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ANSWER KEY

DATE: 26-11-2018

COURSE
NUCLEUS

JEE-MAIN MOCK TEST-4
XII

TEST CODE				
1	1	2	6	9

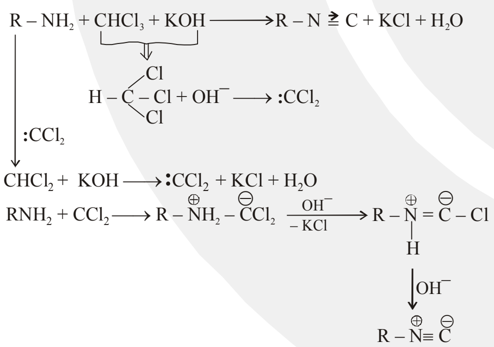
	IOC	OC	PC	IOC	OC	PC	IOC	OC	PC	IOC	OC	PC	IOC	OC	PC
Q.No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans	1	3	2	2	1	4	3	4	1	2	2	2	2	2	2
	IOC	OC	PC	IOC	OC	PC	IOC	OC	PC	IOC	OC	PC	IOC	OC	PC
Q.No.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans	1	3	4	2	3	1	2	2	1	1	1	3	3	3	4
Q.No.	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Ans	2	4	2	4	1	3	1	1	4	1	3	1	2	2	4
Q.No.	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans	4	3	3	4	4	3	1	1	3	4	4	4	4	4	4
Q.No.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
Ans	3	4	3	3	1	4	3	1	2	3	3	1	2	1	1
Q.No.	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
Ans	2	1	3	2	3	3	3	2	1	4	3	3	4	3	2

HINTS & SOLUTIONS

CHEMISTRY

Q.1 Ag, Au (due to less electropositive)

Q.2



3 mole of KOH are required in carbylamine test.

Q.3 Shortest distance between cation and anion =

$\frac{a}{2} = 500$ pm

a = 1000 pm

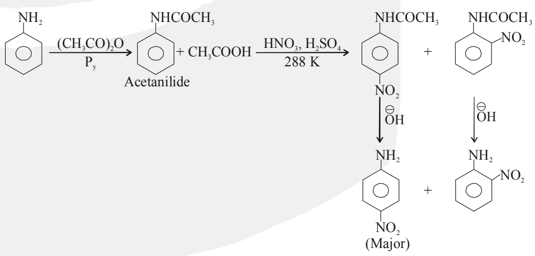
$d_{ideal} = \frac{z.M}{a^3.N_A} = \frac{(4)(120)}{(1000\times10^{-10})^3.6\times10^{23}}$

$= 0.8$ g/cm^3

$d_{actual} = 0.8 \times 0.98 = 0.784$ g/cm^3

Q.4 Sulphide ore of Cu, Pb, Hg are reduced by self reduction.

Q.5



Q.6 Vapour pressure of aquoeus solution of urea

$= 5 \times 10^{-3} \times 0.08 \times 300$ $(\because \pi = CRT)$

= 0.12 atm

$= 0.12 \times 760 = 91.2$ torr

$$\text{R.L.V.P} = \frac{P^\circ - P}{P} = X_{solute}$$

$$\frac{114 - 91.2}{114} = 0.2 = X_{Solute}$$

Q.7 Ag is 3000 times more soluble in Zn in compression of Pb.

$$(Pb - Ag) + Zn \longrightarrow (Ag - Zn)\,Pb \xrightarrow[\text{separation}]{\text{after}} Zn - Ag$$

$$\downarrow \text{distillation}$$

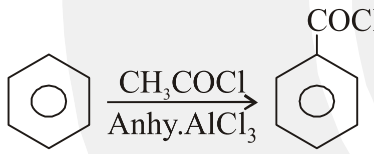
$$Ag + Zn\,(s)\uparrow$$

Q.8 

Yellow coloured dye

Q.9 $Z_x = 4$
$Z_y = 4$
$Z_z = 4$

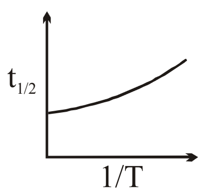
Q.10 $FeS + O_2\text{ (limited)} \longrightarrow FeO + SO_2$
$FeO + SiO_2\text{ (Flux)} \longrightarrow FeSiO_3\text{ (Slag)}$

Q.11 

It is Friedel Craft's acylation not Friedel crafts alkylation.

Q.12 $K = Ae^{-\frac{E_a}{RT}}$; $\frac{\ln 2}{t_{1/2}} = Ae^{\frac{-E_a}{RT}}$

$$t_{1/2} = \frac{\ln 2}{A} = e^{\frac{E_a}{RT}}$$



Q.13 Theory based

Q.14 Cellulose is a linear polymer of D-glucose units joined by β-glycosidic linkage.

