

2. 

3. 

Let coefficient of viscosity & thickness of film $\rightarrow \eta$ & x respectively

Viscous force on element ring is

$$dF = \eta\ (2\pi r dr) \left[\frac{wr - 0}{x}\right]$$

$$d\tau = (dF)\ r$$

$$\therefore\ \tau = \frac{\eta 2\pi w}{t}\int_0^R r^3 . dr = \left(\frac{\eta(2\pi)w}{4x}\right) R^4$$

$$\therefore\ \tau \propto R^4$$

4. Pascal's law

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow F_1A_2 = F_2A_1 \;;$$

as $A_1 < A_2$
So $F_1 < F_2$

5. On the system of 'wedge+block' horizontal force = 0. so momentum will conserve in horizontal direction

$\Rightarrow 10\,v + 5\,(-6) = 0 \Rightarrow v = 3$ m/s

6. $y_{n^{th}} = \frac{n\lambda_1 D}{2d}$; n = 2, 4, 6, 8.....

$y_{m^{th}} = \frac{m\lambda_2 D}{2d}$; m = 2, 4, 6, 8......

$y_{n^{th}} = y_{m^{th}} = \frac{d}{2}$ from central fringe.

7. $v_e = \sqrt{\frac{2GM}{R}}$ $\qquad v_0 = \sqrt{\frac{GM}{r}}$

Now $\Delta KE = 0 - \frac{1}{2}mv^2$

$$= \left(-\frac{GMm}{R}\right) - \left(-\frac{GMm}{r}\right)$$

$$-\frac{1}{2}mv^2 = -\frac{1}{2}mv_e^2 + mv_0^2$$

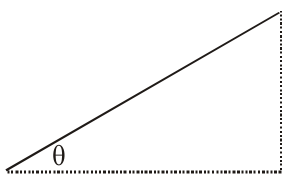
$$v = (v_e^2 - 2v_0^2)^{1/2}$$

8. The speed at angle θ is given by conservation of energy

$\Rightarrow \frac{1}{2}mv^2 = mgh \Rightarrow \frac{1}{2}mv^2 = mg\,R\cos\theta$

$\Rightarrow v = \sqrt{2gR\cos\theta}$ R will cancel out in the final answer.

So we have the following setup of projectile motion.



$v = \sqrt{2gR\cos\theta}$

$v_x = v\cos\theta$

$v_y = v\sin\theta$

The time of flight is twice the time to get to the top $\Rightarrow t = 2\left(\frac{v_y}{g}\right)$

$$\Rightarrow x = v_x t = v_x\left(\frac{2v_y}{g}\right) = \frac{2v_xv_y}{g} = \frac{2v_xv_y}{g}$$

$$= \frac{2(v\cos\theta)(v\sin\theta)}{g} = \frac{2v^2\sin\theta\cos\theta}{g}$$

$$= \frac{2(2gR\cos\theta)\sin\theta\cos\theta}{g} = 4R\cos^2\theta\sin\theta$$

Maximise this $\Rightarrow$ take the derivative and equate it to zero

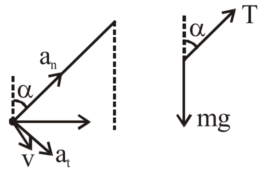
$$\frac{dx}{d\theta} = -8R\cos\theta\sin^2\theta + 4R\cos^3\theta = 0$$

$$\Rightarrow 2\sin^2\theta = \cos^2\theta \Rightarrow \tan^2\theta = \frac{1}{2} \Rightarrow \tan\theta = \frac{1}{\sqrt{2}}$$

9. $C_1 = \frac{\in_0 S}{d}$

$$C_2 = \frac{4\in_0 S/2}{d} + \frac{\in_0 S/2}{\frac{d}{2\times 2} + \frac{d}{2\times 4}}$$

$$= \frac{2\in_0 S}{d} + \frac{4\in_0 S}{3d} = \frac{10\in_0 S}{3d}$$

10. 

