ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

DISTANCE LEARNING PROGRAMME

(Academic Session : 2018 - 2019)

LEADER TEST SERIES / JOINT PACKAGE COURSE

TARGET : JEE (Main + Advanced) 2019

Test Type : MAJOR | Test Pattern : JEE-Advanced

TEST # 06 | **TEST DATE : 03 - 03 - 2019**

PART-1 : PHYSICS | **ANSWER KEY**

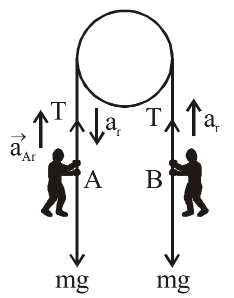
	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	A	A,B,C	B,D	B,C	A,B,C,D	A,B,C	B	C	B	D
SECTION-II	Q.	1	2	3	4	5	6	7	8		
	A.	9.00	2.00	3.00	35.35	1.00	25.00	9.00	16.00		

SOLUTION

1. **Ans. (A)**

Sol. For man B, $T - mg = ma_B = ma_r$ ....(i)

For man A, $T - mg = ma_A = m(a_{Ar} - a_r)$ ....(ii)



From equations (i) & (ii)

$ma_r = ma_{Ar} - ma_r$

or, $a_{Ar} = 2a_r$

so, $a_r = \frac{10}{2} = 5\ m/s^2$

2. **Ans. (A,B,C)**

Sol. $U_{total} = \frac{1}{2} \times F\Delta\ell$

$= \frac{1}{2}\times 100\times 10\times 10^{-3} + \frac{1}{2}\times 500\times 40\times 10^{-3} + 40\times 10^{-3}\times 1000$

$= 5 + 10 + 40 = 55$ J

Work done in streching wire from Q to R

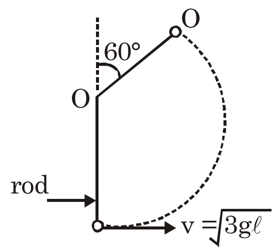
$55 - 5 = 50$ J

Strain $= \frac{\Delta\ell}{\ell} = \frac{50\times 10^{-3}}{2} = 25\times 10^{-3} = 0.25$

Q is a proportionality limit → at this point it will contain its elastic property & come back to orignal shape

3. **Ans. (B, D)**

Sol.



By applying W.E.T.

$0 - \frac{1}{2}mv^2 = -mg\ell(1+\cos\theta)$

$v = \sqrt{3g\ell} \Rightarrow \cos\theta = \frac{1}{2}$

$\theta = 60°$

$H = \ell + \ell\cos 60° = \frac{3\ell}{2}$

$a_T = g\sin 60° = \frac{g\sqrt{3}}{2}$

4. **Ans. (B,C)**

5. **Ans. (A,B,C,D)**

Sol. (A) $\tau = \vec{M}\times\vec{B} = I\alpha$

$i\ell^2 B\sin 90° = \frac{2}{3}m\ell^2\alpha \Rightarrow \alpha = \frac{3Bi}{2m}$

(B) Initial potential energy $= -\vec{M}.\vec{B} = MB\cos 90° = 0$

Final potential energy =

$-\vec{M}\vec{B} = -M.B\cos 60° = \frac{-MB}{2} = \frac{-i\ell^2 B}{2}$

Change in potential energy = Change in rotational kinetic energy

$$\frac{Bi\ell^2}{2} = \frac{1}{2}\times\frac{2}{3}m\ell^2\times\omega^2$$

$$\omega = \sqrt{\frac{3Bi}{2m}}$$

(C) $\tau = \vec{M}\times\vec{B} = i\ell^2 B\sin 30° = \frac{Bi\ell^2}{2}$

(D) τ = MBsin θ = Iα

$\alpha = \frac{MB\sin\theta}{1}$

θ increases 90° to 180°

$\Rightarrow$ sin θ decreases from 1 to 0

$\therefore$ α decreases

6. Ans. (A,B,C)

Sol. $V_{eff} = \sqrt{2gh} = \sqrt{2\times10\times3.6} = 6\sqrt{2}$ m/s

Discharge flow rate = $AV = \frac{\pi d^2}{4}\times V$

$= 96\sqrt{2}\times10^{-4}$ m^3/s

Apply Bernoulli equation between surface and A

$$P_0 = P_A + \frac{1}{2}\rho V^2 + \rho gh$$

$$10^5 = P_A + \frac{1}{2}1000\times\left(6\sqrt{2}\right)^2 + 1000\times10\times1.8$$

