ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) — path to success

DISTANCE LEARNING PROGRAMME

(Academic Session : 2018 - 2019)

LEADER TEST SERIES / JOINT PACKAGE COURSE

TARGET : JEE (Main + Advanced) 2019

Test Type : MAJOR Test Pattern : JEE-Advanced

TEST # 10 **TEST DATE : 14 - 04 - 2019**

PAPER-1

PART-1 : PHYSICS **ANSWER KEY**

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,C	B,C,D	A,C	A,D	A,C	A,C,D	A,B,C	B	C	D
	Q.	11	12	13							
	A.	B	C	D							
SECTION-III	Q.	1	2	3	4	5					
	A.	7	4	7	4	3					

SOLUTION

SECTION-I

1. Ans. (A,C)

Sol. $E = 13.6 \times 4\left(1-\frac{1}{4}\right) = 40.8\ \text{eV}$

$\frac{h}{\lambda} = mv$

$\frac{h\nu}{C} = mv$

$\frac{p^2}{2m} = \frac{h^2\nu^2}{2mc^2}$

2. Ans. (B, C, D)

Sol. $R = \frac{20^2 \times \sin 120^\circ}{g} = 20\sqrt{3} = \frac{\Delta R}{R} = \frac{24U}{U}$

$\Rightarrow \Delta R = \frac{2 \times 5}{100} \times 200\sqrt{3} = 2\sqrt{3}$

$20\sqrt{3} - 2\sqrt{3} < R < 20\sqrt{3} + 2\sqrt{3}$

$\Rightarrow 31.1\text{m} < R < 38.1\ \text{m}$

3. Ans. (A, C)

Sol. At O

$\Delta x_1 = d \sin\theta = 1 \times \sin 30 = \frac{1}{2}\text{mm}$

$\Delta x_1 = n\lambda \Rightarrow$ constructive interference.

$\Rightarrow I = 4\,I_0$

At 4 mm above O

$\Delta x = \Delta x_1 + \Delta x_2$

$= \frac{1}{2}\text{mm} + 1 \times \frac{4}{2\times 10^3}\text{mm}$

$\Delta x = \frac{1}{2}\text{mm} + \frac{2}{10^3}\text{mm} = n\lambda$

$\Rightarrow$ constructive interference

$I = 4I_0$

4. Ans. (A,D)

Sol. $P_1 + \rho g h_1 + \frac{1}{2}\rho V_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho V_2^2$

$h_1 = h_2 = 0$ (horizontal pipe)

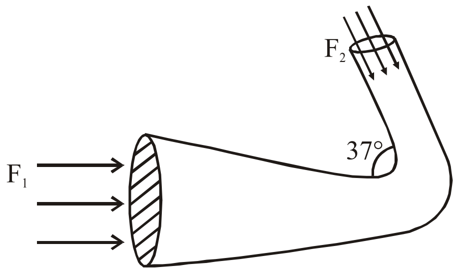
& $A_1V_1 = A_2\,V_2$

$4\,(4) = V_2$

$16 = V_2$

$2.80\times 10^5 + \frac{1}{2}\rho(4)^2 = P_2 + \frac{1}{2}\rho(16)^2$

$2.80\times 10^5 + \frac{1}{2}\rho(16-256) = P_2$



$$2.80\times10^5+\frac{1}{2}900(-240)=P_2$$

$172\times10^3\ N/m^2=P_2$

$F_1=P_1A_1=56\times10^3$

$F_2=P_2A_2=8.6\times10^3$

$$\vec F_1+\vec F_2+\vec F_{pipe}=\frac{d\vec P_{oil}}{dt}$$

$$\vec F_{pipe}=\frac{d\vec P_{oil}}{dt}-\left(\vec F_1+\vec F_2\right)$$

$$\vec F_1=P_1A_1\hat i$$

$$\vec F_2=P_2A_2\left(\cos37^\circ\hat i-\sin37^\circ\hat j\right)$$

$$\frac{d\vec P_{oil}}{dt}=\left(\frac{dm}{dt}\right)\Delta\vec V$$

$$=(\rho A_1V_1)\left(\vec V_2-\vec V_1\right)$$

$$\vec V_2=16\left(\cos37^\circ(-\hat i)+\sin37^\circ(\hat j)\right)$$

$$\vec V_1=4\hat i$$

Solving this for F_{pipe} we get, $|F|=76\times10^3$ N

5. Ans. (A,C)

Sol. Area of PV curve in process 1 > area of PV curve in process 2

Means $w_{process\ 1}>w_{process\ 2}$

As final volume in both process is same $P\propto T$

$T_{final\ process\ 2}<T_{final\ process\ 1}$

6. Ans. (A, C, D)

Sol. $I\vec\alpha=\vec\tau$

$\vec\tau=\vec\mu\times\vec B$

$$\vec\mu=\pi R^2\times I\left(-\hat k\right)\ \&\ I=\frac{MR^2}{2}$$

7. Ans. (A,B,C)

Sol. $v_0=\sqrt{2gh}=\sqrt{2g\ell(1-\cos\theta)}$

$$v_0=\sqrt{4g\ell\sin^2\left(\frac{\theta}{2}\right)}$$

$$T_{after}=w+\frac{mv_0^2}{\ell}$$

8. Ans. (B)

Sol. $y=A\sin\left(2\pi\nu t-\frac{2\pi}{v}\nu x+\phi\right)$

$$\frac{A}{2}=A\sin\left(0-\frac{2\pi}{3}+\phi\right)$$

$$\Rightarrow\frac{\pi}{6}=-\frac{2\pi}{3}+\phi$$

$$\phi=\frac{\pi}{6}+\frac{2\pi}{3}$$

$$\phi=\frac{5\pi}{6}$$

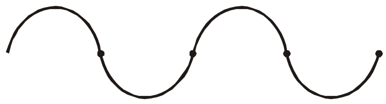
$$y=A\sin\left(2\pi\nu t-\frac{2\pi}{v}\nu x+\frac{5\pi}{6}\right)$$

$$E=\frac{\mu A^2\omega^2}{2}\times\lambda=2\pi^2\mu A^2\nu^2\lambda$$

$$P=\frac{\mu vA^2\omega^2}{2}=2\pi^2\mu A^2\nu^2v$$

9. Ans. (C)

Sol.