Q.15 $\overset{+\frac{2}{1.8}}{Cu_{1.8}}O$ contains Cu^+ and Cu^{2+}

Let total Cu ions = 100
if $Cu^{2+} = x$
$\Rightarrow Cu^+ = (100 - x)$

so $$+\frac{2}{1.8} = \frac{x(+1) + (100 - x)(+2)}{100}$$

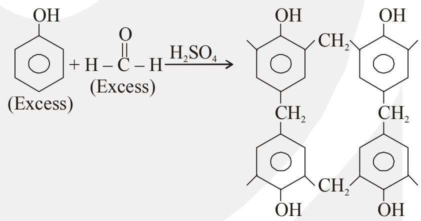
$$1000 = 1800 - 9x$$

$$x = \frac{800}{9} = 88.88\ \%$$

Q.16

d-block cation	**Bead colour in reducing flame**	**in oxidising flame**
Cu^{2+}	Red	Blue
Cr^{3+}	Green	Green
Fe^{3+}	Green	Yellow
Mn^{2+}	Colourless	Violet

Q.17 Bakelite is formed from a condensation reaction of phenol with formaldehyde.



Bakelite

Q.18 $$i = \frac{M_T}{M_O}$$

$$i = \frac{60}{80} = 1 + \alpha\left(\frac{1}{n} - 1\right)$$

$$0.75 = 1 + \alpha\left(\frac{1}{2} - 1\right)$$

$$\alpha = 0.5$$

$\% \alpha = 50 \%$

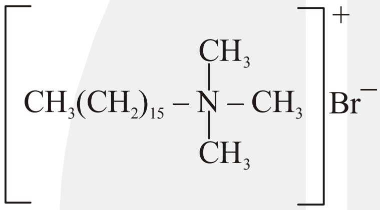
$2CH_3COOH \longrightarrow (CH_3COOH)_2$

$1 \qquad 1-\alpha \qquad \frac{\alpha}{2}$

$$\% \text{ of } CH_3COOH = \frac{\left(\frac{\alpha}{2}\right)}{1-\frac{\alpha}{2}} \times 100$$

$$\text{in dimeric form} = \frac{0.5}{1.5} \times 100 = 33.3 \%$$

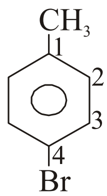
Q.19 $Bi^{3+} + \text{excess } NH_3 \longrightarrow Bi(OH)_3 \downarrow \text{(white)}$
$Al^{3+} + \text{excess } NH_3 \longrightarrow Al(OH)_3 \downarrow \text{(white)}$
$Zn^{2+} + \text{excess } NH_3 \longrightarrow Zn(NH_3)_3 \text{ (clear)}$
$Hg^{2+} + \text{excess } NH_3 \longrightarrow HgO.HgNH_2 \downarrow$
$Pb^{2+} + \text{excess } NH_3 \longrightarrow Pb(OH)_2 \downarrow$
$Cu^{2+} + \text{excess } NH_3 \longrightarrow Cu(NH_3)_4^{2+} \text{ (clear)}$
$Cd^{2+} + \text{excess } NH_3 \longrightarrow Cd(NH_3)_4^{2+} \text{ (clear)}$

Q.20 

Cetyltrimethyl ammonium bromide is cationic detergent.

Q.21 If vapour pressure is less compared to that calculate from Raoult's law, then solution shows negative deviation and for that solution $\Delta V_{mix} < 0$; $\Delta S_{mix} > 0$; $\Delta G_{mix} < 0$; $\Delta H_{mix} > 0$

Q.22 Theory based

Q.23 

4– Bromo toluene

Q.24 $A = \lambda N$

$$3.7 \times 10^{10} = \lambda \left(\frac{1}{226} \times 6 \times 10^{23}\right)$$

$$\lambda = \frac{3.7 \times 10^{10} \times 226}{6 \times 10^{23}}$$

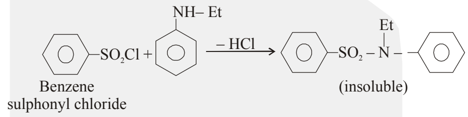
$$t_{mean} = \frac{1}{\lambda}$$

$$= \frac{6 \times 10^{23}}{3.7 \times 10^{10} \times 226 \times 3600 \times 24 \times 365}$$

≈ 2270 years

Q.25 Theory based

Q.26 Hinsberg test



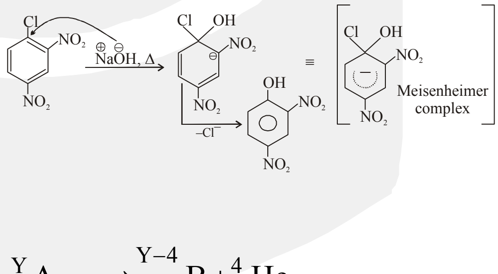
Q.27	$AB_2(aq) \longrightarrow A(g) + 2B(l)$		
initial moles	a		
t = 20 min	a – x	x	2x
t = ∞	–	1	2a

$$K = \frac{1}{t} \ln \frac{a}{a-x}$$

$$K = \frac{1}{20} \ln\left(\frac{40}{20}\right)$$

$$= \frac{0.693}{20} = 3.46 \times 10^{-2} \text{ min}^{-1}$$

Q.28 $CuSO_4 + \text{excess KI} \longrightarrow CuI \downarrow + KI_3 / I_2$
(white) (Brown)

Q.29 

Q.30 $^{Y}_{X}A \longrightarrow ^{Y-4}_{X-2}B + ^{4}_{2}He$

$$^{Y-4}_{X-2}B \longrightarrow ^{Y-4}_{X-1}C + ^{0}_{-1}e$$

$$^{Y-4}_{X-1}C \longrightarrow ^{Y-4}_{X}D + ^{0}_{-1}e$$

$$^{Y-4}_{X}D \longrightarrow ^{Y-8}_{X-2}E + ^{4}_{2}He$$

PHYSICS

Q.31 From the graphs

$\lambda = 9cm$

T = 3 sec

$\Rightarrow \quad v = \frac{\lambda}{T} = \frac{9}{3}$ cm/sec = 3cm/sec.