$10^5 = P_A + 36000 + 18000$

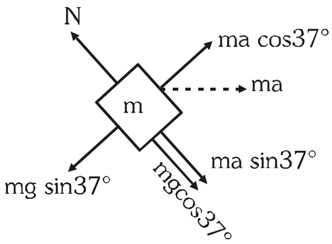
$1\times10^5 = P_A + 0.54\times10^5$

$P_A = 0.46\times10^5$ N/m^2

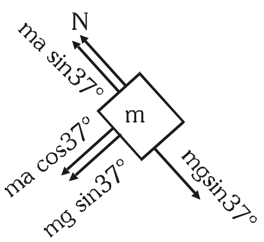
Water will rise as h_A = 1.8 (< 10)

7. Ans. (B)

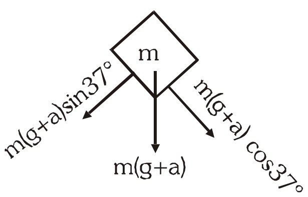
Sol. (A)



(B)



(C)

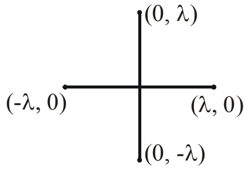


8. Ans. (C)

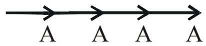
Sol. Apply Gauss law in all cases

9. Ans. (B)

Sol. For S_1, S_2, S_3, S_4



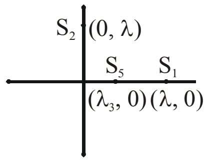
there is no phase difference, so



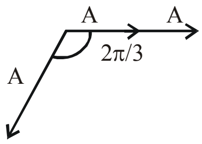
A_{max} = 4A

E_{max} = 16I

for S_1, S_2, S_5



S_1 & S_2 have no phase difference S_5 have $\frac{2\pi}{3}$ Phase diff. with S_1 & S_2



$$A_{max} = \sqrt{4A^2 + A^2 + 2A^2\cos\frac{\pi}{3}}$$

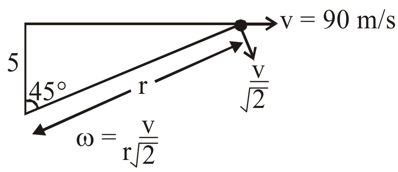
$$A_{max} = \sqrt{3}A$$

10. Ans. (D)

SECTION-II

1. Ans. 9.00

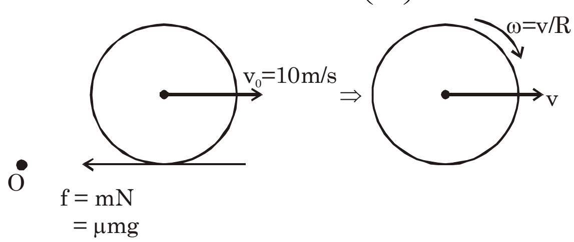
Sol.



2. Ans. 2.00

Sol. About 'O'

$$m(10)R = m(v)R + \frac{2}{5}mR^2\left(\frac{v}{R}\right)$$



$$10 = v + \frac{2v}{5}$$

$\boxed{v = \frac{50}{7}}$

$v = v_0 - \frac{10}{7}(t)$

$\frac{50}{7} = 10 - \frac{10}{7}(t)$

$\boxed{t = 2\,sec}$

3. Ans. 3.00

Sol. From work energy theorem (W=ΔKE)

Kinetic energy

$K = \int_0^x F_x dx = \int_0^x (2\alpha x - 3x^2)dx = \alpha x^2 - x^3$

At maximum kinetic energy condition

$F = 0 \Rightarrow x = \frac{2\alpha}{3}$

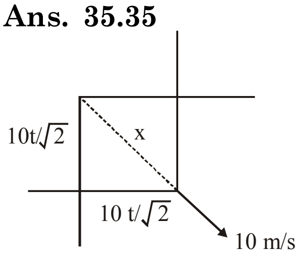
$K_{max} = \alpha\left(\frac{2\alpha}{3}\right)^2 - \left(\frac{2\alpha}{3}\right)^3 = 4$