$$y=A\sin(kx)\cos(\omega t)=A\sin\left(\frac{2\pi}{\ell}\times2x\right)\cos(\omega t)$$

$$\ell=2\lambda\Rightarrow\lambda=\frac{\ell}{2}$$

$$y=A\sin\left(\frac{4\pi}{\ell}x\right)\cos(\omega t)$$

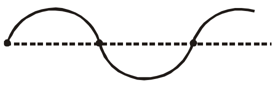
$$E=\int_0^\lambda\frac{1}{2}(dm)(\omega^2)\left(A^2\sin^2\left(\frac{4\pi}{\ell}x\right)\right)$$

$$=\frac{1}{2}\mu\omega^2A^2\int_0^\lambda\sin^2\left(\frac{4\pi}{\ell}x\right)dx$$

$$=\frac{1}{2}\mu\omega^2A^2\left[\frac{1}{2}\int_0^\lambda1-\cos\left(\frac{8\pi x}{\ell}\right)dx\right]$$

$$=\frac{1}{2}\mu\omega^2A^2\frac{\lambda}{2}=\frac{\mu\omega^2A^2\lambda}{4}$$

10. Ans. (D)

Sol.  $\ell = \frac{5\lambda}{4}$

$y = A\sin(kx)\cos(\omega t)$

$$y = A\sin\left(\frac{2\pi}{4\ell}\times 5x\right)\cos(\omega t)$$

$$= A\sin\left(\frac{5\pi}{2\ell}x\right)\cos(\omega t)$$

$$E = \int \frac{1}{2}(\mu dx)\omega^2 A^2 \cos^2\left(\frac{5\pi}{2\ell}x\right)$$

$$= \frac{1}{2}\mu\omega^2 A^2 \int_0^{\lambda} \cos^2\left(\frac{5\pi}{2\ell}x\right)dx$$

$$= \frac{1}{2}\mu\omega^2 A^2 \frac{1}{2}\cdot\lambda = \frac{1}{4}\mu\omega^2 A^2\lambda$$

11. Ans. (B)

12. Ans. (C)

13. Ans. (D)

Sol. (Q. 11 to 13)

Time after slipping ceases

$$t = \frac{2v_0 m}{\mu mg\left(1 + \frac{mR^2}{I}\right)}$$

$I_{Ring} > I_{Sphere} > I_{Disk} > I_{Solid\ sphere}$

$$KE_i = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv_0^2 + \frac{1}{2}\frac{Iv_0^2}{R^2}$$

$$\Rightarrow KE_i = \frac{1}{2}v_0^2\left[m + \frac{I}{R^2}\right]$$

$$\Rightarrow KE_f = \frac{1}{2}v^2\left[m + \frac{I}{R^2}\right]$$

from angular momentum conservation (LAMC)

$$v = \frac{(mR^2 - I)v_0}{(mR^2 + I)}$$

SECTION-III

1. Ans. 7

Sol. $\lambda_{min} = \frac{hc}{eV} = \frac{6.63\times10^{-34}\times3\times10^8}{1.6\times10^{-19}\times20\times10^3} = 0.62\,\text{Å}$

$$\left(\text{OR } \lambda_{min} = \frac{hc}{eV} = \frac{12400}{20000} = 0.62\text{Å}\right)$$

Also, $\frac{1}{\lambda_K} = R(Z-1)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$$\Rightarrow \frac{1}{\lambda_K} = 1.09\times10^7(41-1)^2\left[1 - \frac{1}{4}\right] \Rightarrow \lambda_K = 0.76\,\text{Å}$$

Now,

$$\lambda_K - \lambda_{min} = 0.76 - 0.62 = 0.14\,\text{Å}$$

$$= 0.14\times10^{-10}\text{m} = 14\times10^{-12}\text{m}$$

2. Ans. 4

Sol. $f = \frac{v}{2(\ell + 2e)}$ where e = end correction = 0.6 r

$$\therefore f = \frac{v}{2(\ell + 2\times0.6r)} = \frac{v}{2(\ell + 1.2r)}$$

$$\therefore \frac{\Delta f}{f} = \frac{\Delta v}{v} - \frac{\Delta(\ell + 1.2r)}{\ell + 1.2r}$$

$$= \frac{\Delta v}{v} - \frac{\Delta\ell + 1.2\Delta r}{\ell + 1.2r}$$

here $\frac{\Delta v}{v}$ = 0 (given) $\frac{\Delta f}{f}\times 100$

$$= -\frac{\Delta\ell + 1.2\Delta r}{\ell + 1.2r}\times 100$$

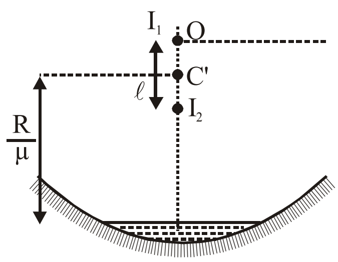
for maximum % error : $\Delta\ell = 0.1$, $\Delta r = 0.05$

$$\left(\frac{\Delta f}{f}\times100\right)_{max} = \frac{0.1 + 1.2\times0.05}{94 + 1.2\times5}\times 100$$

$$= 0.16\%$$

3. Ans. 7

Sol.



On pouring water the new center of curvature is at R/μ.

$$\frac{1}{v}+\frac{1}{-R}=\frac{2\mu}{-R}$$

$$\therefore \quad v=\frac{R}{1-2\mu} \text{ for } I_2;\ v=-5$$

$\mu = 1.4$

4. Ans. 4

Sol. $\Delta P = B\frac{\Delta V}{V} \Rightarrow \frac{2T}{R}=B\frac{\Delta V}{V}$

$$V=\frac{4}{3}\pi R^3$$

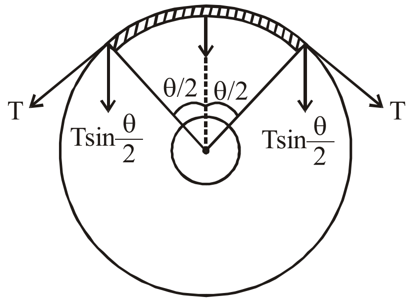
$$\frac{\Delta V}{V}=\frac{3\Delta R}{R}$$

$$\Rightarrow \frac{2T}{R}=B\left(\frac{3\Delta R}{R}\right)$$

$$\Rightarrow \Delta R=\frac{2T}{3B}=\frac{2\times 0.075}{3\times 1.25\times 10^8}=4\text{Å}$$

5. Ans. 3

Sol.



$$2T\sin\frac{\theta}{2}+\frac{GM(dm)}{(3R)^2}=(dm)\omega^2(3R)$$

$$\Rightarrow T\theta+\frac{GM\left(\frac{m}{2\pi}\cdot\theta\right)}{9R^2}=\left(\frac{m\theta}{2\pi}\right)\omega^2(3R)$$

$$T=\frac{m}{2\pi}\left[\frac{GM}{9R^3}\cdot 3R-\frac{GM}{9R^2}\right]$$

$$T=\frac{GMm}{9\pi R^2}$$

PART-2 : CHEMISTRY — ANSWER KEY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,B,C	B,D	B,C	B,C	B,D	A,B,C,D	B,C	B	C	D
	Q.	11	12	13							
	A.	B	A	A							
SECTION-III	Q.	1	2	3	4	5					
	A.	5	6	3	2	2					