Q.32 Combination of isoboric, isochoric & isothermal.

Q.33 $\frac{4L}{5} = \lambda \Rightarrow \lambda = 8cm$

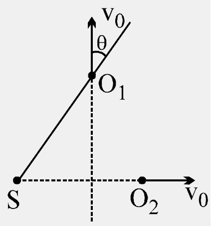
thus 2 cm corresponds to $\Delta\phi = z/2$

1 cm corresponds to $\Delta\phi = z/4$

So $y = Asm\ \pi/4 = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2}$

Q.34 $f_1 = f\left[\frac{v - v_0\cos\theta}{v}\right]$ ...(1)

$f_2 = f\left[\frac{v - v_0}{v}\right]$ ...(2)



$\therefore \quad \frac{f_1}{f_2} = \frac{v - v_0\cos\theta}{v - v_0} > 1$

Q.35 $ms_A (15 - 10) = ms_B (25 - 15)$

$s_A = 2s_B$

$ms_B (30 - 25) = ms_C (40 - 30)$

$s_B = 2s_C \qquad \Rightarrow s_A = 4s_C$

$ms_A (T - 10) = ms_C (40 - T)$

$\Rightarrow 4(T - 10) = 40 - T$

T = 16°C

Q.36 For ring just slides on to the steel rod the diameter of rod and ring should be equal to each other and suppose due to $\Delta\theta$ increment in temperature the diameter of both are equal then

$4\ (1 + \alpha_s\ \Delta\theta) = 3.992\ (1 + \alpha_{Brass}\ \Delta\theta)$

$4 + 4 \times 11 \times 10^{-6} \times \Delta\theta = 3.992 + 3.992 \times 20 \times 10^{-6} \times \Delta\theta$

$4 + 44 \times 10^{-6}\ \Delta\theta = 3.992 + 79.84 \times 10^{-6} \times \Delta\theta$

$0.008 = 35.84 \times 10^{-6}\ \Delta\theta$

$\frac{8\times10^3}{35.84} = \Delta\theta\ ;\ \Delta\theta = \frac{8000}{35.84} = 283$

so if temperature increased by 223°C then ring will start to slide and this temperature will equal to

$\theta = 30° + \Delta\theta = 30 + 253 = 283°C$

$\theta = 283°C \approx 280°C$

Q.37 From N.Law of collision

$\ell n\,(T - T_0) = -kt + \ell n\,(T_i - T_0)$ $(y = -mx + x)$

equation of straight line.

Q.38 By average form of Newton's Law of cooling:

$\frac{80-50}{5} = k\left(\frac{80+50}{2} - 20\right)$ .... (i)

$\frac{60-30}{t} = k\left(\frac{60+30}{2} - 20\right)$ .... (ii)

Solving (i) and (ii) we get t = 9 minute

We should apply actual result.

By Newton's law of cooling :

$\frac{T_{initial} - T_{surrounding}}{T_{final} - T_{surrounding}} = e^{kt}$ when k is const.

$\frac{80-20}{50-20} = e^{k\times5}$

$\Rightarrow \quad (2)^{1/5} = e^k$ .... (i)

$\frac{60-20}{30-20} = e^{kt}$

$\Rightarrow \quad (4)^{1/t} = e^k$ .... (ii)

From (i) and (ii) we get $2^{1/5} = 2^{2/t}$

$\Rightarrow \quad \frac{1}{5} = \frac{2}{t}$

$\Rightarrow \quad t = 10$ min.

Q.39 Let equation of wave as it is moving along – ve x-axis is

$y = A\sin(kx + \omega t + \alpha)$

But, $y(\lambda/4, t) = A\sin\omega t$

Comparing then

$kx + \alpha = 0 \Rightarrow \alpha = -\pi/2$

Q.40 $\frac{PV}{T} = \tan\theta = nR$

$\therefore$ slope α no. of moles

Q.41 $\Delta U = U_f - U_i$

$= \frac{3}{2} nR\Delta T = \frac{3}{2}[P_C V_C - P_A V_A]$

$= \frac{3}{2}$ [150 × 10^{-6} × 200 × 10^3 – 100 × 10^{-6} × 100 × 10^3]

= 30 J

Q.42 $\Sigma F_z = 0$

$(T + dT) + \mu g dz - T = 0$

$dT = -\mu g dz$ .... (i)

also $T = \mu v^2$

$dT = d\mu v^2 + 2vdv\, d\mu$

As v is independent of z

$dv = 0$

$dT = v^2 d\mu$ .... (ii)

from equation (1) and (2) we get

$$\mu\int \frac{d\mu}{\mu} = -\frac{g}{v^2}\int_0^z dz$$

or $\mu = \mu_0\, e^{-(g/v^2)z}$

Q.43 $Q_{abd} - Q_{acd}$

$= (W_{abd} - W_{acd}) + (DU_{abd} - DU_{acd})$

$= W_{abd} - W_{acd} + 0$

(internal energy change is same for two paths)

