



**Useful Data**
**PHYSICS**

|                             |  |
|-----------------------------|--|
| Acceleration due to gravity | $g = 10 \text{ m/s}^2$                                       |
| Planck constant             | $h = 6.6 \times 10^{-34} \text{ J-s}$                        |
| Charge of electron          | $e = 1.6 \times 10^{-19} \text{ C}$                          |
| Mass of electron            | $m_e = 9.1 \times 10^{-31} \text{ kg}$                       |
| Permittivity of free space  | $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N-m}^2$ |
| Density of water            | $\rho_{\text{water}} = 10^3 \text{ kg/m}^3$                  |
| Atmospheric pressure        | $P_a = 10^5 \text{ N/m}^2$                                   |
| Gas constant                | $R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$                |

**CHEMISTRY**

|                   |       |   |   |
|-------------------|-------|---|---|
| Gas Constant      | R     | = | $8.314 \text{ J K}^{-1} \text{ mol}^{-1}$             |
|                   |       | = | $0.0821 \text{ Lit atm K}^{-1} \text{ mol}^{-1}$      |
|                   |       | = | $1.987 \approx 2 \text{ Cal K}^{-1} \text{ mol}^{-1}$ |
| Avogadro's Number | $N_a$ | = | $6.023 \times 10^{23}$                                |
| Planck's constant | h     | = | $6.625 \times 10^{-34} \text{ J-s}$                   |
|                   |       | = | $6.625 \times 10^{-27} \text{ erg-s}$                 |
| 1 Faraday         |       | = | 96500 coulomb   |
| 1 calorie         |       | = | 4.2 joule   |
| 1 amu             |       | = | $1.66 \times 10^{-27} \text{ kg}$                     |
| 1 eV              |       | = | $1.6 \times 10^{-19} \text{ J}$                       |

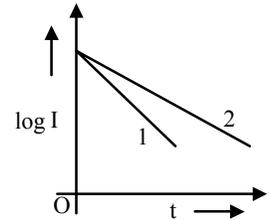
Atomic No: H=1, He = 2, Li=3, Be=4, B=5, C=6, N=7, O=8, N=9, Na=11, Mg=12, Si=14, Al=13, P=15, S=16, Cl=17, Ar=18, K =19, Ca=20, Cr=24, Mn=25, Fe=26, Co=27, Ni=28, Cu = 29, Zn=30, As=33, Br=35, Ag=47, Sn=50, I=53, Xe=54, Ba=56, Pb=82, U=92.

Atomic masses: H=1, He=4, Li=7, Be=9, B=11, C=12, N=14, O=16, F=19, Na=23, Mg=24, Al = 27, Si=28, P=31, S=32, Cl=35.5, K=39, Ca=40, Cr=52, Mn=55, Fe=56, Co=59, Ni=58.7, Cu=63.5, Zn=65.4, As=75, Br=80, Ag=108, Sn=118.7, I=127, Xe=131, Ba=137, Pb=207, U=238.

**Physics****PART – I****SECTION – A****Single Correct Choice Type**

This section contains **7 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

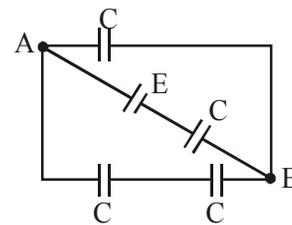
- An electric bulb consumes 55 W when operated at 110 volts AC. It is connected to a 220 V, 50 Hz line through an inductor (coil) in series. The inductance of the coil for which the device gets correct voltage is  
 (A)  $0.7\sqrt{3}$  H (B)  $7\sqrt{3}$  H  
 (C)  $0.07\sqrt{3}$  H (D)  $\sqrt{3}$  H
- A bowler throws a ball horizontally along east direction with speed of 144 km/hr. The batsman hits the ball such that it deviates from its initial direction of motion by  $74^\circ$  north of east direction, without changing its speed. If mass of the ball is  $\frac{1}{3}$  kg and time of contact between bat and ball is 0.02 s. Average force applied by batsman on ball is  
 (A) 800 N,  $53^\circ$  East of North (B) 800 N,  $53^\circ$  North of East  
 (C) 800 N,  $53^\circ$  North of West (D) 800 N,  $53^\circ$  West of North.
- A capacitor of capacity C is charged to a steady potential difference V and connected in series with an open key and a pure resistor R. At time  $t = 0$ , the key is closed. If  $I =$  current at time  $t$ , a plot of  $\log I$  versus  $t$  is as shown as (A) in the graph. Later one of the parameters i.e., V, R or C is changed keeping the other two constant, and the graph (B) is recorded. Then  
 (A) C is increased (B) C is reduced  
 (C) R is increased (D) R is reduced