SOLUTION

SECTION–I

1. Ans. (A,B,C)

2. Ans. (B, D)

Sol. $MnO_4^- + 5e^- + 8H^+ \rightarrow Mn^{+2} + 4H_2O$

$$E=E^0-\frac{.059}{5}\log\frac{[Mn^{+2}]}{[MnO_4^-]}+\frac{.059}{5}\log[H^+]^8$$

$$E=E^0-\frac{.059}{5}\log\frac{[Mn^{+2}]}{[MnO_4^-]}-\frac{.059}{5}(8)\text{ pH}$$

$Cr_2O_7^{2-} + 6e^- + 14H^+ \rightarrow 2Cr^{+3} + 7H_2O$

$$E=E^0-\frac{.059}{6}\log\frac{[Cr^{+3}]^2}{[Cr_2O_7^{-2}]}-\frac{.059}{6}(14)\text{ pH}$$

3. Ans. (B,C)
4. Ans. (B,C)
5. Ans. (B,D)
6. Ans. (A,B,C,D)
7. Ans. (B,C)
8. Ans. (B)
9. Ans. (C)
10. Ans. (D)
11. Ans. (B)
12. Ans. (A)
13. Ans. (A)

SECTION–III

1. Ans. (5)
2. Ans. (6)
3. Ans. (3)
4. Ans. (2)
5. Ans. (2)

PART-3 : MATHEMATICS

ANSWER KEY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,B,C	B,D	A,B,C	B	A,C,D	A,C,D	A,B,C,D	B	A	C
	Q.	11	12	13							
	A.	C	A	D							
SECTION-III	Q.	1	2	3	4	5					
	A.	2	2	3	3	3					

SOLUTION

SECTION–I

1. **Ans. (A,B,C)**

f is derivable

$\Rightarrow f$ is continuous

$f(2) = 6$

$f(3) = 9 \Rightarrow$ From IMVT

$f(x) = 8$ atleast once in (2, 3)

Further let $g(x) = f(x) - 3x$

$g(1) = 0$

$g(2) = 0$

$g(3) = 0$

$\Rightarrow g'(x) = 0$ once in (1, 2) & once in (2, 3) from Rolle's theorem

$\Rightarrow g'(x) = f'(x) - 3 = 0$ twice in (1, 3)

$\Rightarrow f'(x) = 3$ twice say at $x = a$ & $x = b$ in (1, 3)

$\therefore f''(x) = 0$ atleast once in (1, 3)

2. **Ans. (B,D)**

$\because \alpha z^3 + \beta z^2 + \gamma = 3 \Rightarrow |\alpha z^3 + \beta z^2 + \gamma| = 3$

$\because |z_1 + z_2 + z_3| \le |z_1| + |z_2| + |z_3|$

$\therefore 3 \le |z^3| + |z^2| + 1$ ($\because |\alpha|, |\beta|, |\gamma| \le 1$)

$\Rightarrow |z|^3 + |z|^2 - 2 \ge 0$

$(|z|-1)\underbrace{(|z|^2 + 2|z| + 2)}_{+ve} \ge 0 \quad \Rightarrow |z| \ge 1$

3. **Ans. (A,B,C)**

Let $H(x) = f(x) - f\left(x + \frac{1}{4}\right)$

$$\left.\begin{array}{l} H(0) = f(0) - f\left(\frac{1}{4}\right) \\ H\left(\frac{1}{4}\right) = f\left(\frac{1}{4}\right) - f\left(\frac{1}{2}\right) \end{array}\right\}$$ $H(0)$ & $H\left(\frac{1}{4}\right)$ are of opposite sign

Similarly $H(x) = 0$ has one root in $\left[\frac{1}{2}, \frac{3}{4}\right]$

Hence (A,B,C)

4. **Ans. (B)**

$$f(x) = \int_2^x \sin(t^3)dt$$

$$f(x^4) = \int_2^{x^4} \sin(t^3)dt \quad \text{...(i)}$$

$$\lambda = \int_5^{x^4} \sin t^3 dt$$

$$-\lambda = \int_{x^4}^5 \sin(t^3)dt \quad \text{...(ii)}$$

adding (i) + (ii) $f(x^4) - \lambda = \int_2^5 \sin t^3 dt$

$f(x^4) - \lambda = f(5)$

$\lambda = f(x^4) - f(5)$

5. **Ans. (A,C,D)**

$4a^2 + 9b^2 + c^2 = 2a^2 + 6ab + 2ac$

$\Rightarrow 2a^2 - 6ab + 9b^2 + c^2 - 2ac = 0$

$\Rightarrow (a - 3b)^2 + (a - c)^2 = 0$

$\Rightarrow a = 3b = c \quad \therefore \Delta ABC$ is isosceles.

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{18b^2 - b^2}{2.3b.3b} = \frac{17}{18}$$

$$\therefore \angle B = \cos^{-1}\left(\frac{17}{18}\right)$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{b^2}{2b.3b} = \frac{1}{6}$$

$$\therefore \sin A = \frac{\sqrt{35}}{6} \Rightarrow \angle A = \sin^{-1}\left(\frac{\sqrt{35}}{6}\right).$$

6. Ans. (A,C,D)

$$\lim_{x\to 0^+}\left[1+x+\frac{f(x)}{x}\right]^{1/x}=e^3 \Rightarrow e^{\lim_{x\to 0^+}\left(x+\frac{f(x)}{x}\right)\frac{1}{x}}=e^3$$

where $\lim_{x\to 0^+}\frac{f(x)}{x}=0$

$$\Rightarrow \lim_{x\to 0^+}\left(1+\frac{f(x)}{x^2}\right)=3 \Rightarrow \lim_{x\to 0^+}\frac{f(x)}{x^2}=2$$