= area of abdca = 15 J]

$Q_{abd} = Q_{ab} + Q_{bd}$ = 60 + 20 = 80 J

$Q_{acd} = Q_{abd}$ –15= 65 J

Alternative :

From the First law of Thermodynamics, one has

$\Delta U_{a\to c\to d} = Q_{a\to c\to d} + W_{a\to c\to d}$ = (60 J + 20 J) + [–(8Pa) (3m^3)] ⇒ 56 J. Since energy is a state variable,

$\Delta U_{a\to c\to d} = Q_{a\to c\to d} + W_{a\to c\to d}$ ⇒ 56 J

= Q + [–(3Pa) (3m^3)] ⇒$Q_{a\to c\to d}$ = 65 J

Q.44 $\lambda = \frac{v}{f} = \frac{330}{500} = 0.66\text{ m} = \frac{4\ell}{2n-1}$

⇒ n = 3

Q.45 Friction force = 0.5 × 25 ×10 = 125 N

distance moved = 2 × 10^3

$\therefore$ work done against friction = 250 × 10^3 J

$\therefore$ Heat given to the body = 125 × 10^3 J

$\therefore T = \frac{125\times10^3}{25\times1000\times0.1\times4.2} = \frac{50}{42} = \frac{250}{21}$

= 11.9 K

Q.46 All dimension will increase

Q.47 To keep Buoyent force constant volume of submerged part must increase.

Q.48 $\frac{dQ}{dt} = \frac{kA}{\ell}(T_2 - T_1)$

$\frac{dQ}{dt}$ max if $\frac{A}{\ell}$ is max.

⇒ parallel to CD, AB, FG or EH.

$\frac{dQ}{dt}$ min. If $\frac{A}{\ell}$ is min.

⇒ parallel to CH / BG / AF / DE

⇒ [C]

Q.49 $u = \sigma eAT^4$ and σ_1e and T are constant

$\therefore \frac{u_2}{u_1} = \frac{A_2}{A_1} = \frac{(2\pi R^2 + \pi R^2)\times 2}{4\pi R^2} = \frac{3}{2}$

Q.50 PV × V = C

TV = C

$T' = \frac{T}{2}$

Q.51 $PV = \frac{m}{M}RT$

V α m

$V_1 < V_2 \Rightarrow m_1 < m_2$

Q.52 Theree must be 3 half loops.

Q.53 Frequency observed by man is same as "observed" by wall and it reflects the same and as man and wall are relatively at rest, hence man observers same frequency of reflected sound. Hence no beat frequency

Q.54 $pV = N_A kT$

$N_A = \frac{pV}{kT}$

Q.55 $1° R = 1°C$

$1° S = \frac{100}{70} = \frac{10}{7}°C$

$1° U = \frac{100}{75} = \frac{4}{3}°C$

$1°S > 1°U > 1°R$

$\Rightarrow \quad x_2 > x_3 > x_1$

Q.56 $\frac{dQ}{dt} = \frac{dmL}{dt} = \frac{kA\Delta T}{L}$

Q.57 Power recived by earth from sun $\propto \frac{1}{r^2}$

Q.58 $\because \quad PV = nRT$

$10^5 \times \frac{4\pi}{3}r^2 = \frac{N}{N_V}RT$

Q.59 $P_2 = 2P_1 \quad V_2 = 4V_1 \quad n = 1$

$C = Cv + \frac{PdV}{dT}$

$dw = PdV = \text{Area} = \frac{1}{2}[(P_1 + P_2)(V_2 - V_1)]$

$= \frac{1}{2}(3P_1 \times 3V_1) = \frac{9}{2}P_1V_1$

$dT = T_2 – T_1 = \frac{P_2V_2}{R} - \frac{P_1V_1}{R}$

$= \frac{2P_1 \times 4V_1}{R} - \frac{P_1V_1}{R} = \frac{7P_1V_1}{R}$

$C = \frac{5}{2}R + \frac{9}{2}\frac{P_1V_1R}{7P_1V_1}$

$= \frac{5}{2}R + \frac{9R}{14} = \frac{44R}{14} = \frac{22R}{7}$ **Ans.**

Q.60 $0.6 = \frac{\text{workdone}}{Q_{input}} = \frac{Q_{input} - Q_{reject}}{Q_{input}}$

$= 1 - \frac{Q_r}{Q_i}$

$0.6 = 1 - \frac{20}{Q_i}$

$Q_i = 50$

$W = Q_i - Q_r = 30$ J

MATHEMATICS

Q.61 $f(x) = \frac{1}{g(x)}$; $f'(x) = \frac{-1}{g^2(x)} \cdot g'(x)$

$f'(1) = \frac{-1}{g^2(1)}\, g'(1) = \frac{-1}{9}\left(\frac{1}{f'(3)}\right)$

$= \frac{-1}{9}\left(\frac{1}{2}\right) = \frac{-1}{18}$. **Ans.**]

Q.62 $l = ln \lim_{t\to 0} \frac{\int_0^t (1+2\sin 3x)^{4/x}\, dx}{t}$

$= ln \lim_{t\to 0} (1+2\sin 3t)^{4/t}$

(using L'Hospital's rule)

$= ln\, e^{\lim_{t\to 0}\frac{4}{t}(2\sin 3t)} =$

$\lim_{t\to 0} \frac{2\cdot 3\cdot 4\cdot \sin 3t}{3t} = 24$ Ans.]