$a_t = g\sin\alpha$

$a_n = \frac{v^2}{\ell}$

$mgl(\cos\alpha) + \frac{1}{2}mv^2 \therefore \frac{v}{\ell} = 2g\cos\alpha = a_n$

$\tan\alpha = \frac{a_n}{a_t} = \frac{2\cos\alpha g}{g\sin\alpha} \Rightarrow \tan\alpha = \sqrt{2}$

$\therefore \cos\alpha = \frac{1}{\sqrt{3}}$

11. $\frac{\partial y}{\partial t} = 2A\omega\sin\left(\frac{\pi x}{L}\right)\cos\omega t$

For antinode (at $x = \pm = \frac{L}{2}$)

$\frac{\partial y}{\partial t} = 2A\omega\cos\omega t$

now for the given particle

$$\frac{\partial y}{\partial t} = 2A\omega\sin\left(\frac{\pi x}{L}\right) = A\omega$$

$\therefore \sin\frac{\pi x}{L} = \frac{1}{2}$

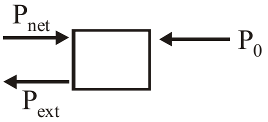
$\therefore x = \frac{L}{6}$

Hence : answer is $\frac{L}{3}$

12. $F_{net} = mg\sin\theta - \mu mg\cos\theta$

$F_{net} = 5$ N

$P_{ext} = \frac{5}{2.5\times10^{-4}} = 0.2$ Atm



$P_{ext} = P_{net} + P_0$

$= 0.2\,P_0 + P_0$

$= 1.2\,P_0$

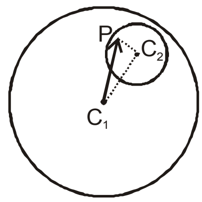
13. $E = \frac{d\phi}{dt}$, $\phi = B\pi r^2$ and

$\frac{dr}{dt}$ = constant so E is constant

17. Acceleration of +3Q = $\frac{3QE}{m}$ $(\downarrow)$

Acceleration of –2Q = $\frac{3QE}{m}$ $(\uparrow)$

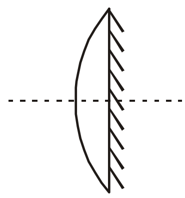
18. $\vec{E}_1 = \frac{\rho}{2\varepsilon_0}\overrightarrow{C_1P}$



$\vec{E}_1 = \frac{\rho}{2\varepsilon_0}\overrightarrow{C_2P}$

$\vec{E} = \vec{E}_1 - \vec{E}_2 = \frac{\rho}{2\varepsilon_0}\overrightarrow{C_1C_2} = \frac{\rho}{2\varepsilon_0}\cdot\vec{a}$

22. $\frac{1}{F_{eq}} = \frac{1}{F_m} - \frac{2}{F_\ell}$; $\frac{1}{-28} = -\frac{2}{F_\ell}$



$\Rightarrow F_\ell = 56$ cm.

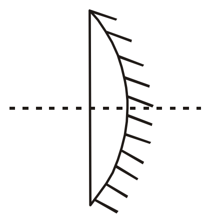
$\frac{1}{-10} = \frac{1}{F_m} - \frac{2}{56}$

$\Rightarrow \frac{1}{F_m} = \frac{1}{28} - \frac{1}{10} = \frac{10-28}{28.10} = -\frac{18}{28.10}$

$\Rightarrow F_m = -\frac{280}{18} = -\frac{140}{9}$ as $F_m = \frac{R}{2}$

$\Rightarrow R = -\frac{280}{9}$

Now for lens $\frac{1}{56} = (\mu - 1)\left[\frac{1}{\infty} - \frac{1}{(-280/9)}\right]$



$\Rightarrow \frac{1}{56} = (\mu - 1)\,\frac{9}{280}$

$\Rightarrow \mu - 1 = \frac{280}{56\times9} \Rightarrow \mu = 14/9$

23. $T\lambda = (T + 1)(\lambda - 0.01\lambda)$

$\therefore\ T = (T + 1)(0.99)$

$\therefore\ \frac{T+1}{T} = \frac{1}{0.99} = \frac{100}{99}$

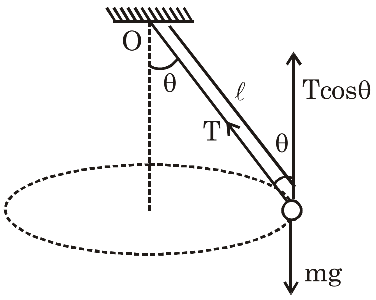
$\therefore\ 1 + \frac{1}{T} = 1 + \frac{1}{99}$