Now $\lim_{x\to 0^+}\left[1+\frac{f(x)}{\tan^2 x}\right]^x = (1+2)^0=1$

and $\lim_{x\to\infty}\left(1+xf\left(\frac{1}{x}\right)\right)^x = e^{\lim_{x\to\infty} xf\left(\frac{1}{x}\right).x} = e^{\lim_{x\to\infty} x^2 f\left(\frac{1}{x}\right)}$

$$= e^{\lim_{x\to 0^+}\frac{f(x)}{x^2}}=e^2$$

Also $\lim_{x\to 0^+}\left(1+(+1)^{\frac{f(x)}{x}}\right)=1+(+1)^0=2$

7. Ans. (A,B,C,D)

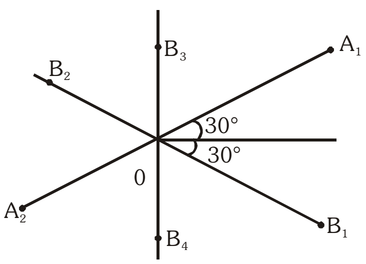
These are possible triangles

$O\,A_1\,B_1$

$O\,A_1\,B_3$

$O\,A_2\,B_2$

$O\,B_1\,B_4$



$\therefore\quad B_3 \equiv (0, a)$

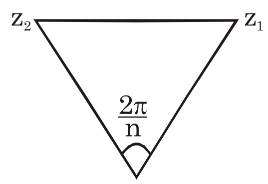
$B_4 \equiv (0, -a)$

$B_2 \equiv \frac{\sqrt{3}a}{2}, -\frac{a}{2}$

$B_1 \equiv \left(-\frac{\sqrt{3}}{2}a, \frac{a}{2}\right)$

Solution for Q.8 to 10

$z^n - 1 = 0$ has roots $z_1, z_2, \ldots\ldots z_n$



$|z_i - z_j|_{min} = 2\sin\left(\frac{\pi}{n}\right)$

$n = 4,\ |z_i - z_j|_{min} = 2\sin\left(\frac{\pi}{4}\right)=\sqrt{2}$

$n = 6,\ |z_i - z_j|_{min} = 2\sin\left(\frac{\pi}{6}\right)=1$

$n = 18,\ |z_i - z_j|_{min} = 2\sin\left(\frac{\pi}{18}\right)$

$n = 12,\ |z_i - z_j|_{min} = 2\sin\left(\frac{\pi}{12}\right)=\sqrt{2-\sqrt{3}}$

Area of polygon $= \frac{n}{2}\sin\left(\frac{2\pi}{n}\right)$

$n = 4$, Area $= \frac{4}{2}\times\sin\left(\frac{2\pi}{4}\right)=2$

$n = 6$, Area $= \frac{6}{2}\times\sin\left(\frac{2\pi}{6}\right)=3\times\frac{\sqrt{3}}{2}$

$n = 18$, Area $= \frac{18}{2}\times\sin\left(\frac{2\pi}{6}\right)$

$n = 12$, Area $= \frac{12}{2}\times\sin\left(\frac{2\pi}{12}\right)=6\times\frac{1}{2}=3$

8. Ans. (B)

9. Ans. (A)

10. Ans. (C)

Solutions for Q.11 to 13

(I) $\frac{dy}{dx}+\frac{2}{x}y=x-3$

$\Rightarrow\ yx^2=\int x^2(x-3)\,dx$

$\Rightarrow\ yx^2=\frac{x^4}{4}-x^3+c$

$\because\ x = 0,\ y = 4 \Rightarrow c = 0$

$\therefore\ y=\frac{x^2}{4}-x$

$A=\left|\int_0^1 \frac{x^2}{4}-x\,dx\right|=\frac{5}{12}$ (i)

$\left.\frac{dy}{dx}\right|_{x=2}=\left.\frac{x}{2}-1\right|_{x=2}=0$ (R)

(II) $\frac{y''}{y'}=\frac{2x}{x^2+1}$

$\Rightarrow \ell ny' = \ell n(x^2 + 1) + \ell nc$

$\Rightarrow y' = c(x^2 + 1)$

$\because\ y'(0) = 3 \Rightarrow c = 3$

$\therefore\ y' = 3(x^2 + 1) \Rightarrow y = x^3 + 3x + c_1$

$\because\ y(0) = 1 \Rightarrow c_1 = 1\ \therefore\ y = x^3 + 3x + 1$

$A=\int_0^1 x^3+3x+1\,dx=\frac{11}{4}$ (iv)

$\left.\frac{dy}{dx}\right|_{x=2}=3(x^2+1)\Big|_{x=2}=15$ (S)

(III) $ydx - xdy = x^2(xdy + ydx)$

$-d\left(\frac{y}{x}\right)=d(xy)$

$\Rightarrow\ -\frac{y}{x}=xy+c$

$\because\ x = 1,\ y = 1, \Rightarrow c = -2$

$\Rightarrow\ y=\frac{2x}{x^2+1}$

$A=\int_0^1 \frac{2x}{x^2+1}\,dx=\ell n(x^2+1)\Big|_0^1 = \ell n2$ (ii)

$y'\Big|_{x=2}=\left.\frac{2-2x^2}{(x^2+1)^2}\right|_{x=2}=\frac{-6}{25}$ (Q)

(IV) $\frac{1}{y^2}\frac{dy}{dx}+\frac{1}{y}.\frac{1}{x}=\frac{1}{x^2}$

put $\frac{1}{y}=z\ \Rightarrow\ -\frac{1}{y^2}\frac{dy}{dx}=\frac{dz}{dx}$

$\Rightarrow\ -\frac{dz}{dx}+\frac{1}{x}.z=\frac{1}{z^2}\ \Rightarrow\ \frac{dz}{dx}+\left(-\frac{1}{x}\right)z=\frac{-1}{x^2}$

$\Rightarrow\ \frac{z}{x}=-\int\frac{1}{x^3}dx+C=\frac{1}{x}+C$

$\Rightarrow\ C=\frac{1}{2}$

$\therefore\ y=\frac{2x}{x^2+1}$ $\qquad y'=\frac{-6}{25}$ (Q)

$A=\int_0^1\frac{2x}{x^2+1}\,dx=\ell n2$ (ii)

11. Ans. (C)

12. Ans. (A)

13. Ans. (D)

SECTION–III

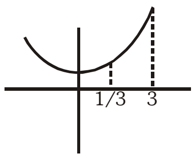
1. Ans. 2

$x^2y + xy + y = x^2 - x + 1$

$x^2(y-1) + x(y + 1) + (y - 1) = 0 \quad (x \in R)$

$D \geq 0 \quad (y + 1)^2 - 4(y - 1)^2 \geq 0$

$(y-3)(3y-1) \leq 0$



$\frac{1}{3} \leq y \leq 3$

for $z = y^2 + 2y + 3$

$z_{min} = \frac{1}{9} + \frac{2}{3} + 3 = \frac{1+6+27}{9} = \frac{34}{9} = \lambda_2$