Q.63 $g(x^3 + 1) = x^6 + x^3 + 2 = (x^3 + 1)^2 – x^3 + 1$

$=(x^3+1)^2–(x^3+1–1)+1=(x^3+1)^2–(x^3+1)+2$

Put $\quad x^3 + 1 = t$

So, $\quad g(t) = t^2 – t + 2$

$\Rightarrow \quad g(x^2 – 1) = (x^2 – 1)^2 – (x^2 – 1) + 2$

$= x^4 – 3x^2 + 4$. **Ans.**]

Q.64 $g(x) = f(-x + f(f(x)))$;

$f(0) = 0; \quad f'(0) = 2$

$g'(x) = f'(-x + f(f(x)))\cdot[-1 + f'(f(x))\cdot f'(x)]$

$g'(0) = f'(f(0))\cdot[-1 + f'(0)\cdot f'(0)]$

$= f'(0)\,[– 1 + (2)(2)]$

$= (2)(3) = 6$ **Ans.**]

Q.65 $I = \int_1^\infty \frac{dx}{(e\cdot e^x + e^3 \cdot e^{-x})} = \int_1^\infty \frac{e^x\, dx}{e(e^{2x} + e^2)}$

(multiply N^r and D^r by e^x)

put $e^x = t \quad \Rightarrow \quad e^x\, dx = dt$

$I = \frac{1}{e}\int_e^\infty \frac{dt}{t^2 + e^2} = \frac{1}{e^2}\tan^{-1}\frac{t}{e}\Big|_e^\infty$

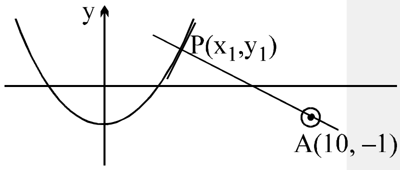
$= \frac{1}{e^2}\left[\frac{\pi}{2} - \frac{\pi}{4}\right] = \frac{\pi}{4e^2}$ Ans.]

Q.66 $\left.\frac{dy}{dx}\right|_P = \frac{2x_1}{4} = \frac{x_1}{2}$

$\Rightarrow$ slope of normal $= -\frac{2}{x_1}$

$\Rightarrow \quad -\frac{2}{x_1} = \frac{y+1}{x_1-10}$

$\Rightarrow 20 - 2x_1 = x_1y_1 + x_1$



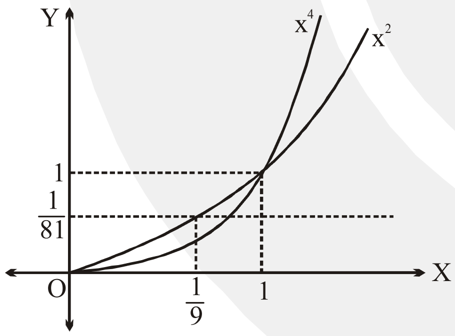
$\Rightarrow \quad 3x_1 + x_1y_1 = 20 \quad ....(1)$

also $\quad y_1 = \frac{x_1^2}{4} - 2$

$\Rightarrow \quad 4y_1 = x_1^2 - 8 \quad ....(2)$

only (D) satisfies (1) and (2) both.]

Q.67 Clearly $f(x) = \begin{cases} \frac{1}{81}, 0 \le x \le \frac{1}{9} \\ x^2, \frac{1}{9} < x \le 1 \\ x^4, x > 1 \end{cases}$



Clearly f(x) is non differentiable at $x = \frac{1}{9}, 1$

$\therefore$ sum of squares of reciprocals

$= 9^2 + 1 = 82$ Ans.]

Q.68 f is not differentiable at $x = \frac{1}{2}$

g is not continuous in [0, 1] at x = 0 & 1

h is not continuous in [0, 1] at x = 1

$k(x) = (x+3)^{\ln_2 5} = (x+3)^p$ where $2 < p < 3$

Q.69 $f(2) = 10$, hence $2ae^{-2b} = 10$

$\Rightarrow \quad ae^{-2b} = 5 \quad ....(1)$

$f'(x) = a[e^{-bx} - bx\, e^{-bx}] = 0$

$f'(2) = 0$

$a(e^{-2b} - 2be^{-2b}) = 0$

$ae^{-2b}(1 - 2b) = 0$

$\Rightarrow \quad b = 1/2$ or $a = 0$ (rejected)

from (1) if $b = 1/2$; $\quad a = 5e$

$\therefore \quad a = 5e$ and $b = 1/2$ Ans.]

Q.70 $f(x, n) = \sum_{k=1}^{n} \log_x\left(\frac{k}{x}\right)$

$= \log_x\left(\frac{1}{x}\right) + \log_x\left(\frac{2}{x}\right) + ...\log_x\left(\frac{n}{x}\right) = \log_x\left(\frac{n!}{x^n}\right)$

given: f(x, 10) = f(x, 11)

$\Rightarrow \log_x\left(\frac{10!}{x^{10}}\right) = \log_x\left(\frac{11!}{x^{11}}\right) \Rightarrow \frac{10!}{x^{10}} = \frac{11!}{x^{11}}$

$\Rightarrow x = 11$ Ans.]