**Rough work**

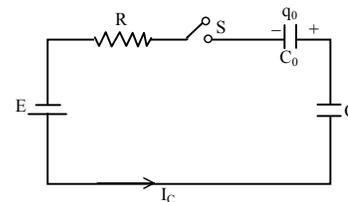
4. For the circuit shown, find the potential difference between A and B

- (A)  $E$  (B)  $\frac{2E}{3}$   
 (C)  $\frac{2E}{5}$  (D) zero



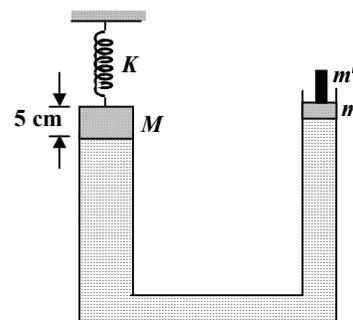
5. In the circuit shown, the switch S is closed at  $t=0$ . Initially the capacitor C is uncharged but  $C_0$  has a charge  $q_0$  at  $t = 0$ . The current immediately after the switch S is closed is  $I_C$ .

- (A)  $I_C = 0$  (B)  $I_C \rightarrow \infty$   
 (C)  $I_C = \frac{E - \frac{q_0}{C_0}}{R}$  (D)  $I_C = \frac{E}{R}$



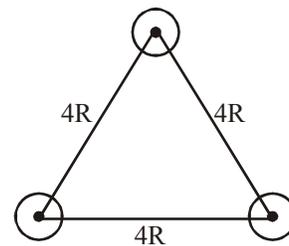
6. For the system shown in the figure, the cylinder on left has a mass (M) of 25 kg cross-sectional area  $20 \text{ cm}^2$  and thickness 5 cm. It is connected to a spring of spring constant 1400 N/m. The piston on the right has mass  $m (= 5 \text{ kg})$  and cross-sectional area  $4 \text{ cm}^2$ . Initially heights of water column in both cylinders are equal and pistons are in equilibrium. The minimum mass  $m'$  to be kept on  $m$  so that water just spills out from the left is ( $g = 10 \text{ m/s}^2$ )

- (A) 1.4 kg (B) 2.0 kg  
 (C) 2.5 kg (D) 3.0 kg



7. Three solid spheres each of mass  $m$  and radius  $R$  are released from the position shown in figure. The speed of any one sphere at the time of collision would be

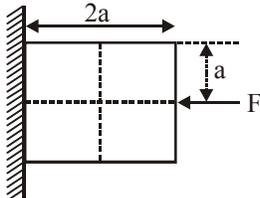
- (A)  $\sqrt{\frac{2Gm}{R}}$  (B)  $\sqrt{\frac{3Gm}{R}}$   
 (C)  $\sqrt{\frac{Gm}{3R}}$  (D)  $\sqrt{\frac{Gm}{2R}}$



**Rough work**

## Multiple Correct Answer(s) Type

This section contains **4 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct**.

8. The coefficient of linear expansion of an inhomogeneous rod changes linearly from  $\alpha_1$  to  $\alpha_2$  from one end to the other end of the rod. The effective coefficient of linear expansion of rod is
- (A)  $\alpha_1 + \alpha_2$  (B)  $\frac{\alpha_1 + \alpha_2}{2}$   
 (C)  $\sqrt{\alpha_1 \alpha_2}$  (D)  $\alpha_1 - \alpha_2$
9. A particle is projected from the ground in earth's gravitational field at an angle  $\theta$  with the horizontal then
- (A) center of curvature of projectile's trajectory at the highest point is below the ground level if  $\theta < \tan^{-1} \sqrt{2}$   
 (B) centre of curvature of projectile's trajectory at the highest point is above the ground level if  $\theta > \tan^{-1} \sqrt{2}$   
 (C) center of curvature of projectile's trajectory at the highest point is above ground level if  $\theta < \tan^{-1} 2$   
 (D) center of curvature of projectile's trajectory at the highest point is below the ground level is  $\theta > \tan^{-1} 2$
10. A block of mass  $m$  is held stationary against a rough wall by applying a force  $F$  as shown in figure. Which one of the following statements is incorrect ?
- (A) frictional force  $f = mg$   
 (B) normal reaction  $N = F$   
 (C)  $F$  will not produce a torque  
 (D)  $N$  will not produce any torque
- 
11. X ray from a tube with a target A of atomic number  $Z$  shows strong  $K_\alpha$  lines for target A and weak  $K$  lines for impurities. The wavelength of  $K_\alpha$  lines is  $\lambda_z$  for target A and  $\lambda_1$  and  $\lambda_2$  for two impurities.
- $$\frac{\lambda_z}{\lambda_1} = 4 \text{ and } \frac{\lambda_z}{\lambda_2} = \frac{1}{4}.$$
- Screening constant of  $K_\alpha$  lines to be unity. Select the correct statement(s)
- (A) The atomic number of first impurity is  $2z - 1$ .  
 (B) The atomic number of first impurity is  $2z + 1$ .  
 (C) The atomic number of second impurity is  $\frac{(z+1)}{2}$ .  
 (D) The atomic number of second impurity is  $\frac{z}{2} + 1$ .