$\lambda_1 = 9 + 6 + 3 = 18$

$|2\lambda_1 - 9\lambda_2| = 2$

2. Ans. 2

tr(A) = sum of roots

$10 + \beta = 0 \Rightarrow \beta = -10$

and product of roots = $|A|$

$\Rightarrow \quad 72 = 1(9\beta - 35) - 2(9\alpha + 30) + 3(7\alpha + 6\beta)$

put $\beta = -10$

$\Rightarrow \quad 72 = -90 - 35 - 18\alpha - 60 + 21\alpha - 180$

$\Rightarrow \quad 3\alpha = 365 + 72$

$\Rightarrow \quad 3\alpha = 437$

then $3\alpha + \beta - 425 = 2$

3. Ans. 3

Planes intersect XOY plane at point with $z = 0$

$\therefore \quad 9x - 3y = 3,\ x + 2y = 5\ \&\ 21x + ay = 3$

Solving, we get $x = 1$, $y = 2$ & $a = -9$

Planes intersect XOZ plane at point with $y = 0$

$\therefore \quad 9x + 4z = 3,\ x + 2z = 5\ \&\ 21x + bz = 3$

gives $x = -1$, $z = 3$ & $b = 8$

Now area of ΔAOB, $k = \frac{1}{2}\left|\overrightarrow{OA} \times \overrightarrow{OB}\right|$

$= \frac{1}{2}\left|\left(\hat{i} + 2\hat{j}\right) \times \left(-\hat{i} + 3\hat{k}\right)\right| = \frac{7}{2}$

$\therefore \quad k + \left(\frac{a+b}{2}\right) = \frac{7}{2} + \left(\frac{-9+8}{2}\right) = 3$

4. Ans. 3

Since normals are concurrent

$\Rightarrow \begin{vmatrix} 1 & a & 0 \\ 1 & -b & 3 \\ 1 & 1 & -3 \end{vmatrix} = 0 \Rightarrow \quad 3b - 3 + 6a = 0$

$\Rightarrow \quad b + 2a = 1 \quad$ ....(i)

Also if slopes of concurrent normals are m_1, m_2, m_3 then $m_1 + m_2 + m_3 = 0$

$\Rightarrow \quad a - b + 1 = 0 \quad$ ....(i)

from (i) & (ii)

$a = 0$ & $b = 1$

$\Rightarrow \quad \alpha = 3\ \&\ \beta = 0$

$\alpha + \beta = 3$

5. Ans. (3)

$x^4 - x^2 = x^2(x^2 - 1) < 0$ in immediate vicinity of $x = 0$

and $\quad x^4 - x^6 = x^4(1 - x^2) > 0$ in immediate vicinity of $x = 0$

Hence $\lim_{x \to 0} f(x^4 - x^2) = \lim_{t \to 0^-} f(t) = 3$

and $\lim_{x \to 0} f(x^4 - x^6) = \lim_{t \to 0^+} f(t) = 2$

Therefore $2 \times 3 = \lambda(2)$

$\lambda = 3$

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

DISTANCE LEARNING PROGRAMME
(Academic Session : 2018 - 2019)

LEADER TEST SERIES / JOINT PACKAGE COURSE

TARGET : JEE (Main + Advanced) 2019

Test Type : MAJOR Test Pattern : JEE-Advanced

TEST # 10 **TEST DATE : 14 - 04 - 2019**

PAPER-2

PART-1 : PHYSICS **ANSWER KEY**

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	B	C	B	C	C	D	A	C,D	A,B	A,B,D
	Q.	11	12	13	14	15	16	17	18		
	A.	B,C	A,C	A,B,C	A,B,C	B	B	A	B		

SOLUTION

SECTION-I

1. Ans. (B)

Sol. Total energy of hydrogen atom is ground state = –13.6 eV

$$\therefore M_H = M_P + M_e - \frac{13.6\text{eV}}{C^2}$$

2. Ans. (C)

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow g = 4\pi^2\frac{\ell}{T^2}$

$$\frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + \frac{2\Delta \ell}{T}$$

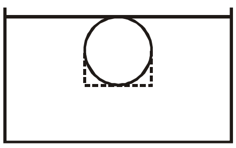
3. Ans. (B)

Sol. If $v_2 t > A_1B_2$, then refracted wavefront will be absent. (Not possible to draw wavefront)

4. Ans. (C)

5. Ans. (C)

Sol.



Force exerted on lower half

$$= \rho g(2R)\pi R^2 - \frac{1}{3}\pi R^3 \rho g$$

$$= \frac{5\rho g\pi R^3}{3}$$

Force exerted by upper half

$$\frac{5}{3}\rho p R^3 g - \frac{4}{3}\rho\pi R^3 g = \frac{1}{3}\rho\pi R^3 g$$

Ratio = 5

6. Ans. (D)

7. Ans. (A)

Sol. $eV = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2eV}{m}}$ and $T = \frac{2\pi m}{eB}$

so $S = vT = \frac{2\pi m}{eB}\sqrt{\frac{2eV}{m}}$

$$\left[S \Rightarrow 2\sqrt{\frac{20mV}{eB^2}}\right]$$

8. Ans. (C, D)

Sol. $\delta_1 + \delta_2 > 0 \Rightarrow \mu_1 > \mu_2; \theta > 0; \theta_1 - \theta_2 > 0$

9. Ans. (A,B)

Sol. Use $I = 1/2\ \rho\omega^2 A^2\ v$

$P = 1/2\ \mu\omega^2 A^2\ v$

$ED = 1/2\ \rho\omega^2 A^2$

10. Ans. (A, B, D)

Sol. $8 = \frac{1}{2}kA^2$

$A = 6$ cm

$$k = \frac{16}{(6\times10^{-2})^2} = \frac{16}{36}\times10^4 = \frac{40}{9}\times10$$

$$= 4.44\ \text{kN/m}$$

mass con't be found k_{max} at mean.

11. Ans. (B, C)

Sol. In steady state current in the branch having capacitor becomes zero.

$\therefore$ Reading of $A_1 = 0$, $V_1 = \frac{Q}{C} = \frac{4\times10^{-3}}{100\times10^{-6}}$

Also potential drop across $(900 + A_2) = 40$

$\Rightarrow i$ in $A_2 = \frac{1}{25}A$

From circuit analysis, EMF of ideal cell must be less than 48 V.