Q.71 $T_r = \frac{1}{\sqrt{\frac{r}{n}} \cdot n\left(3\sqrt{\frac{r}{n}} + 4\right)^2}$

$S = \frac{1}{n}\sum_{1}^{4n} \frac{1}{\left(3\sqrt{\frac{r}{n}} + 4\right)^2 \cdot \sqrt{\frac{r}{n}}}$

$= \int_0^4 \frac{dx}{\sqrt{x}(3\sqrt{x}+4)^2}$

put $\quad 3\sqrt{x} + 4 = t$

$\Rightarrow \quad \frac{3}{2}\frac{1}{\sqrt{x}}dx = dt$

$= \frac{2}{3}\int_4^{10} \frac{dt}{t^2} = \frac{2}{3}\left[\frac{1}{t}\right]_{10}^{4} = \frac{2}{3}\left[\frac{1}{4} - \frac{1}{10}\right] = \frac{2}{3} \cdot \frac{6}{40} = \frac{1}{10}$

Q.72 We have $\sin^{-1}\left[\frac{\pi x}{6}\right] > 0 \Rightarrow \left[\frac{\pi x}{6}\right] = 1$

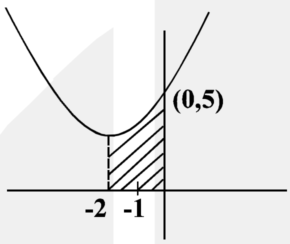
$\Rightarrow \quad 1 \le \frac{\pi x}{6} < 2 \Rightarrow \frac{6}{\pi} \le x < \frac{12}{\pi}$

$\therefore \quad x = 2, 3$ only.

Hence two integral solution will satisfy above equation.]

Q.73 $y = x^2 + 4x + 5 = (x+2)^2 + 1$

$$A = \int_{-2}^{0}\left(x^2+4x+5\right)dx = \left.\frac{x^3}{3}+2x^2+5\right]_{-2}^{0}$$



$$= -\left[-\frac{8}{3}+8-10\right] = 2+\frac{8}{3} = \frac{14}{3} = 4\frac{2}{3}$$

Q.74 equation $(x-a)^2 + y^2 = (x-b)^2$ $[S = (a, 0)$; $D : x = b]$

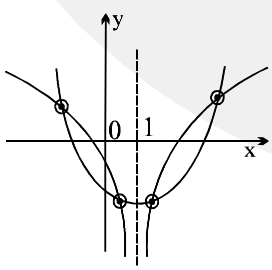
$$y^2 = (b^2 - a^2) + 2x(a - b)$$

differentiate twice to get $y\frac{d^2y}{dx^2} + \left[\frac{dy}{dx}\right]^2 = 0$;

$$y\frac{d^2y}{dx^x} + \left(\frac{dy}{dx}\right)^2 = 0.$$

Q.75 $x^2 - 2x - 3 = \log_2|1 - x|$

4 points]



Q.76 $F(x) = \int\frac{3x+2}{\sqrt{x-9}}dx$; let $x - 9 = t^2$

$\Rightarrow \quad dx = 2t\,dt$

$\therefore \quad F(x) = \int\left(\frac{3(t^2+9)+2}{t}\cdot 2t\right)dt$

$= 2\int\left(29+3t^2\right)dt = 2\,[29t + t^3]$

$F(x) = 2\left[29\sqrt{x-9}+(x-9)^{3/2}\right] + C$

given $F(10) = 60 = 2\,[29 + 1] + C$

$\Rightarrow \quad C = 0$

$\therefore \quad F(x) = 2\left[29\sqrt{x-9}+(x-9)^{3/2}\right]$

$F(13) = 2\,[29 \times 2 + 4 \times 2]$

$= 4 \times 33 = 132$ Ans.]

Q.77 $\lim_{x\to 0} f(x) = 0$

$\left(\because \lim_{x\to 0} x^2 = 0 \text{ and } \left\{e^{1/x}\right\} \text{ is a bounded function}\right)$

$\therefore \quad k = 0$

Now, $f'(0) = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$

$= \lim_{h\to 0} h\left\{e^{1/h}\right\} = 0 \Rightarrow f'(0) = 0$ **Ans.**

Note that $f(x)$ is discontinuous at

$x = \pm\frac{1}{\ln 2}, \pm\frac{1}{\ln 3}$ and so on.]

Q.78 $\sin x = t$; $I = \int\frac{\left(1-t^2\right)\left(2-t^2\right)}{t^2\left(1+t^2\right)}dt$;

$f(t) = \int\frac{(y-1)(y-2)}{y(1+y)} = 1 + \frac{2(1-2y)}{y(y+1)}$; $y = t^2$

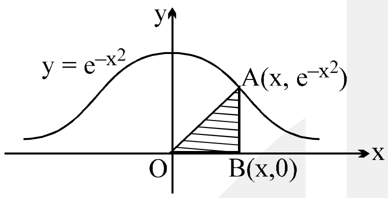
$= 1 + 6\left[\frac{1}{3y} - \frac{1}{y+1}\right]$; $\int\left(1+\frac{2}{t^2}-\frac{6}{1+t^2}\right)dt$]

Q.79 $A = \frac{x e^{-x^2}}{2}$;