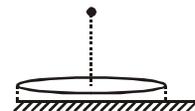
**Rough work**

**Comprehension Type**

This section contains **2 paragraphs**. Based upon one of paragraphs **2 multiple choice questions** and based on the other paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices (A), (B), (C) and (D) out of **which ONLY ONE** is correct.

**Paragraph for Question Nos. 12 to 13**

A thin biconvex lens of refractive index  $\frac{3}{2}$  is placed on a horizontal plane mirror as shown in the figure. The space between lens and the mirror is then filled with water of refractive index  $\frac{4}{3}$ . It is found that when a point object is placed 15 cm above the lens on the principal axis the object coincides with its own image.



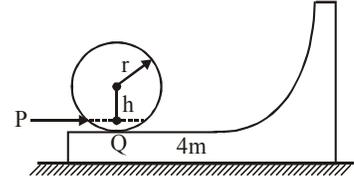
12. If the same setup is used but water is not used to fill up the space between the mirror and the lens, then where should the object be placed on the axis of the system so that its image coincides with it? Assume that the radius of curvature of the lens is R.
- (A) 1.5 R (B) R  
(C) 2R (D)  $\frac{R}{2}$
13. What is value of R from above information
- (A) R = 10 cm (B) R = 15 cm  
(C) R = 5 cm (D) R = 20 cm

**Rough work**

**Paragraph for Question Nos. 14 to 16**

If no external force is acting on the system, net linear momentum of the system is conserved. If system is acted upon by some external force, the component of momentum of the system, along which no external force is present or their vector sum is zero, is conserved. If a sharp blow is given to a body its linear momentum changes immediately. Change in angular momentum not only depends on the magnitude of the blow but also on point of application. In the case of symmetrical body we take the axis of rotation through center of the body.

A wedge of mass  $4m$  is placed at rest on a smooth horizontal surface. A uniform solid sphere of mass  $m$  and radius  $r$  is placed at rest on the flat portion of the wedge at the point  $Q$  as shown in the figure. A sharp horizontal impulse  $P$  is given to the sphere at a point below  $h = 0.4r$  from the center of the sphere. The radius of curvature of the curved portion of the wedge is  $R$ . Coefficient of friction to the left side of point  $Q$  is  $\mu$  and to the right side of point  $Q$  is zero. For a body to roll on a surface without slipping, there should be no relative velocity between the points of contact.



14. The maximum height to which the center of mass of the sphere will climb on the curved portion of the wedge is
- (A)  $\frac{2P^2}{5m^2g}$  (B)  $\frac{P^2}{5m^2g}$
- (C)  $\frac{P^2}{2m^2g}$  (D) none of these
15. Kinetic energy of the system when sphere is at the highest point is
- (A)  $\frac{P^2}{10m}$  (B)  $\frac{P^2}{5m}$
- (C)  $\frac{3P^2}{10m}$  (D)  $\frac{3P^2}{5m}$
16. Speed of the wedge when sphere reaches the flat portion again
- (A)  $\frac{2P}{5m}$  (B)  $\frac{3P}{5m}$
- (C)  $\frac{8P}{5m}$  (D)  $\frac{P}{5m}$

**Rough work**

**SECTION –C**

**Integer Answer Type**

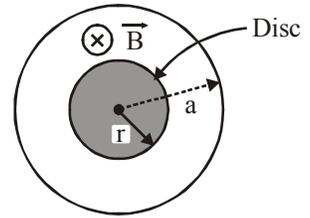
This section contains **7 questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (both inclusive).