12. Ans. (A, C)

Sol. $I = \frac{dV}{dR} = \frac{Edr}{\frac{dr}{\sigma 4\pi r^2}} = kE^2 4\pi r^2$ ...... (i)

dV = E.dr

from (1) $E^2 = \frac{I}{4\pi k r^2}$

$\Rightarrow dV = \int_a^b \sqrt{\frac{I}{4\pi k}}\frac{1}{r}dr$

13. Ans. (A,B,C)

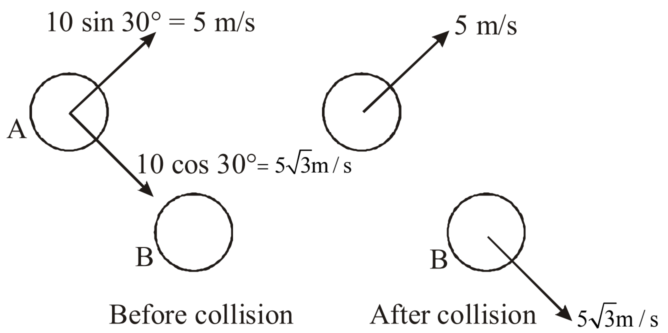
Sol. If 'S' open, V $\to$ same w/i outer sphere

If 'S' closed, $V_{\text{inner sph.}} = 0$

$\Rightarrow \frac{kQ}{2R} + \frac{kq}{R} = 0 \Rightarrow q = -\frac{Q}{2}$

14. Ans. (A,B,C)

Sol.



Before collision After collision

Velocity components in common tangent direction will remain unchanged while velocity components in common normal direction are interchanged in case of an elastic collision. Hence both A and B move at right angles after collision with $v_A = 5$ m/s and $v_B = 5\sqrt{3}$ m/s. Kinetic energy is conserved in an elastic collision. Whether it is a head on collision or and oblique collision.

15. Ans. (B)

Sol. $\frac{P}{100} = \frac{dN}{dt}\frac{hc}{\lambda}$ for 3 minute $\Rightarrow \frac{dN}{dt} = \frac{\lambda}{hc}\times1$

$\Rightarrow dN = \int_0^{80}\frac{\lambda}{hc}dt$; $N' = \frac{3.71\times10^{21}}{20}$

for one hour N = N' × 20 = 3.71×10^{21}

16. Ans. (B)

Sol. $\left(\frac{dN}{dt}\right) = \frac{P\lambda}{hc} = \frac{100}{hc}(3000+40t)$

17. Ans. (A)

Sol. $156 = f\left(\frac{325}{300}\right)\times\frac{360}{335}$

$\therefore f = 134$ Hz

18. Ans. (B)

Sol. $\frac{x}{325} + \frac{x - 25\times\frac{11}{3}}{335} = \frac{11}{3}$

$\therefore x = 650$ m

PART-2 : CHEMISTRY

ANSWER KEY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	D	C	D	B	D	C	B,D	B,C,D	A,B,C
	Q.	11	12	13	14	15	16	17	18		
	A.	A,B,C,D	A,B,D	A,B,C,D	A,B,C,D	C	B	C	A		

SOLUTION

SECTION–I

1. Ans.(D)

Sol. Moles of Cl_2 = $\dfrac{\frac{380}{760}\times 96}{0.08\times 300} = 2$

$Ca + Cl_2 \rightarrow CaCl_2$

4 mole   2 mole
↓
(LR)

mass of $CaCl_2$ formed = 2 mol = 222 gm

2. Ans.(D)

S-I : effective molarity = 0.4 M

S-II : effective molarity = 0.2 + x = 0.24 M

S-III : effective molarity = 0.2 M

$BOH \rightleftharpoons B^+ + OH^-$

0.2-x   x   x

$\dfrac{x^2}{0.2-x} = 0.01 \Rightarrow x = 0.04$

$HCl + BOH \rightleftharpoons BCl + H_2O$

0.2 mole   0.2 mol   0

Final   0   0   0.2 mole

$\therefore$ [BCl] $\dfrac{0.2}{2} = 0.1M$

Now, conc. S- III < S - II < S- I

3. Ans. (C)

4. Ans. (D)

5. Ans. (B)

6. Ans. (D)

7. Ans. (C)

8. Ans. (B,D)

9. Ans. (B,C,D)

10. Ans. (A,B,C)

11. Ans. (A,B,C,D)

12. Ans. (A,B,D)

13. Ans. (A,B,C,D)

14. Ans. (A,B,C,D)

15. Ans. (C)

$$t = \frac{2.303}{6.93\times 10^{-4}}\log\frac{10^{-4}}{5\times 10^{-5}}$$

$$= \frac{2.303\times 0.30}{6.93\times 10^{-4}} = 1000\,\text{sec}.$$

16. Ans. (B)

r = k[A] [B]

= $6.93 \times 10^{-4} \times 0.25 \times 1.25$

17. Ans. (C)

18. Ans. (A)

PART-3 : MATHEMATICS — ANSWER KEY

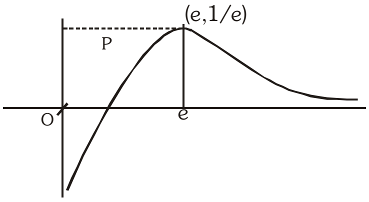
SECTION-I										
Q.	1	2	3	4	5	6	7	8	9	10
A.	A	D	A	A	D	B	C	B,C,D	A,B,C,D	A,B,C,D
Q.	11	12	13	14	15	16	17	18		
A.	A	A,B,C,D	C,D	A,C	A	D	D	A		

SOLUTION

SECTION–I

1. Ans. (A)

$A = \dfrac{\ell n x}{x}$



$A' = \dfrac{1-\ell n x}{x^2} = 0$ at $x = e$

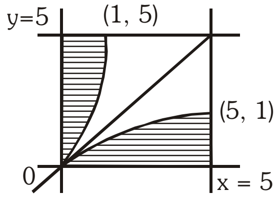
$A'' = \dfrac{-x-2x(2-\ell n x)}{x^2}$

$A'' < 0$ at $x = e \Rightarrow$ maxima

$A_{\max|x=e} = \dfrac{1}{e}$

2. Ans. (D)

$A = \int_0^1 (5-3x^3-2x)\,dx$



$= \dfrac{13}{4}$

3. Ans. (A)

$\int_{-1}^{x} f(t)dt + f'''(3)\int_x^0 dt = \int_1^x t^3dt - f'(1)\int_0^x t^2dt + f''(2)\int_x^3 t\,dt$

$f(x) - f'''(3) = x^3 - f'(1).\ x^2 - f''(2)x$

$f(x) = x^3 - f'(1)\ x^2 - f''(2)x + f'''(3)$

$f'(x) = 3x^2 - 2f'(1)\ .\ x - f''(2)$ ...(i)

$f''(x) = 6x - 2f'(1)$ ...(ii)

In (i) put x = 1

$f'(x) = 3 - 2f'(1) - f''(2)$

$3f'(1) + f''(2) = 3$ ...(iii)