$\Rightarrow \frac{4}{9}\alpha^3 - \frac{8}{27}\alpha^3 = 4$

$\Rightarrow \frac{4}{27}\alpha^3 = 4 \Rightarrow \alpha^3 = 27 \Rightarrow \alpha = 3$

4. Ans. 35.35

Sol.



x = 10t

$\phi = B\left[\frac{10t}{\sqrt{2}}\right]^2$

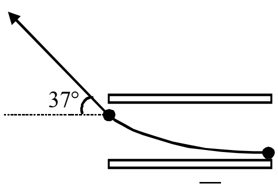
$\frac{d\phi}{dt} = 100Bt = 100 \times (.10) \times (.10) = 1V$

$R = (.01) \times 4\left(\frac{10t}{\sqrt{2}}\right)$

$i = \frac{1}{R}\frac{d\phi}{dt} = 35.35 \approx 35\ A$

5. Ans. 1.00

Sol. $\tan 37° = \frac{V_y}{V_x} = \frac{V_y}{V_0}$



$V_y = a_y t = \frac{qE_y}{m} \times \frac{\ell}{V_0}$

$E_y = \frac{V}{d} = \frac{iR}{d} = \frac{\varepsilon R}{(R+r)d}$

$\Rightarrow \frac{3}{4}V_0 = V_y = \frac{q\ell}{mv_0} \times \frac{\varepsilon R}{(R+r)d}$

$\frac{3}{4}V_0^2 = \frac{16}{91} \times 10^{12} \times \frac{3 \times R}{(R+2)10^{-3}} \times 0.182$

2.5 R + 5 = 3R

5 = 0.5 R ; R = 10Ω

6. Ans. 25.00

Sol. Current through R_1 = 1 4mg

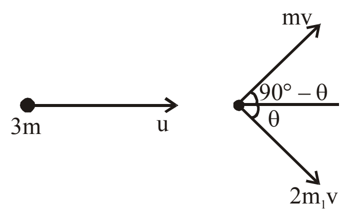
Total heat loss = $(1)^2 \times 60 \times t$

so $(1)^2 \times 60 \times t = \left(\frac{240gm}{1000}\right) \times (4200)\Delta T$

ΔT = 25° C

7. Ans. 9.00

Sol. From momentum conservation,



3mu = 2mvcosθ + mvsinθ ...... (i)

Also,

mvcosθ = 2mv sin θ .....(ii)

From (i) & (ii), $\tan\theta = \frac{1}{2}$

$\therefore\ 3u = 2v \times \frac{2}{\sqrt{5}} + v \times \frac{1}{\sqrt{5}}$

$\Rightarrow\ 3u = \sqrt{5}v \Rightarrow v = \frac{3}{\sqrt{5}}u$

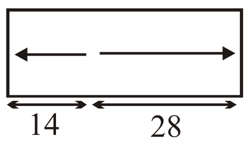
$\therefore\ k_f = \frac{1}{2} \times 3m \times v^2 = \frac{1}{2}.3m.\left(\frac{3}{\sqrt{5}}u\right)^2 = \frac{9}{5}\left[\frac{1}{2}.3m.u^2\right]$

$k_i = \frac{1}{2}.3m.u^2$

$\therefore\ \frac{k_f}{k_i} = \frac{9}{5}$

8. Ans. 16.00

Sol. Δx = 28 m = nλ



$\lambda = \frac{343}{196} = 1.75m$

$n = \frac{28}{1.75} = 16$

$\Rightarrow \Delta\phi = 2\pi \times 16 = 32\pi$ **Ans.**

PART-2 : CHEMISTRY **ANSWER KEY**

	Q.	1	2	3	4	5	6	7	8	9	10
SECTION-I	A.	A,D	C	A,B,C,D	A,C,D	A,C,D	A,B,C,D	A	A	D	A
SECTION-II	Q.	1	2	3	4	5	6	7	8		
	A.	6.00	0.50	68.40	6.00	4.00	2.00	4.00	1.00		

SOLUTION

SECTION-I

1. **Ans.(A,D)**

2. **Ans. (C)**

$2NH_3 \rightleftharpoons N_2 + 3H_2$

a 0 0

a – 2x x 3x

moles $\propto$ pressure

a $\propto$ 31 equilibrium

a + 2x $\propto$ 50 after equilibrium

$\Rightarrow$ Also , $\frac{15}{300} = \frac{P}{620}$ (At constant V)