- The equation of a transverse wave is given by  

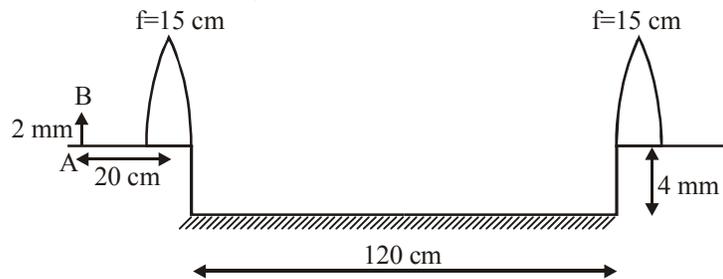
$$\psi = 10^{-2} \sin \pi[30t - x\sqrt{3} - y]$$
 where  $x$ ,  $y$  and  $\psi$  are in metre, and  $t$  in second.  
 If phase difference between two points  
 $A(2\sqrt{3}\text{m}, 2\text{m})$  and  $B(3\sqrt{3}\text{m}, 3\text{m})$  be  $n\pi$ . Find value of  $n$
- An unknown cell balances 300 cm of wire in a potentiometer circuit, but the balance length reduces to 280 cm when a resistance of  $28\Omega$  is added in parallel with the cell. A standard cell of emf 1.1 V balances 220 cm on the same potentiometer. The balance length (in cm) when  $48\Omega$  resistance is connected across the unknown cell is  $72n$ . Find the value of  $n$ .
- The diameter of a wire of length 100 cm is measured with the help of a screw gauge. The main scale reading is 1mm and circular scale is reading is 25. Pitch of the screw gauge is 1 mm and the total number of divisions on the circular scale is 100. This wire is used in an experiment of determination of Young's modulus of a wire by Searle's method. The following data are available : elongation in the wire  $l = 0.125$  cm under the tension of 50 N, least count for measuring normal length of wire is .01 cm and for elongation in the wire is .001 cm. The maximum error (in order of  $10^{-2}$ ) in the calculating value of Young's modulus ( $Y$ ), assuming that the force is measured very accurately, is  $\frac{8n}{10}\%$ , where  $n$  is very nearly an integer. Find the value of  $n$ .
- The emissivity of tungsten is approximately 0.35. A tungsten sphere 1 cm in radius is suspended within a large evacuated enclosure whose walls are at 300 K. What power input is required to maintain the sphere at a temperature of 3000 K if heat conduction along the supports is neglected? Express your answer in kW after rounding off the nearest integer. Take  $\sigma = \frac{17}{3} \times 10^{-8}$  S. I. units and  $\pi = \frac{213}{68}$ .

**Rough work**

5. A uniform disc of radius  $r$  and mass  $m$  is charged uniformly with the charge  $q$ . This disc is placed flat on a rough horizontal surface having coefficient of friction  $\mu$ . A uniform magnetic field is present in a circular region ( $a > r$ ) but varying as  $kt^3$  as shown in figure. Find the time in second after which the disc begins to rotate. (Given  $r = 1$  m,  $m = 18$  kg,  $q = 1$  C,  $\mu = 0.1$  m,  $K = 4$ ,  $g = 10$  m/s<sup>2</sup>)



6. A convex lens of focal length 15 cm is split into two halves and the two halves are placed at a separation of 120 cm. Between the two halves of convex lens a plane mirror is placed horizontally and at a distance of 4 mm below the principal axis of the lens halves. An object AB of length 2 mm is placed at a distance of 20 cm from one half lens as shown in figure. The final image of the point A is formed at a distance of  $\frac{n}{3}$  mm from the principal axis. Determine the value of  $n$ .



7. A gas consists of identical hydrogen like atoms in the lowest (ground) energy level A, and some atoms in a particular upper (excited) energy level B, but there are no atoms in any other energy level in the gas. The atoms of the gas make transitions to a higher energy level by absorbing monochromatic light of photon energy 3.0 eV. Subsequently, the atoms emit radiation of only six different photon energies. Some of the emitted photons have energy of 3.0 eV, some have more energy, and some have energy less than 3.0 eV. The ionization energy for the gas atoms is 4n eV, where  $n$  is an integer. Find  $n$ .

**Rough work**

# Chemistry

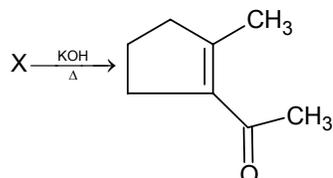
## PART – II

### SECTION – A

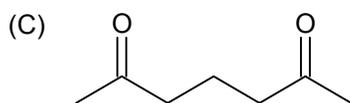
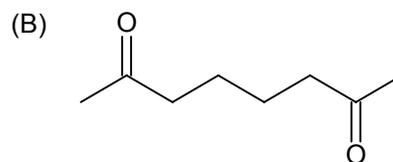
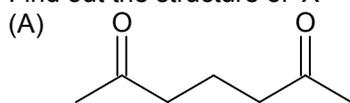
#### Straight Objective Type

This section contains 7 multiple choice questions numbered 1 to 7. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

1.

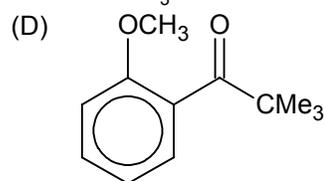
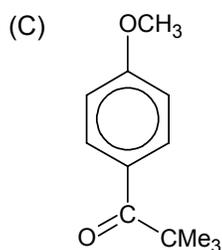
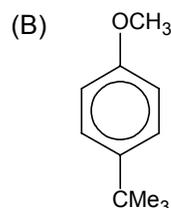
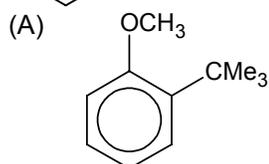
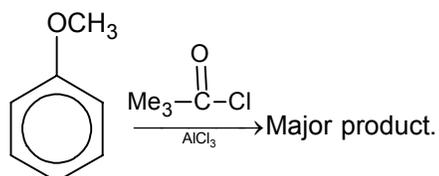