In (ii) put x = 2

$2f'(1) + f''(2) = 12$ ...(iv)

(iii) - (iv)

$f'(1) = -9.$

4. Ans. (A)

Let P (h, k) be a point on the circle, $15x^2 + 15y^2 - 48x + 64y = 0$

Let PT_1 & PT_2 be lengths of tangents from P (h, k) to

$5x^2 + 5y^2 - 24x + 32y + 75 = 0$ &

$5x^2 + 5y^2 - 48x + 64y + 300 = 0$

$PT_1 = \sqrt{h^2+k^2-\frac{24}{5}h+\frac{32}{5}k+15}$

$PT_2 = \sqrt{h^2+k^2-\frac{48}{5}h+\frac{64}{5}k+60}$

Since (h, k) lies on

$15x^2 + 15y^2 - 48x + 64y = 0$

$\therefore\ h^2 + k^2 = \frac{48}{15}h - \frac{64}{15}k$

$PT_1 = \sqrt{\frac{48}{15}h-\frac{64}{15}k-\frac{24}{15}h+\frac{32}{15}k+15}$

$= \sqrt{\frac{32}{15}k-\frac{24}{15}h+15}$

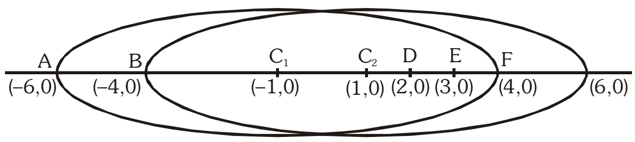
$PT_2 = \sqrt{\frac{48}{15}h-\frac{64}{15}k-\frac{48}{5}h+\frac{64}{5}k+60}$

$= \sqrt{-\frac{96}{15}h+\frac{128}{15}k+60}$

$= 2\sqrt{-\frac{24}{15}h+\frac{32}{15}k+15} = 2(PT_1)$ $\quad \dfrac{PT_1}{PT_2} = \dfrac{1}{2}$

5. Ans. (D)

$|z-z_1| + |z-z_2| = 2a$ is the equation of an ellipse with foci at z_1 and z_2 and whose major axis = 2a



For $|z+4| + |z-2| = 10$

centre is at $C_1(-1, 0)$

foci at (–4, 0) and (2, 0) and vertices at (– 6, 0) and (4, 0)

for $|\omega + 1| + |\omega - 3| = 10$

centre is at $C_2(1, 0)$

foci at (–1, 0) and (3, 0) and vertices at (– 4, 0) and (6, 0)

$\therefore$ m = 0 and M = AG = 12

$\Rightarrow$ m + M = 12

6. Ans. (B)

$$\int_{1/e}^{\tan x} \frac{t}{1+t^2}\,dt + \int_{1/e}^{\cot x} \frac{dt}{t(1+t^2)}$$

$t^2 = y$ $\qquad\qquad$ $1/t^2 = z$

$$\Rightarrow \frac{1}{2} \times \int_{1/e^2}^{\tan^2 x} \frac{1}{1+y}\,dy - \frac{1}{2} \times \int_{1/e^2}^{\tan^2 x} \frac{1}{1+z}\,dz$$

$$= \frac{1}{2}\left[\left|\ell n\,(1+y)\right|_{1/e^2}^{\tan^2 x} - \left|\ell n\,(1+z)\right|_{1/e^2}^{\tan^2 x} \right]$$

$$= \frac{1}{2}$$

$$\left[\ell n\left(1+\tan^2 x\right) - \ell n\frac{(e^2+1)}{e^2} - \ell n\left(1+\tan^2 x\right) + \ell n\left(1+e^2\right)\right]$$

$$= \frac{1}{2}(\ln e^2) = \frac{1}{2}(2) = 1$$

7. Ans. (C)

Co-ordinate of point P(acosθ, bsinθ).

Also, $PF_1 + PF_2 = 17$ .....(i)

Given $\frac{1}{2} PF_1.PF_2 = 30 \Rightarrow PF_1.PF_2 = 60$.....(ii)

From (i) & (ii) $PF_1 = 5$ & $PF_2 = 12$

$\therefore (F_1F_2)^2 = (PF_1)^2 + (PF_2)^2$

$= 5^2 + 12^2 \qquad \Rightarrow F_1F_2 = 13$

8. Ans. (B,C,D)

3R – W = 124

R + W = 84

4R = 208 $\Rightarrow$ R = 52

W = 32

Number of ways = $^{84}C_{32}$

$(1 + x^4)^{84}$

coefficient is $^{84}C_{32}$ for x^{128}

= coefficient of x^{208} in $(1 + x^4)^{84}$

(D) 3R – W = 44

R + W = 84

4R = 128 $\Rightarrow$ R = 32

W = 52

Number of ways = $^{84}C_{32}$

9. Ans. (A,B,C,D)

Sample space

11	12	13	14	15	16
21	22	23	24	25	26
31	32	33	34	35	36
41	42	43	44	45	46
51	52	53	54	55	56
61	62	63	64	65	66

$\longleftrightarrow$

0	1	2	3	4	5
1	0	1	2	3	4
2	1	0	1	2	3
3	2	1	0	1	2
4	3	2	1	0	1
5	4	3	2	1	0

$$P(1) = \frac{10}{36} = \frac{5}{18}$$

$$P(1) = \frac{5}{18}, P(2) = \frac{4}{18}, P(3) = \frac{3}{18}, P(4) = \frac{2}{18},$$

$$P(5) = \frac{1}{18}$$

P(1) > P(2) > P(3) > P(4) > P(5)

$$P(0) + P(2) + P(4) = \frac{6}{36} + \frac{8}{36} + \frac{4}{36} = \frac{1}{2}$$

10. Ans.(A,B,C,D)

$A^{-1}B = B^{-1}$

Taking inverse on both sides, we get

$(A^{-1}B)^{-1} = B$

$\Rightarrow B^{-1}A = B$

$\Rightarrow BB^{-1}A = B^2$ or $A = B^2$

$$\therefore A = \begin{bmatrix} -2 & -2 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -2 & -2 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 4 \\ 2 & 2 \end{bmatrix}$$

Also $kA^{-1} = 2B^{-1} + I$

Multiplying by B as post factor, we get

$kA^{-1}B = 2I + B$

or $kB^{-1} = 2I + B$

$\Rightarrow kI = 2B + B^2$

$$\Rightarrow \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} -4 & -4 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 4 \\ 2 & 2 \end{bmatrix}$$