P = 31 atm

$\therefore \frac{a+2x}{a} = \frac{50}{31} \Rightarrow x = \frac{19}{62}a$

$\therefore$ % of NH_3 decomposed

$= \frac{2x}{a} \times 100 = 61.3\%$

3. **Ans. (A,B,C,D)**

4. **Ans. (A, C, D)**

5. **Ans. (A, C, D)**

6. **Ans. (A, B, C, D)**

7. **Ans. (A)**

8. **Ans. (A)**

9. **Ans. (D)**

10. **Ans. (A)**

SECTION-II

1. **Ans. 10^{-6} OMR ans. (6.00)**

Sol. $CH_3COOH \rightleftharpoons CH_3COO^- + H^+$

$\frac{0.1}{2}M$ $\frac{1}{2}M$

$\frac{0.1}{2} - x$ x $\frac{1}{2} + x$

$\simeq \frac{0.1}{2}$ x $\frac{1}{2}$

$K_a = 10^{-5} = \frac{x \times \frac{1}{2}}{\frac{0.1}{2}}$

$x = 10^{-6}$

2. **Ans. (0.50)**

Sol. $H_2 + H_2O + \frac{1}{2} O_2 \rightarrow 2H^+ + 2OH^-$

$\Delta G^o = -256.5 + 2 \times 80 = -96.5$ kJ

$-\Delta G^o = nFE^o$

$+96.5 \times 1000 = 2 \times 96500 \times E^o$

$E^o = 0.5$ Volt.

3. **Ans. 68.40**

$\Delta T_f + \Delta T_b = 5 = (K_b + k_f)m$

$5 = 2.5 \times \frac{w \times 1000}{342 \times 100}$

$W = \frac{342 \times 2}{10} = \frac{684}{10} = 6.8$

4. **Ans. (6.00)**
5. **Ans. (4.00)**
6. **Ans. (2.00)**
7. **Ans. (4.00)**
8. **Ans. (1.00)**

x = 6

y = 5

PART-3 : MATHEMATICS — ANSWER KEY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,C	B,C	A,B,C,D	C,D	A,B,C,D	A,C	D	A	B	D
SECTION-II	Q.	1	2	3	4	5	6	7	8		
	A.	0.81 or 0.82	8.00	1.41 or 1.42	3.00	1.57 or 1.58	0.00	1.57 or 1.58	8.00		

SOLUTION

SECTION–I

1. Ans. (A,C)

Since $\vec{a}, \vec{b}$ and $(\vec{a}\times\vec{b})$ are non coplanar

$\therefore \quad \vec{r} = x\vec{a} + y\vec{b} + z(\vec{a}\times\vec{b})$ ...... (i)

for some scalars x, y, z

Now $\vec{b} = \vec{r}\times\vec{a}$

$\therefore \quad \vec{b} = \{x\vec{a} + y\vec{b} + z(\vec{a}\times\vec{b})\}\times\vec{a}$

$= x(\vec{a}\times\vec{a}) + y(\vec{b}\times\vec{a}) + z\{(\vec{a}\times\vec{b})\times\vec{a}\}$

$= 0 + y(\vec{b}\times\vec{a}) + z\{(\vec{a}\,.\,\vec{a})\vec{b} - (\vec{a}\,.\,\vec{b})\vec{a}\}$

$\therefore \quad \vec{b} = y(\vec{b}\times\vec{a}) + z\,(\vec{a}\,.\,\vec{a})\vec{b} \qquad \{\because\ \vec{a}\,.\,\vec{b} = 0\}$

Comparing the coefficients, we get

$$y = 0 \text{ and } z = \frac{1}{(\vec{a}\,.\,\vec{a})}$$

Putting the values of y and z in (i), we get

$$\vec{r} = x\vec{a} + \frac{1}{(\vec{a}\,.\,\vec{a})}(\vec{a}\times\vec{b})$$

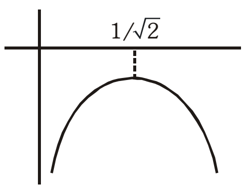
Similarly option C is also true.