Find out the structure of 'X'

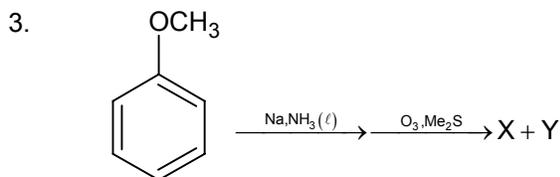


(D) None of these

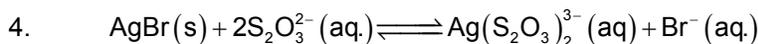
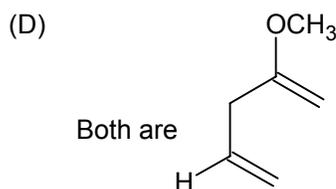
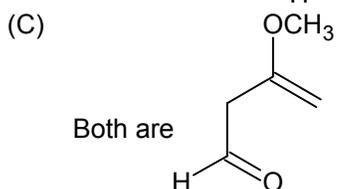
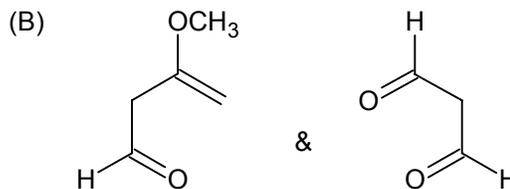
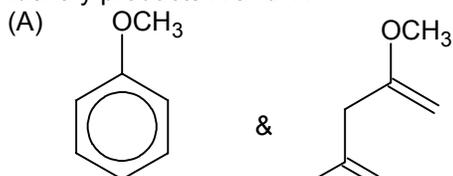
2.



*Rough Work*



Identify products X and Y.



Given  $K_{sp}(\text{AgBr}) = 5 \times 10^{-13}$ ,  $K_f(\text{Ag}(\text{S}_2\text{O}_3)_2^{3-}) = 5 \times 10^{13}$

What is the molar solubility of AgBr in 0.1 M  $\text{Na}_2\text{S}_2\text{O}_3$ ?

- (A) 0.5 M (B) 0.45 M  
(C) 0.045 M (D) None of these

5. A gas expands adiabatically at constant pressure such that

$$T \propto \frac{1}{\sqrt{V}}$$

The value of  $\gamma (C_p / C_v)$  of gas will be

- (A) 1.30 (B) 1.50  
(C) 1.70 (D) 12

6. The specific conductance of saturated solution of silver chloride is  $\kappa$  ( $\text{ohm}^{-1} \text{cm}^{-1}$ ). The limiting ionic conductance of  $\text{Ag}^+$  and  $\text{Cl}^-$  ions are  $x$  and  $y$  respectively. The solubility of AgCl in  $\text{g litre}^{-1}$  is (mol mass of AgCl =  $143.5 \text{ g mol}^{-1}$ )

- (A)  $\kappa \times \frac{1000}{(x-y)}$  (B)  $\frac{\kappa}{(x+y)} \times 143.5$   
(C)  $\frac{\kappa \times 1000 \times 143.5}{x+y}$  (D)  $\frac{(x+y)}{\kappa} \times \frac{1000}{143.5}$

7. The pair in which both species have same magnetic moment is:

- (A)  $[\text{Cr}(\text{H}_2\text{O})_6]^{2+}$ ,  $[\text{Co}(\text{Cl})_4]^{2+}$  (B)  $[\text{Cr}(\text{H}_2\text{O})_6]^{2+}$ ,  $[\text{Fe}(\text{H}_2\text{O})_6]^{2+}$   
(C)  $[\text{Mn}(\text{H}_2\text{O})_6]^{2+}$ ,  $[\text{Cr}(\text{H}_2\text{O})_6]^{2+}$  (D)  $[\text{Co}(\text{Cl})_4]^{2-}$ ,  $[\text{Fe}(\text{H}_2\text{O})_6]^{2+}$

**Rough Work**

**Multiple Correct Choice Type**

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out which **ONE OR MORE** is/are correct.

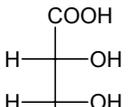
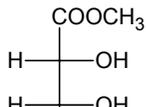
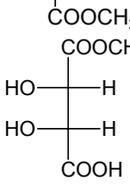
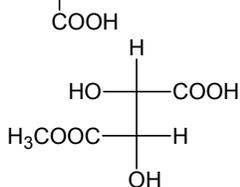
8. Which of the following possess two lone pairs at central atom and has square planar shape?