$\therefore\ T = 99\ K$

Alternative : $\lambda_{max} T = 0.99\ \lambda_{max} (T + 1)$

$\Rightarrow T = 0.99\ T + 0.99 \quad \Rightarrow T = 99\ K$

24. $\frac{T\cos\theta}{T\sin\theta} = \frac{mg}{m\ell\sin\theta\omega^2}$



$\omega = \sqrt{\frac{g}{\ell\cos\theta}}$

$\ell\cos\theta$ is same for two particle

25. V voltage of source drop across resistance is V_R, inductor is V_L and capacitor V_C

Applying Krichhof's law

$\varepsilon V = 0$

$V_R + V_L = V_C - V = 0$

26. Maximum height = $H = \frac{u_y^2}{2g}$

$T = \frac{2u_y}{g}$

Given, $H_A < H_B$

$\Rightarrow u_{yA} < u_{yB} \Rightarrow T_A < T_B$

31. $KClO_3 \rightarrow KCl + \frac{3}{2}O_2$ m mass

decomposed$= \frac{50m}{100}$ $\quad \frac{3}{2}\times\frac{50m}{100\times122.5}$

moles $= \frac{50m}{100\times122.5}$ $\quad = \frac{67.2}{22.4}$,

m = 490 gm

33. $\frac{3}{2}\times1\times R\times T = \frac{3}{2}\times1\times R\times400$

T = 400 K

35. $12C(s) + 11H_2(g) + \frac{11}{2}O_2(g) \rightarrow C_{12}H_{22}O_{11}$

$\Delta H_f = ?$

$\Delta H_f^o = \Sigma\Delta H_C(\text{reactant}) - \Sigma\Delta H_C^o\ (\text{Product})$

$= -12 \times 394 - 11\times286 + 5665 = -2209$ kJ/mol

40. (1) Brownian movement is one of the reason for stability of sols.

(2) Catalyst can undergo qualitative/physical change but cannot undergo any quantitative change.

(3) Both heating & freezing cause de–emulsification.

(4) Excessive dialysis causes co–agulation in colloids making them unstable.

42. $Be_2 \Rightarrow$ Because according to MOT bond order is zero.

43. LiCl > NaCl > KCl < RbCl < CsCl

(Solubility in water)

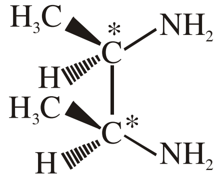
LiCl < CsCl < RbCl < KCl < NaCl

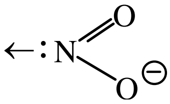
(mp)

CsCl < RbCl < KCl < NaCl < LiCl

(Lattice energy)

45. Due to the presence of chiral centre in bn ligand



46. M ←  $\qquad$ $M \leftarrow O^- - N = O$

$M \leftarrow S^- - C \equiv N \qquad M \leftarrow N^- = C = S$

47. Both Ag and Au are processed by this cyanide method.

61. Clearly point of intersection of lines is (1, 2, 3). Angle bisector must pass through this point.

Hence $\frac{0}{-1} = \frac{2+\alpha}{1} = \frac{3+\beta}{4} \Rightarrow \alpha = -2, \beta = -3$

62. $\left(\sqrt{3h}\right)^2 = \left(\sqrt{3k+2}+1\right)^2$

or $3h = 3k + 2 + 1 + 2\sqrt{3k+2}$

$\Rightarrow$ $9(h - k - 1)^2 = 4(3k + 2)$

or $9(h^2 + k^2 + 1 - 2hk - 2h + 2k) - 12k - 8 = 0$

$\Rightarrow$ $9(h^2 + k^2) - 18hk - 18h + 6k + 1 = 0$

$H^2 = AB,\ \Delta \neq 0$

$\therefore$ Locus is a parabola.

63. Let $A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$= a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{21}a_{32}a_{13} - (a_{13}a_{22}a_{31} + a_{11}a_{23}a_{32} + a_{12}a_{21}a_{33})$

$= x - y$ (let)

Now$|A|$ is maximum, when $y = 0$.