2. Ans. (B,C)

Let $f(x) = \ell nx - x^2, x > 0$

$\Rightarrow \quad f'(x) = \frac{1}{x} - 2x = 0 \quad \Rightarrow \quad x = \frac{1}{\sqrt{2}}$

$f''(x) = -\frac{1}{x^2} - 2 < 0$



$\Rightarrow$ Maxima at $x = \frac{1}{\sqrt{2}}$

$\Rightarrow$ (B) is correct

Let normal to C_1 at (t, t^2) be

$y - t^2 = -\frac{1}{2t}(x - t) \quad \Rightarrow \quad 2ty - 2t^3 = t - x$

$\Rightarrow \quad x + 2ty = t + 2t^3$ .... (i)

Let normal to C_2 at $(\alpha, \ell n\alpha)$ be

$y - \ell n\alpha = -\alpha(x - \alpha)$

$\Rightarrow \quad \alpha x + y = \ell n\alpha + \alpha^2$ ....(ii)

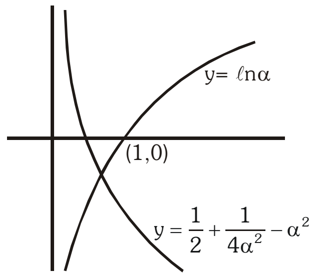
Comparing (i) & (ii)

$\frac{1}{\alpha} = 2t = \frac{t + 2t^3}{\ell n\alpha + \alpha^2} \quad \Rightarrow \quad 2(\ell n\alpha + \alpha^2) = 1 + 2t^2$

$\Rightarrow \quad \ell n\alpha + \alpha^2 = \frac{1}{2} + t^2 \Rightarrow \ell n\alpha = \frac{1}{2} + \frac{1}{4\alpha^2} - \alpha^2$

Draw the graphs of $y = \ell n\alpha$

& $\quad y = \frac{1}{2} + \frac{1}{4\alpha^2} - \alpha^2$ to



Hence there is one value of $\alpha \in (0, 1)$

$\therefore$ (C) is correct

3. Ans. (A,B,C,D)

From the first equation $\frac{dy}{dx} = b$

and from the second equation $\frac{dy}{dx} = -\frac{x}{y}$

$\Rightarrow \quad b\left(-\frac{x}{y}\right) = -1 \Rightarrow y = bx$

So the curves are perpendicular for every value of

a & b.

4. Ans. (C,D)

$(1 + x)^6 + (1 + x)^7 + (1 + x)^8 + ... + (1 + x)^{19}$

coefficient $x^7 = 0 + {}^7C_7 + {}^8C_7 + ... + {}^{19}C_7$

$= {}^{20}C_8 = {}^{20}C_{12} = {}^nC_r$

$\therefore \quad n + r = \begin{cases} 20 + 8 = 28 \\ 20 + 12 = 32 \end{cases}$

5. Ans. (A,B,C,D)

Since the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ cuts the three given circles at the extremities of a diameter, the common chords will pass through the centre of the given circles, so that

$2gx + 2fy + c + 4 = 0$ passes through (0, 0)

$\Rightarrow \quad c = -4$ .... (i)

Next $\quad 2gx + 2fy + c + 6x + 8y - 10 = 0$ passes through (3, 4)

$\Rightarrow \quad (2g + 6)3 + (2f + 8)4 - 14 = 0$

$\Rightarrow \quad 3g + 4f + 18 = 0$ .... (ii)

and $2gx + 2fy + c - 2x + 4y + 2 = 0$ passes through (–1, 2)

$\Rightarrow \quad (2g - 2)(-1) + (2f + 4)2 - 2 = 0$

$\Rightarrow \quad g - 2f - 4 = 0$ .... (iii)

From (ii) and (iii) $g = -2$ and $f = -3$

$\Rightarrow \quad g + f - c = -1$

and $g^2 + f^2 - c = 4 + 9 + 4 = 17$

and $gf = 6$

6. Ans.(A,C)

$\bar{z}_1 = \bar{z}_2$

$5y^2 - 2ix - 19 = 4y^2 + 6i + 6$

$(y^2 - 25) - 2i(x + 3) = 0$

Since complex number is zero

$y^2 - 25 = 0$ and $x + 3 = 0$

$\therefore x = -3$ and $y = \pm 5$

$z = x + iy$

$z = -3 \pm 5i$

7. Ans. (D)

(P) Required probability $= \dfrac{1}{2}$

(Q) Required probability

$$= \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4 - \left(\frac{1}{2}\right)^7 = \frac{47}{2^7}$$