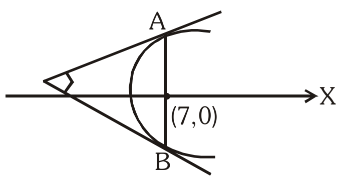
gives $k = 2$

11. Ans. (A)

$x = 7 + ty$ is a focal chord of $y^2 = 28x$

Tangents at the ends of focal chord are always $\perp$

$\therefore$ Circumcentre is the mid point of AB



Let mid point is (h, k)

$\Rightarrow T = S_1$

$ky - 14(x + h) = k^2 - 28h$

compare with $x = 7 + ty$

$$14 = \frac{k}{t} = \frac{28h - 14h - k^2}{7}$$

$\Rightarrow 98 = 28h - 14h - k^2$

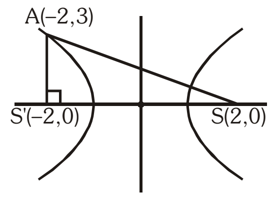
$y^2 = 14(x - 7)$

12. Ans. (A,B,C,D)

$S \equiv (2, 0)$, $S' \equiv (-2, 0)$

Using reflection property of hyperbola,

S'A is incident ray.



Equation of incident ray

S'A is $x = -2$

Equation of reflected ray

SP is $3x + 4y = 6$.

Now $2ae = 4 \quad \Rightarrow ae = 2$ ....(i)

Point (–2, 3) lies on hyperbola,

$$\therefore \quad \frac{4}{a^2} - \frac{9}{b^2} = 1 \quad \Rightarrow \quad \frac{4}{a^2} - \frac{9}{4 - a^2} = 1$$

on solving it we get $a = 4$ (reject), $a = 1$ .....(ii)

$\therefore$ Using (i) & (ii), we get $e = 2$

length of latus rectum $= 2a(e^2 - 1) = 6$

13. Ans. (C,D)

$\because$ f is a continuous function

$\therefore$ for any value of x say 0.1

$f(0.2) = f(1.1)$

$\therefore$ Using Rolle's theorem $f'(x) = 0$ between 0.2 & 1.1 if we start taking all real values of x between say 0.1 & say 0.9 then $f(a) = f(b)$ will hold for infinite pairs (a,b).

Hence $f'(x) = 0$ for infinite values similarly $f''(x) = 0$ for infinite values of x.

14. Ans. (A,C)

$a^2 + b^2 + c^2 = ca + ab\sqrt{3}$

$$\Rightarrow \underbrace{a^2 - ab\sqrt{3} + \frac{3}{4}b^2}_{} + \underbrace{\frac{b^2}{4} - ca + c^2}_{} = 0$$

$$\Rightarrow \left(a-\frac{\sqrt{3}}{2}b\right)^2+\left(\frac{b}{2}-c\right)^2=0$$

$$\therefore \frac{a}{b}=\frac{\sqrt{3}}{2} \quad \& \quad \frac{c}{b}=\frac{1}{2} \quad \Rightarrow \quad a = b . \frac{\sqrt{3}}{2}$$

$$\& \quad c=\frac{b}{2}$$

$$\cos B=\frac{a^2+c^2-b^2}{2ac}=\frac{\frac{3}{4}b^2+\frac{b^2}{4}-b^2}{2ac}=0$$

$\therefore$ $\angle B = 90°$

Similarly $\angle A = 60°$ & $\angle C = 30°$

$\therefore$ ΔABC is right angled but not isosceles

$$[\tan A+\tan C]=\left[\sqrt{3}+\frac{1}{\sqrt{3}}\right]=\left[\frac{4}{\sqrt{3}}\right]=2$$

$$\left\{\frac{r_2}{r_1}\right\}=\left\{\frac{r\tan B/2}{r\tan A/2}\right\}=\left\{\frac{\tan 45°}{\tan 30°}\right\}$$

$$=\{\sqrt{3}\}=\sqrt{3}-1$$

Solutions for Question 15 & 16

15. Ans. (A)

$S_1(n) + S_2(n) + \ldots\ldots.. S_n(n) = (1 + 1^2 + \ldots..+1^n) +(2 + 2^2 + \ldots\ldots..+2^n)+ (3+ 3^2 + \ldots.+ 3^n)+\ldots\ldots+ (n + n^2 +\ldots\ldots..+n^n)$

$$=n+\frac{2(2^n-1)}{2-1}+\frac{3(3^n-1)}{3-1}+\ldots\ldots..+\frac{n(n^n-1)}{n-1}$$

$$=n+\sum_{r=2}^{n}\frac{r(r^n-1)}{r-1}=a$$

16. Ans. (D)

$$\lim_{n\to\infty}\frac{\left[\frac{S_3(n)}{n^4}\right]^2}{\frac{S_2(n)}{n^3}\times\frac{S_4(n)}{n^5}}=\frac{(1/4)^2}{1/3\times 1/5}=\frac{15}{16}$$

using above obtained value in Q.No.15

Solutions for Questions 17 to 18

$$f(x)=\left(\frac{t^{1-x}}{1-x}\right)_1^{\infty}$$

$$f(x)=\lim_{t\to\infty}\frac{t^{1-x}}{1-x}-\left(\frac{1}{1-x}\right)$$

for limit to be finite $x > 1 \Rightarrow f(x)=\frac{1}{x-1}$

17. Ans. (D)

$$h(x)=\begin{cases}(x-1)^3, & x\le 1\\ \dfrac{1}{\left(\frac{1}{1-x}\right)^4+\frac{1}{(1-x)^2}+1}, & x>1\end{cases}$$

$h(1^-) = 0 = h(1)$

$h(1^+) = 0 \Rightarrow$ 'h' is continuous at x = 1

$h'(1^-) = 0$

$$h'(1^+)=\lim_{h\to 0}\frac{\frac{1}{\frac{1}{h^4}+\frac{1}{h^2}+1}}{h}=\lim_{h\to 0}\frac{h^4}{(1+h^2+h^4)h}=0$$

$\Rightarrow$ 'h' is derivable at x = 1.

18. Ans. (A)

$y = e^{1/(x-1)} \Rightarrow y' = -y.\frac{1}{(x-1)^2}$

$\Rightarrow (x-1)^2 . y' = -y$ .... (1)

$\Rightarrow (x-1)^2 y'' + 2(x-1)y' = -y'$

$\Rightarrow (x-1)^2 y'' + y'\{2x-1\} = 0$

$$\Rightarrow \frac{y''}{y'}=\frac{1-2x}{(x-1)^2}$$

from (1) $\frac{y'}{y}=\frac{-1}{(x-1)^2}$

$$\frac{y''}{y'}=\frac{(2x-1).y'}{y} \quad \Rightarrow \quad \frac{y.y''}{(y')^2}=(2x-1)$$