$A' = \frac{1}{2}\left[e^{-x^2} - 2x^2 \cdot e^{-x^2}\right]$

$= \frac{e^{-x^2}}{2}\left[1 - 2x^2\right] = 0$

$\Rightarrow \; x = \frac{1}{\sqrt{2}}$ gives $A_{max.}$



$\therefore \quad A_{max} = \frac{e^{-1/2}}{2\sqrt{2}} = \frac{1}{\sqrt{8e}}$]

Q.80 f(x) will be continuous where
$3 \sin x + a^2 - 10a + 30 = 4\cos x$

or $\underbrace{a^2 - 10a + 30}_{\geq 5} = \underbrace{4\cos x - 3\sin x}_{\leq 5}$

or $(a-5)^2 + 5 = 4\cos x - 3\sin x$
$\therefore$ $a = 5$ and $4\cos x - 3\sin x = 5$

$\Rightarrow \; \frac{4}{5}\cos x - \frac{3}{5}\sin x = 1$

or $\cos(x + \theta) = 1$, where $\tan\theta = \frac{3}{4}$

$\therefore \; x = 2n\pi - \theta = 2n\pi - \tan^{-1}\frac{3}{4}, \; n \in I.$

Q.81 $x = -\pi/4$; $y = \cos\frac{\theta}{2}$; where $\cos\frac{\theta}{2} = \frac{1}{8}$ and

$\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}} = \frac{3}{4}$]

Q.82 Solve graphically]

Q.83

$P'(x) = f(x)\, g'(x) + g(x)\, f'(x)$
$P'(2) = f(2)\, g'(2) + g(2)\, f'(2)$
$= (1)(2) + 4(-1)$
$= -2$

$Q'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{g^2(x)}$

$Q'(2) = \frac{(4)(-1) - (1)(2)}{16} = -\frac{6}{16} = -\frac{3}{8}$

$C'(x) = f'(g(x))g'(x)$
$C'(2) = f'(4) \cdot 2 = 3 \cdot 2 = 6$]

Q.84 Let $\int_0^x f(t)\,dt = T(x) \;\Rightarrow\; T'(x) = f(x)$

$\therefore$ On differentiating b.t.s. w.r.t. x, we get
$f(x) = T'(x)$
Hence

$G(x) = \int e^x\left(\int_0^x f(t)\,dt + f(x)\right)dx$

$= \int e^x\,(T(x) + T'(x))dx = e^x\,T(x) + C$

$\Rightarrow \quad G(x) = e^x \int_0^x f(t)dt + C$

Now on differentiating

$G'(x) = e^x \int_0^x f(t)dt + e^x f(x)$

$\Rightarrow$ G'(0) = f(0) = 1 **Ans.**]

Q.85 $\lim_{n\to\infty} \frac{e^n}{\left(1+\frac{1}{n}\right)^{n^2}} = \lim_{n\to\infty} \frac{e^n}{e^{n^2 \ln\left(1+\frac{1}{n}\right)}}$

$= \lim_{n\to\infty} e^{n - n^2 \ln\left(1+\frac{1}{n}\right)}$; Put $n = \frac{1}{y}$

$= \lim_{y\to 0} e^{\frac{y - \ln(1+y)}{y^2}} = e^{\frac{1}{2}} = \sqrt{e}$ **Ans.**]

Alternatively: $L = \lim_{n\to\infty} \dfrac{e^n}{\left(1+\frac{1}{n}\right)^{n^2}}$

$\Rightarrow \ln L = \lim_{n\to\infty}\left(n - n^2 \ln\left(1+\frac{1}{n}\right)\right)$

Put $n = \dfrac{1}{y}$,

we get $\ln L = \lim_{y\to 0} \dfrac{y - \ln(1+y)}{y^2}$

$\Rightarrow \ln L = \lim_{y\to 0} \dfrac{y - \left(y - \frac{y^2}{2} + ...\right)}{y^2} = \dfrac{1}{2}$

$\Rightarrow L = e^{\frac{1}{2}} = \sqrt{e}$ **Ans.**]

Q.86 We have $\dfrac{dy}{dx} = (e^y - x)^{-1} \Rightarrow \dfrac{dx}{dy} = e^y - x$

$\Rightarrow \dfrac{dx}{dy} + x = e^y$; So I.F. $= e^{\int dy} = e^y$

$\therefore$ General solution is given by

$x\,e^y = \dfrac{1}{2}e^{2y} + C \Rightarrow x = \dfrac{e^y}{2} + Ce^{-y}$

As $y(0) = 0$, so $C = \dfrac{-1}{2}$

$\therefore\ x = \dfrac{e^y}{2} - \dfrac{1}{2}e^{-y} \Rightarrow e^y - e^{-y} = 2x$

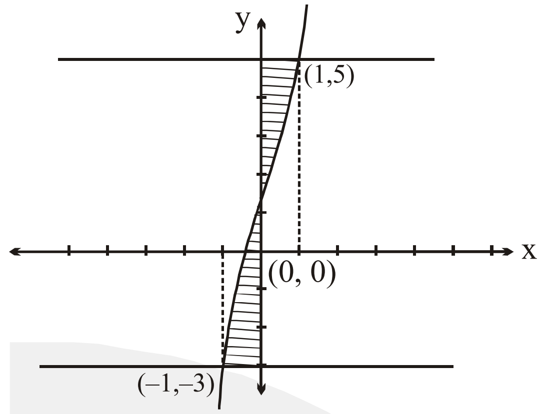
$\Rightarrow e^{2y} - 2xe^y - 1 = 0 \Rightarrow 2e^y = 2x \pm \sqrt{4x^2+4}$