(R) Required probability $= \dfrac{4}{2^7}$

(S) There are five cases 0, 1, 2, 3, or 4 wins

Required probability

$$= \frac{1 + 7 + 15 + 10 + 1}{2^7} = \frac{34}{2^7}$$

8. Ans. (A)

Sol. (P)

$$\int \frac{2x^7 + 3x^2}{x^{10} - 2x^5 + 1} dx = \int \frac{x^6\left(2x + \frac{3}{x^4}\right)}{x^6\left(x^4 - \frac{2}{x} + \frac{1}{x^6}\right)} dx = \int \frac{2x + \frac{3}{x^4}}{\left(x^2 - \frac{1}{x^3}\right)^2} dx$$

Let $\quad x^2 - \dfrac{1}{x^3} = t \Rightarrow \left(3x + \dfrac{3}{x^4}\right) dx = dt$

$$= \int \frac{dt}{t^2} = -\frac{1}{t} + c = -\frac{x^3}{x^5 - 1} + c = \frac{x^3(x^5 - 1)}{(x^5 - 1)^2} + c = \frac{x^3 - x^8}{(x^5 - 1)^2} + c$$

(Q) $\quad \left| f\left((1 + i\sqrt{3})^n\right) \right|$

$$\left| f\left(2^n \cos n\frac{\pi}{3} + i \sin\frac{n\pi}{3}\right) \right| = 2^n \left| \cos\frac{n\pi}{3} \right|$$

$$\sum_{n=1}^{6} \log_2\left(2^n \left| \cos\frac{n\pi}{3} \right|\right)$$

$$= \sum_{n=1}^{6} n + \log_2 \left| \cos\frac{n\pi}{3} \right|$$

$$= \frac{6}{2}(6 + 1) + \left[-1 - 1 + 0 - 1 - 1 + 0\right]$$

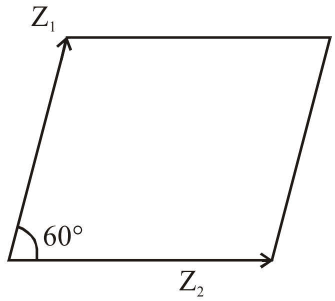
$= 3 \times 7 - 4$

$= 17$

(R) $\quad |z_1| = 2, \ |z_2| = 3$

$$\frac{z_1}{z_2} = \frac{2}{3} e^{i\pi/3}$$

$$\frac{z_1}{z_2} = \frac{2}{3}\left(\frac{1 + i\sqrt{3}}{2}\right)$$



$$\left| \frac{z_1 + z_2}{z_1 - z_2} \right| = \left| \frac{\frac{2}{3}\left(\frac{1 + i\sqrt{3}}{2}\right) + 1}{\frac{2}{3}\left(\frac{1 + i\sqrt{3}}{2}\right) - 1} \right| = \left| \frac{4 + i\sqrt{3}}{-2 + i\sqrt{3}} \right|$$

$$= \sqrt{\frac{16 + 3}{4 + 3}} = \sqrt{\frac{19}{7}} = \frac{\sqrt{133}}{7}$$

(S) $f(z) = z^4 + a_1z^3 + a_2z^2 + a_3z + a_4 = 0$

$f(ki) = k^4 - a_1k^3i - a_2k^2 + a_3ki + a_4 = 0$

$k^4 - k^2a_2 + a_4 = 0$ ...(1)

$k^3a_1 - ka_3 = 0$ ...(2)

$\Rightarrow \quad k^2 = \frac{a_3}{a_1}$ as $k \neq 0$

$\therefore \quad \frac{a_3^2}{a_1^2} - \frac{a_3a_2}{a_1} + a_4 = 0$

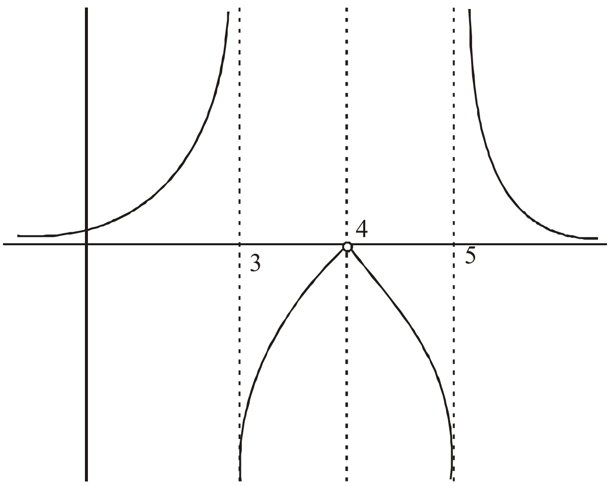
$\frac{a_3^2}{a_1^2} + a_4 = \frac{a_3a_2}{a_1} \Rightarrow \frac{a_3}{a_1a_2} + \frac{a_1a_4}{a_2a_3} = 1$