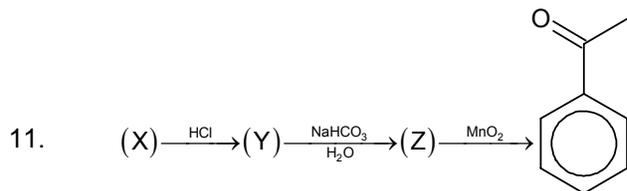
- (A)  $SF_4$  (B)  $XeO_4$   
 (C)  $XeF_4$  (D)  $ICl_4^-$

9. Select the correct statements among following:

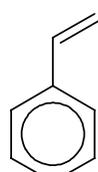
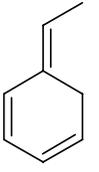
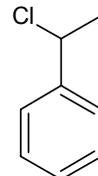
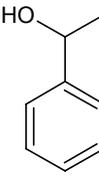
- (A) Nearest neighbour distance of NaCl =  $a/2$  (B) Nearest neighbour distance in  $CaF_2 = \frac{a\sqrt{3}}{4}$   
 (C) Nearest neighbour distance in  $Na_2O = \frac{a\sqrt{3}}{4}$  (D) Nearest neighbour distance in  $CsCl = \frac{a\sqrt{3}}{2}$   
 (a = edge length of unit cell)

10. Which of the following are identical molecules?

- (A)  (B)   
 (C)  (D) 



Which of the following compound can be a part of above reaction sequence:

- (A)  (B)   
 (C)  (D) 

**Rough Work**

**Comprehension Type**

This section contains **2 paragraphs**. Based upon the first paragraph 2 multiple choice questions and based upon the second paragraph **3 multiple choice questions** have to be answered. Each of these questions has four choices A), B), C) and D) out of **WHICH ONLY ONE CORRECT**.

**Paragraph for Question Nos. 12 to 13**

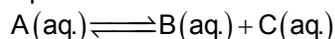
The compound  $X_2(g)$  decomposed to its monomer  $X(g)$  at high temperature. The decomposition follows first order kinetics. When certain amount of dimer is rapidly introduced into the flask at  $127^\circ\text{C}$ ; after 693 secs the pressure in the vessel observed was 30.0 mm and at the very time observed pressure was 40.0 mm. When the same experiment is carried out at  $227^\circ\text{C}$ ; the pressure after 220 secs observed was 75.0 mm and after a very long time it was observed 90.0 mm. [ $\log 2 = 0.3010$ ,  $\log 3 = 0.4771$ ]

12. The rate constant of the reaction at  $127^\circ\text{C}$  is  
(A)  $1 \times 10^{-3} \text{ min}^{-1}$  (B)  $2 \times 10^{-3} \text{ min}^{-1}$   
(C)  $5 \times 10^{-3} \text{ sec}^{-1}$  (D)  $1 \times 10^{-3} \text{ sec}^{-1}$
13. The partial pressure of  $X_2(g)$  at  $227^\circ\text{C}$ , at 220 sec was:  
(A) 15 mm (B) 45 mm  
(C) 37.5 mm (D) 75 mm

**Rough Work**

**Paragraph for Question Nos. 14 to 16**

When a non-volatile solute (A) dissolves in water, it begins to dissociate slowly to establish following equilibrium:



The progress of the reaction was followed determining the depression in freezing point. When 2 gm of A was dissolved in 500 gm of water the depression in freezing point was  $0.2^\circ$  at zero time and  $0.25^\circ$  after 30 minutes from the time of dissolution at which equilibrium established. The back ward reaction is however prevented completely by the presence of another solute "X" isomeric with A. When 1 gm "X" is dissolved together with 2 g of A in 500 gm water the depression in freezing point after 30 minutes from the point of dissolution was  $0.4^\circ$ . ( $K_f = 1.86$ )

14. Value of equilibrium constant of reaction

$A(aq.) \rightleftharpoons B(aq.) + C(aq.)$  is:

- (A)  $8.96 \times 10^{-3}$  (B)  $8.33 \times 10^{-2}$   
 (C)  $8.96 \times 10^{-2}$  (D)  $8.33 \times 10^{-4}$

15. How we can calculate rate constant ( $K_f$ ) of reaction

$A(aq.) \longrightarrow B(aq.) + C(aq.)$ , if we do not add "X"?

- (A)  $\frac{-d[A]}{dt} = K_f [A]$ , here  $\frac{-d[A]}{dt} = \frac{0.25}{2 \times 30}$  and  $[A] = \frac{0.75}{37.2}$ , at 30 mins. therefore, we can calculate  $K_f$

- (B) can be calculate by eq.  $K_f = \frac{2.303}{t} \log \frac{A_o}{A_t}$ , as we know the values of  $A_o$ (initial moles) and  $A_t$  at 30 mins.