But $e^y = x - \sqrt{x^2+1}$

(Rejected)

Hence $y = \ln\left(x + \sqrt{x^2+1}\right)$]

Q.87



Area

$= \int_{-1}^{0}\left((x^3+3x+1) - (-3)\right)dx + \int_{0}^{1}\left(5 - (x^3+3x+1)\right)dx$

$= \dfrac{9}{2}$ **Ans.**]

Q.88 In the integral J, substitute $x + 1 = t$

$\Rightarrow dx = dt$ and $x^2 + 2x = (t^2 - 1)$

Now $J = \int_1^e \dfrac{e^{\frac{t^2-2}{2}}}{t}dt$ and $K = \int_1^e t\ln t\, e^{\frac{t^2-2}{2}}dt$

Hence $(J + K) = \int_1^e e^{\frac{t^2-2}{2}}\left(\dfrac{1}{t} + t\ln t\right)dt$

$= \left(e^{\frac{t^2-2}{2}}\ln t\right)_{t=1}^{t=e} = e^{\frac{e^2-2}{2}} = \left(\sqrt{e}\right)^{e^2-2}$

Q.89 Let (x_1, y_1) and (x_2, y_2) are two of these points given $y = x^3 + 2x - 1$ and $y = 2x^3 - 4x + 2$

$\therefore\quad y_1 = 2x_1^3 - 4x_1 + 2$ ....(1)

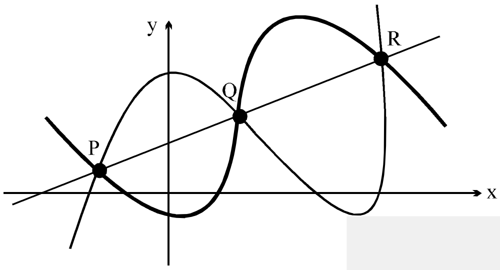
and $\quad 2y_1 = 2x_1^3 + 4x_1 - 2$ ....(2)

(2) – (1)

$y_1 = 8x_1 - 4$ ....(3)

|||ly $\quad y_2 = 8x_2 - 4$ ....(4)

$y_2 - y_1 = 8(x_2 - x_1)$



$$\frac{y_2 - y_1}{x_2 - x_1} = 8 \;\; \textbf{Ans.} \;]$$

Q.90 $T_r = \frac{5(r+1) - 3r}{r(r+1)} \cdot \left(\frac{3}{5}\right)^{r+1}$

$$= \left(\frac{5}{r} - \frac{3}{r+1}\right)\left(\frac{3}{5}\right)^{r+1}$$

$$= \frac{5}{r} \cdot \frac{3}{5} \cdot \left(\frac{3}{5}\right)^{r} - \frac{3}{r+1}\left(\frac{3}{5}\right)^{r+1}$$

$$= 3\left[\frac{1}{r} \cdot \left(\frac{3}{5}\right) - \frac{1}{r+1}\left(\frac{3}{5}\right)^{r+1}\right]$$

$$\therefore \; S_n = \sum_{r=1}^{n} T_r$$

$$T_1 = 3\left[\frac{1}{1}\left(\frac{3}{5}\right)^{1} - \frac{1}{2}\left(\frac{3}{5}\right)^{2}\right]$$

$$T_2 = 3\left[\frac{1}{2}\left(\frac{3}{5}\right)^{2} - \frac{1}{3}\left(\frac{3}{5}\right)^{3}\right]$$

:

$$T_n = 3\left[\frac{1}{n}\left(\frac{3}{5}\right)^{n} - \frac{1}{n+1}\left(\frac{3}{5}\right)^{n+1}\right]$$

$$S_n = 3\left[\frac{3}{5} - \frac{1}{(n+1)}\left(\frac{3}{5}\right)^{n+1}\right]$$

$$\therefore \; \lim_{n\to\infty} S_n = \frac{9}{5}. \; \textbf{Ans.}$$

Aliter: $T_r = \left(\frac{2r+5}{r(r+1)}\right)\left(\frac{3}{5}\right)^{r+1}$

$$= \left(\frac{5(r+1) - 3r}{r(r+1)}\right)\left(\frac{3}{5}\right)^{r+1}$$

$$= 3\left[\frac{1}{r}\left(\frac{3}{5}\right)^{r} - \frac{1}{r+1}\left(\frac{3}{5}\right)^{r+1}\right]$$

$$= 3(r_r - v_{r+1})$$

So, $\sum_{r=1}^{n} T_r = 3\left[\sum_{r=1}^{n} v_r - \sum_{r=1}^{n} v_{r+1}\right]$

$$\Rightarrow \; S_n = 3\,(v_1 - v_{n+1}) = \frac{9(n+1)5^n - 3^{n+2}}{(n+1)\,5^{n+1}}$$

So, $\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{9(n+1)5^n - 3^{n+2}}{(n+1)\,5^{n+1}} = \frac{9}{5}.$