9. Ans. (B)

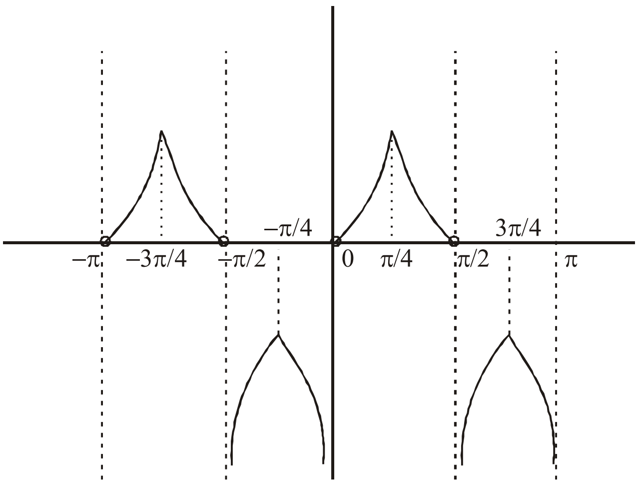
(P) $\lim\limits_{x\to 0}\left[\min.\left((y+1)^2+7\right).\frac{\sin x}{x}\right] = 6$

(Q) $\lim\limits_{x\to 0}\frac{\sin^{-1}\left(\frac{2x}{1+x^2}\right)}{\left(\frac{2x}{1+x^2}\right)}.\frac{2x}{x.(1+x^2)} = 1.\frac{2}{1+0} = 2$

(R)



(S)



10. Ans. (D)

(P) $2f(x).f(y) = f(x+y) + f(x-y)$

at $x = y = 0 : 2(f(0))^2 = 2f(0) \Rightarrow f(0)=1$

$x = 0 : 2f(y) = f(y) + f(-y)$

$\Rightarrow \quad f(y) = f(-y)$

$\Rightarrow \quad f(x) = f(-x)$

$\Rightarrow \quad f'(x) = f'(-x) = 0.$

(Q) $I = \int_0^\infty \frac{dx}{(x^2+1)(x^5+1)}$ ...(i)

$x = \frac{1}{t} \Rightarrow I = -\int_\infty^0 \frac{t^5dt}{(t^2+1)(t^5+1)}$

$\Rightarrow I = \int_0^\infty \frac{x^5}{(x^2+1)(x^5+1)}dx$ ...(ii)

(i) + (ii) $\Rightarrow \ 2I = \int_0^\infty \frac{dx}{x^2+1} = \tan^{-1}x\Big|_0^\infty = \frac{\pi}{2}$

$\Rightarrow I = \frac{\pi}{4}$

(R) $\cos^7x = 1 - \sin^4x$

$\Rightarrow \cos^7x = \cos^2x(1 + \sin^2x)$

$\Rightarrow \cos x = 0;\ \cos^5x = 1 + \sin^2x$

$\Rightarrow \begin{cases}\cos x = 1\\ \sin x = 0\end{cases}$

$\therefore \ x = -\frac{\pi}{2}, 0, \frac{\pi}{2}$

(S) $\sqrt{3} + i = ac - bd + i(bc + ad)$

$\Rightarrow ac - bd = \sqrt{3}; bc + ad = 1$

$\tan^{-1}\left(\frac{a}{b}\right) + \tan^{-1}\left(\frac{c}{d}\right) = \pi + \tan^{-1}\left(\frac{ad+bc}{bd-ac}\right)$

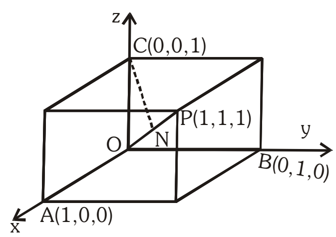
$= \pi + \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