- (C)  $K_f = 8.33 \times 10^{-2}$

- (D) can not calculate as rate constant of back ward reaction is not known

16. Rate constant of the reaction between B and C ( $K_b$ ) is:

- (A) can not calculate

- (B) as we calculate  $K_f$  according to previous question, we know the value of equilibrium constant

$$K_{eq}, K_b = \frac{K_f}{K_{eq}}$$

- (C)  $2.58 \text{ min}^{-1}$

- (D)  $9.3 \times 10^1 \text{ min}^{-1}$

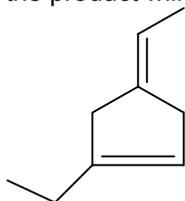
**Rough Work**

## SECTION – C

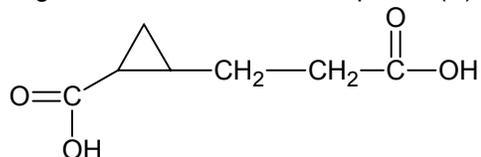
## Integer Answer Type

This section contains **7 questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (both inclusive).

1. A 0.0020 m aqueous solution of an ionic compound  $\text{Co}(\text{NH}_3)_5\text{NO}_2$  freezes at  $-0.00732^\circ\text{C}$ . What will be the number of moles of ions which 1 mol of ionic compound produces ( $K_f = -1.86^\circ\text{C/m}$ ).
2. What will be the mass of glucose that should be dissolved in 50 g of water in order to produce the same lowering of vapour pressure as produced by dissolving 1 g of urea in the same quantity of water?
3. If the following compound is treated with Pd/C in excess of  $\text{H}_2$  gas, how many stereoisomers of the product will be obtained?



4. A hydrocarbon (X) on oxidative ozonolysis gives following compound, then what will be the degree of unsaturation in compound (X):



*Rough Work*

5.  $2\text{SO}_2 + \text{O}_2 \rightleftharpoons 2\text{SO}_3$   
 Starting with 2 mol of  $\text{SO}_2$  and 1 mol of  $\text{O}_2$  in 1-L flask, mixture required 0.4 mol of  $\text{MnO}_4^-$  in acidic medium. Find the  $K_c$  value.
6. Temperature of one mole of helium gas is increased by  $1^\circ\text{C}$ , find the increase in internal energy in cal.
7. The equilibrium constant for the reaction:  
 $\text{H}_3\text{BO}_3 + \text{Glycerine} \rightleftharpoons \text{complex}$  is 0.90. If  $x/3$  moles of glycerine is added to one litre of 0.10 M  $\text{H}_3\text{BO}_3$  solution to convert 60% of  $\text{H}_3\text{BO}_3$  in to complex, what must be value of 'x'?
- 

*Rough Work*

**Mathematics****PART – III****SECTION – A****Straight Objective Type**

This section contains **7 multiple choice questions** numbered 1 to 7. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

1. A function  $F(x)$  satisfies  $f(x) = \sin x + \int_0^x f'(t)(2\sin t - \sin^2 t) dt$ , then  $f(x)$  is/are
- (A)  $\frac{x}{1 - \sin x}$  (B)  $\frac{\sin x}{1 - \sin x}$   
 (C)  $\frac{1 - \cos x}{\cos x}$  (D)  $\frac{\tan x}{1 - \sin x}$
2. If A and B are different matrices satisfying  $A^3 = B^3$  and  $A^2B = B^2A$ , then
- (A)  $\det(A^2 + B^2)$  must be zero (B)  $\det(A - B)$  must be zero  
 (C)  $\det(A^2 + B^2)$  as well as  $\det(A - B)$  must be zero  
 (D) at least one of  $\det(A^2 + B^2)$  or  $\det(A - B)$  must be zero
3. Suppose we are given a point P on the Argand plane represented by the complex number Z, moving anticlockwise along the circle with  $|z| = 2$  from the point of complex number 2 to 2i. If  $\omega = \frac{z+1}{z+2}$ , then the path described by Q which represents the complex number  $\omega$  is/are
- (A) a line with gradient  $\frac{1}{3}$  passing through the origin  
 (B) a circle  
 (C) a segment of line  $4x - 3 = 0$  with y varying from  $\left[0, \frac{1}{4}\right]$   
 (D) a line  $4x - 3 = 0$  with y varying from  $[0, \infty)$

**Rough work**

4. The line which contains all points  $(x, y, z)$  which are of the form  $x\hat{i} + y\hat{j} + z\hat{k} = 2\hat{i} - 2\hat{j} + 5\hat{k} + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$  intersects the plane  $2x - 3y + 4z = 163$  at P and intersects the YZ plane at Q. If the distance PQ is  $a\sqrt{b}$  where  $a, b \in \mathbb{N}$  and  $a > 3$  then  $(a + b)$  equals  
 (A) 23 (B) 95  
 (C) 27 (D) none of these
5. The right hand derivative of  $f(x - 1) = 2x^2 - 3x + 1$  at  $x = k$  where  $k \in \mathbb{I}$ , is/are  
 (A)  $4k + 1$  (B)  $2k + 1$   
 (C)  $2k - 1$  (D)  $4k - 3$
6. The number 916238457 is an example of nine digit number which contains each of the digit 1 to 9 exactly one. It also has the property that the digit 1 to 5 occur in their natural order, while the digit 1 to 6 do not. Number of such numbers are  
 (A) 2268 (B) 2520  
 (C) 2975 (D) 1560
7. The value of  $\lim_{x \rightarrow \infty} \frac{d}{dx} \int_{\sqrt{3}}^{\sqrt{x}} \frac{r^3}{(r+1)(r-1)} dr$ , is  
 (A) 0 (B) 1  
 (C)  $\frac{1}{2}$  (D) non existent

**Multiple Correct Answer(s) Type**

This section contains **4 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct**.