$\Rightarrow$ one possibility is to choose $a_{21} = a_{32} = a_{13} = 0$

& remaining elements to be 3.

$\therefore$ $|A|_{max} = 54$

Similarly $|A|_{min} = -54$

Hence $|A|_{max} - |A|_{min} = 54 - (-54) = 108$

64. Centre of hyperbola is (5, 0), so equation is

$\frac{(x-5)^2}{a^2} - \frac{y^2}{b^2} = 1$

$a = 5,\ ae - a = 8 \Rightarrow e = \frac{13}{5}$

$b^2 = 144$.

So equation is $\frac{(x-5)^2}{25} - \frac{y^2}{144} = 1$.

65. $(1 + i)(1 + 2i)(1 + 3i) \ldots\ldots (1 + ni) = x + iy$

So, $(1 - i)(1 - 2i)(1 - 3i) \ldots\ldots (1 - ni) = x - iy$

Multiply both,

$(1 + i)(1 - i)\ (1 + 2i)(1 - 2i)\ (1 + 3i)(1 - 3i) \ldots\ldots = x^2 + y^2$

$(2)(5)(10) \ldots\ldots (1 + n^2) = x^2 + y^2$

So, (1) is correct .

66. Let $f(x) = x^3 - 6x^2 - 15x + k$

$f'(x) = 3x^2 - 12x - 15$

$f'(x) = 0$ gives $x = -1, 5$

$\Rightarrow$ $f(x)$ is maximum at $x = -1$ and minimum at $x = 5$

$\therefore$ $f(-1) = 8 + k$ & $f(5) = -100 + k$

In order that $f(x) = 0$ has one real and 2 complex roots, $f(-1) \cdot f(5) > 0$

$\Rightarrow$ $k > 100$ or $k < -8$

67. Let $g(x) = |f(x)|$

if $f(x) > 0$ and $f'(x) < 0$

then $g(x) = f(x) \Rightarrow g'(x) = f'(x) < 0$

$\Rightarrow$ $g(x)$ is decreasing

Now if $f(x) < 0$ and $f'(x) > 0$

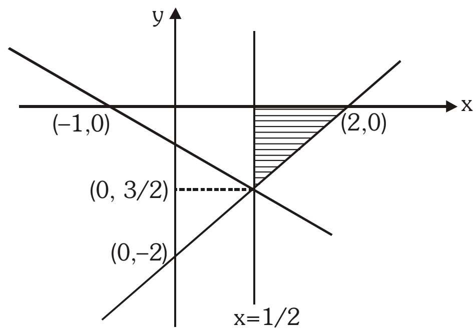
$\Rightarrow$ $g(x) = -f(x) \Rightarrow g'(x) = -f'(x) < 0$

$\Rightarrow$ $g(x)$ is decreasing.

68. Domain : $x + y + 1 > 0$ & $y - x + 2 > 0$

squaring $x + y + 1 > y - x + 2$

$x > \frac{1}{2}$



shaded area indicates required region

$A = \frac{1}{2} \times \frac{3}{2} \times \frac{3}{2} = \frac{9}{8}$ sq. units

69. $\left(-\frac{1}{\sqrt{2}}\right)^n = \left(\frac{1}{3^{5/3}}\right)^{\log_3 8} \Rightarrow \left(-\frac{1}{\sqrt{2}}\right)^n = 8^{\log_3\left(\frac{1}{3}\right)^{5/3}}$

$\Rightarrow \left(-\frac{1}{2}\right)^{\frac{n}{2}} = (8)^{\frac{-5}{3}} \Rightarrow \left(-\frac{1}{2}\right)^{\frac{n}{2}} = \left(\frac{1}{2}\right)^5$