SECTION-II

1. Ans. 0.81 or 0.82

$\overrightarrow{OP} = \hat{i} + \hat{j} + \hat{k} \ \Rightarrow \ \widehat{OP} = \frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$

Now ON is projectionof OC



along OP $= \hat{k}.\widehat{OP} = \hat{k}.(\hat{i}+\hat{j}+\hat{k})\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

In right angle ΔONC, CN $= \sqrt{1-\frac{1}{3}} = \sqrt{\frac{2}{3}}$

2. Ans. 8.00

$\vec{r}.\vec{a} = 27$

$4xy - xy - 3 = 27$

$xy = 10$

(1, 0) (2, 5) (–1, –10) (–2, –5) } 8 ordered pairs.

(10, 1) (5, 2) (–10, –1) (–5, –2)

3. Ans. 1.41 or 1.42

The curve is a rectangular hyperbola.

So eccentricity = $\sqrt{2}$

4. Ans. 3.00

Tangent at P is 2y = 2(x + 1)

Putting x = 0, we get y = 1

Slope of HP × slope of SH = –1

$\angle SHP = \frac{\pi}{2}$

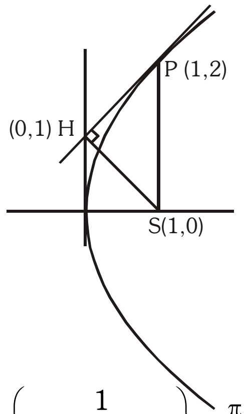
$\Rightarrow$ SP is diameter

so SP = 2

r = 1

A = π

[A] = 3



5. Ans. 1.57 or 1.58

$$\sin^{-1}\sqrt{(x^2+4x+4)} + \cos^{-1}\left(\frac{1}{\sqrt{x^2+4x+4}}\right) = \frac{\pi}{2}$$

$$\sqrt{x^2+4x+4} = \frac{1}{\sqrt{x^2+4x+4}}$$

$\Rightarrow$ $(x+2)^2 = 1$ $\Rightarrow$ $x+2 = \pm 1$ $\Rightarrow$ x = –1 or –3

Now, $\sec^{-1}(2x) + \sin^{-1}(x/2) = \sec^{-1}(-2) + \sin^{-1}\left(-\frac{1}{2}\right)$

$= \frac{2\pi}{3} + \left(-\frac{\pi}{6}\right) = \frac{\pi}{2}$

$\sin^{-1}\frac{x}{2}$ is not defined for x = –3.

6. Ans. 0.00

Let $A_i = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\therefore$ $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$

or $\begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$

$\Rightarrow$ $a^2 + bc = 1$, $(a + d)b = 2$,

$(a + d)c = 0$, $bc + d^2 = 4$

$\therefore$ $c = 0$, $a^2 = 1$, $d^2 = 4$, $(a + d)b = 2$

$\Rightarrow$ 4 possible sets of a, b, c, d are possible

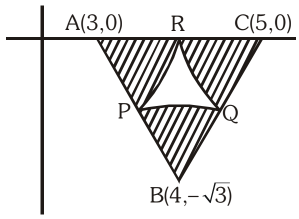
Hence A_i are $\begin{bmatrix} 1 & 2/3 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 1 & -2 \\ 0 & -2 \end{bmatrix}, \begin{bmatrix} -1 & 2 \\ 0 & 2 \end{bmatrix},$

$\begin{bmatrix} -1 & -2/3 \\ 0 & -2 \end{bmatrix}$

$\therefore$ $\sum_{i=1}^{4} \det(A_i) = 2 - 2 - 2 + 2 = 0$

7. Ans. 1.57 or 1.58

In the fig. P, Q, R are the mid points of



AB, BC & CA respectively & ΔABC is equilateral having side 2 units

Then required area $= \left(\frac{60^\circ}{360^\circ} \times \pi(1)^2\right) \times 3$

$= \frac{\pi}{2}$ sq. units

8. Ans. 8.00

$x^2dy + 2xydx = 3x^2dx + dy$

$\Rightarrow$ $d(x^2y) = (x^3 + y)$ $\Rightarrow$ $x^2y = x^3 + y + c$

$\Rightarrow$ c = 0 ($\because$ curve passes through 0,0)

when x = 2, y = a

$\Rightarrow$ 4a = 8 + a $\Rightarrow$ 3a = 8