8. The function defined as  $f(x) = \lim_{n \rightarrow \infty} \begin{cases} \cos^{2n} x & \text{if } x < 0 \\ \sqrt[n]{1+x^n} & \text{if } 0 \leq x \leq 1, \\ \frac{1}{1+x^n} & \text{if } x > 1 \end{cases}$ , which of the following does not hold good?  
 (A) continuous at  $x = 0$  but discontinuous at  $x = 1$   
 (B) continuous at  $x = 1$  but discontinuous at  $x = 0$   
 (C) continuous both at  $x = 1$  and  $x = 0$   
 (D) discontinuous both at  $x = 1$  and  $x = 0$

**Rough work**



13. Visible area of the point Q(11, 0) in square units is  
 (A) 20 (B) 30  
 (C) 40 (D) none of these

**Paragraph for Question Nos. 14 to 16**

**Read the following write up carefully and answer the following questions:**

While finding the sine of a certain angle  $x$ , an absent minded professor failed to notice that his calculator was not in the correct angular mode. However he was lucky to get the right answer. The two least positive values of  $x$  for which the Sine of  $x$  degrees is the same as the Sine of  $x$  radians were found by him as

$\frac{m\pi}{n-\pi}$  and  $\frac{p\pi}{q+\pi}$  where  $m, n, p$  and  $q$  are positive integers. Suppose  $\frac{mn}{pq}$  be denoted by the quantity 'L'.

Now answer the following questions:

14. The value of  $(m + n + p + q)$  is equal to  
 (A) 720 (B) 900  
 (C) 1080 (D) 1260
15. If  $x$  is measured in radians and  $\lim_{x \rightarrow \infty} (\sqrt{Ax^2 + Bx} - Cx) = L$ , the value of  $\frac{BC}{A}$  equals ( $A, B, C \in \mathbb{R}$ )  
 (A) 4 (B) 2  
 (C)  $\frac{1}{2}$  (D) none of these
16. Assume that  $f$  is differentiable for all  $x$ . The sign of  $f$  is as follows:  
 $f'(x) > 0$  on  $(-\infty, -4)$   
 $f'(x) < 0$  on  $(-4, 6)$   
 $f'(x) > 0$  on  $(6, \infty)$   
 Let  $g(x) = f(10 - 2x)$ . The value of  $g'(L)$  is  
 (A) positive (B) negative  
 (C) zero (D) the function  $g$  is not differentiable at  $x = 5$

**Rough work**

## SECTION – C

## Integer Answer Type

This section contains **7 questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (both inclusive).

- Number of zeros of the cubic  $f(x) = x^3 + 2x + k \forall k \in \mathbb{R}$ , is \_\_\_\_\_.
- Number of real solution of equation  $16\sin^{-1}x \tan^{-1}x \operatorname{cosec}^{-1}x = \pi^3$  is/are \_\_\_\_\_.
- The digit at the unit place of the number  $(2003)^{2003}$  is \_\_\_\_\_.
- A circle of radius 320 units is tangent to the inside of a circle of radius 1000. The smaller circle is tangent to a diameter of the larger circle at the point P. Least distance of the point P from the circumference of the larger circle is K then  $\frac{K}{100}$  is equal to \_\_\_\_\_.
- P is an orthogonal matrix and  $A^{4n} = A^4$  &  $A^{5n} = A$  ( $n \in \mathbb{I}$ ) and  $Q = PAP^T$  then  $X = P^T Q^{2005} P$  will be equal to  $A^k$ , then k is equal to \_\_\_\_\_.
- The degree of the differential equations of the family of ellipse centred at origin; principal axis along co-ordinate axis and of length  $\sqrt{a+k}$  and  $\sqrt{b+k}$  where a, b are fixed real numbers and k is a real parameter is \_\_\_\_\_.
- Given  $f(x) = \frac{x^4 - 7x^2 + 9}{x - (3/x) + 1}$ . Its zeroes are of the form  $\frac{a \pm \sqrt{b}}{c}$ , where a, b and c are positive integers. Then the value of  $\frac{(a + b + c)}{4}$ , is \_\_\_\_\_.

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**Rough work**