So n must be even and $\frac{n}{2} = 5 \Rightarrow n = 10$

70. Centre of the required circle is (–4, –5)

Radius of the circle is

$r = \sqrt{(-4-2)^2 + (-5-3)^2} = 10$

$\therefore$ Equation of the circle

$(x + 4)^2 + (y + 5)^2 = 100$

$x^2 + y^2 + 8x + 10y - 59 = 0.$

71. If $f(x) = \begin{cases} Ax+3 & x<1 \\ 2 & x=1 \\ B+x^2 & x>1 \end{cases}$

$f(1^+) = B + 1$

$f(1) = 2$

$f(1^-) = 3 + A$

$\because f(x)$ is continuous at $x = 1$

$B + 1 = 2 \Rightarrow B = 1$

$A + 3 = 2 \Rightarrow A = -1$

72. $I = \int_1^{e^{35}} \frac{\pi \sin(\pi \ell n x)}{x} dx$

$\pi \ell n x = t$

$\frac{\pi}{x} dx = dt$

$I = \int_0^{35\pi} \sin t dt = 2$

73. $a^3b^3c^3 \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = a^3b^3c^3\{-1(0-1)+1(1-0)\}$

$= 2a^3b^3c^3 \qquad \therefore \quad p = q = r = 3$

$p + q + r + 10 = 19$

74. $(K+1)^2 e^{(K+1)x} - 4(K+1)e^{(K+1)x} + 4e^{(K+1)x} = 0$

$(K+1)^2 - 4(K+1) + 4 = 0$

$K + 1 = 2 \quad \Rightarrow \quad K = 1$

75. Given $y'(0) = 0$

$8x + 2yy' + a + by' = 0$

$0 + 0 + a + by' = 0 \Rightarrow y' = -a/b = 0 \Rightarrow a = 0$

also $c = 0$ (passing through 0)

also $4 + 4 - 0 + 2b = 0$ (passing through 0)

$b = -4$

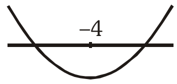
so ellipse is $4x^2 + y^2 - 4y = 0$

$4x^2 + (y-2)^2 = 4$

$\frac{x^2}{1} + \frac{(y-2)^2}{4} = 1$

$1 - e^2 = \frac{1}{4} \quad \Rightarrow \quad e = \frac{\sqrt{3}}{2}$

76. vertex is at $x = -4$



$f(x) = x^2 + 8x - 9$

$f(-4) = -25$ & $f(\infty) \to (\infty)$

$\because f(x)$ is one-one in $(-4, \infty)$

$\therefore$ Range is $(-25, \infty)$

$\therefore$ $B \neq (-25, \infty)$

77. $\int \sqrt{1 + 2\tan x \sec x + 2\tan^2 x}\, dx$

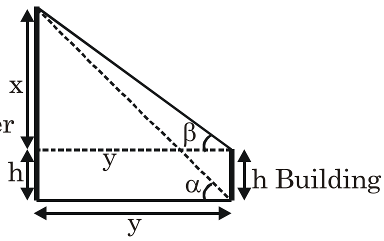
$= \int \sqrt{\sec^2 x + 2\tan x \sec x + \tan^2 x}\, dx$

$= \int (\sec x + \tan x)\, dx$

$= \ln(\sec x + \tan x) + \ln \sec x + C$

Hence (2)

78.



$\tan\beta = \frac{x}{y}$

$\tan\alpha = \frac{x+h}{y}$

$\Rightarrow \tan\alpha = \frac{x+h}{x\cot\beta}$

$\Rightarrow x = \frac{h\cot\alpha}{\cot\beta - \cot\alpha}$

$\therefore \ h + x = \frac{h\cot\beta}{\cot\beta - \cot\alpha}$

79. $\frac{\pi}{2} - \sin^{-1}\left(-\sin\left(\frac{7\pi}{6}\right)\right) = \frac{\pi}{2} + \sin^{-1}\sin\left(\frac{7\pi}{6}\right)$

$= \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}.$

80. $\lim_{x\to 1}\frac{(x+8)-(8x+1)}{5-x-(7x-3)}\left(\frac{\sqrt{5-x}+\sqrt{7x-3}}{\sqrt{8+x}+\sqrt{8x+1}}\right)$

$\Rightarrow \lim_{x\to 1}\frac{7(1-x)}{8(1-x)}\frac{4}{6}=\frac{7}{12}$

81. $N + P + C + M = 7$

where $N \le 6$, $P, C, M \ge 0$

$\Rightarrow \quad 1 \le P + C + M \le 7$

The required number of ways (from begger method)

$= {}^3C_2 + {}^4C_2 + {}^5C_2 + {}^6C_2 + {}^7C_2 + {}^8C_2 + {}^9C_2 = 119$

82. Let the boxes be B_1, B_2, B_3, B_4. Let us assume that two specific balls have been put in box B_i (i = 1,2,3,4).

It means in box B_i we have to put 3 balls from the remaining 18 balls.

Thus the probability that the two specific balls have been put in the particular box

$P(B_i)=\frac{{}^{18}C_3}{{}^{20}C_5}=\frac{5\times 4}{20\times 19}=\frac{1}{19}$

83. If one root is 2i then other root is –2i

$\therefore$ sum of the roots = 0

$\Rightarrow \quad -B/A = 0 \Rightarrow \quad B = 0$

$\therefore \quad B^3 (A^3 - C^3) = 0.$

84. $\left(\sum a_i\right)^2=\sum a_i^2+2\sum a_i a_j$

$\sum a_i a_j=\frac{100-50}{2}=25$ **Hence (2)**

85. $a(x + 2y + 1) + b(2x - y + 2) = 0$

$x + 2y + 1 + \frac{b}{a}(2x - y + 2) = 0$

$L_1 + \lambda L_2 = 0$

$\therefore$ Family of lines passes through the point of intersection of $L_1 = 0$ & $L_2 = 0$

$\left.\begin{array}{l}x+2y+1=0\\2x-y+2=0\end{array}\right\}$ solving these, we get (–1, 0)

Now, equation of the line passing through (–1, 0) and cutting off equal non-zero intercepts on co-ordinate axes (m = –1) is

$y - 0 = -1(x + 1)$

$x + y + 1 = 0$

86. $\Rightarrow \sin x - 1 + \cos x - \sin x\cos x = 0$

$\Rightarrow \quad (\sin x - 1)(1 - \cos x) = 0$

either $\sin x = 1 \quad \Rightarrow \quad x = 2n\pi + \frac{\pi}{2}$

or $\cos x = 1 \Rightarrow x = 2n\pi$

87. $\vec{a}=2\hat{i}+4\hat{j}$ is rotated through 90° and passing through x-axis

hence final vector is also lies on xy-plane.

Let vector be $\vec{r}=x\hat{i}+y\hat{j}$

$\vec{a}\,.\,\vec{r}=0 \Rightarrow \quad 2x+4y=0$

$|\vec{a}|=|\vec{r}| \quad \Rightarrow \quad x^2+y^2=20$

Solving we get x = 4 & y = –2

or x = –4 & y = 2

Initial vector is in Ist quadrant after rotating 90° & passing through x-axis final vector is in IVth quadrant hence final vector is $4\hat{i}-2\hat{j}$.

88. $a^2 - b^2 = c \Rightarrow c = (a - b)(a + b)$

as c is prime number (अभाज्य संख्या) so one of the factor must be equal to 1, but a + b = 1 is not possible as a & b are prime numbers. So a – b = 1 this is possible only when a = 3 & b = 2.

89. $\vec{a}.\left\{(\vec{b}+\vec{c})\times(\vec{a}+\vec{b}+\vec{c})\right\}$

$=\vec{a}.\{\vec{b}\times\vec{a}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+\vec{c}\times\vec{b}\}$

$=[\vec{a}\ \vec{b}\ \vec{c}]+[\vec{a}\ \vec{c}\ \vec{b}]=0$

90. $\frac{1}{20}\sum_{i=1}^{20}\left(x_i-\bar{x}\right)^2=5$

$\sum_{i=1}^{20}\left(x_i-\bar{x}\right)^2=100$

new observations are $2x_1, 2x_2, \ldots\ldots, 2x_{20}$.

Their mean = $\bar{x}_1=\frac{2\left(x_1+x_2+\ldots+x_{20}\right)}{20}=2\bar{x}$

Now, variance $=\frac{1}{20}\sum_{i=1}^{20}\left(2x_i-2\bar{x}\right)^2$

$=\frac{1}{20}\times 4\sum_{i=1}^{20}\left(x_i-\bar{x}\right)^2=\frac{1}{20}\times 4\times 100=20$