PART (C) : MATHEMATICS

SECTION-I : (SINGLE ANSWER CORRECT TYPE)

This section contains **04 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

41. The value of $\lim_{n\to\infty}\frac{1}{n^2}\{[x]+[2x]+[3x]+.....[nx]\}$, where [.] denotes greatest integer function, is

(A) $\frac{x}{4}$ (B) $\frac{x}{3}$ (C) $\frac{x}{2}$ (D) $\frac{x}{5}$

41. **(C)**

Since, $x-1<[x]\le x$

$2x-1<[2x]\le 2x$

$3x-1<[3x]\le 3x$

. . . .

. . . .

$nx-1<[nx]\le nx$

On adding above terms, we get

$x(1+2+.....+n)-n<[x]+[2x]+[3x]+.....+[nx] \le x(1+2+3+.....+n)$

$\Rightarrow \quad x\frac{n(n+1)}{2}-n<[x]+[2x]+[3x]+.....+[nx]\le x\frac{n(n+1)}{2}$

$\Rightarrow \quad \frac{x\frac{n(n+1)}{2}-n}{n^2}<\frac{[x]+[2x]+[3x]+......+[nx]}{n^2}\le\frac{x\frac{n(n+1)}{2}}{n^2}$

$\Rightarrow \quad \lim_{n\to\infty}x\left(\frac{n+1}{2}-1\right)\frac{1}{n}<\lim_{n\to\infty}\frac{[x]+[2x]+[3x]+.....+[nx]}{n^2} \le \lim_{n\to\infty}x\frac{n(n+1)}{2}$

$\Rightarrow \quad \frac{x}{2}<\lim_{n\to\infty}\frac{[x]+[2x]+.....+[nx]}{n^2}\le\frac{x}{2}$

$\Rightarrow \quad \lim_{n\to\infty}\frac{[x]+[2x]+........+[nx]}{n^2}=\frac{x}{2}$

42. The sum $\sum_{r=1}^{n}\text{arc }\tan\left(\frac{2r}{2+r^2+r^4}\right)$ is equal to

(A) $\tan^{-1}(n^2+n+1)+\frac{\pi}{4}$ (B) $\tan^{-1}(n^2+n+1)-\frac{\pi}{4}$

(C) $\tan^{-1}(n^2+2n+1)-\frac{\pi}{4}$ (D) $\tan^{-1}(n^2-n+1)-\frac{\pi}{4}$

42. **(B)**

$\sum_{r=1}^{n}\text{arc }\tan\left(\frac{2r}{2+r^2+r^4}\right)$

$$=\sum_{r=1}^{n}\tan^{-1}\left\{\frac{(r^2+r+1)-(r^2-r+1)}{1+(r^2+r+1)(r^2-r+1)}\right\}$$

$$=\sum_{r=1}^{n}\tan^{-1}(r^2+r+1)-\sum_{r=1}^{n}\tan^{-1}(r^2-r+1)$$

$$=\left[\tan^{-1}3+\tan^{-1}7+\tan^{-1}13+......+\tan^{-1}(n^2-n+1)+\tan^{-1}(n^2+n+1)\right]$$

$$-\left[\tan^{-1}+\tan^{-1}3+\tan^{-1}7+......+\tan^{-1}(n^2-n+1)\right]$$

$$=\tan^{-1}(n^2+n+1)-\tan^{-1}1$$

$$=\tan^{-1}(n^2+n+1)-\frac{\pi}{4}$$

43. If $\sum_{r=1}^{n}T_r=\frac{n(n+1)(n+2)(n+3)}{8}$, then $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{T_r}$ is equal to

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{4}$ (D) $\frac{1}{8}$

43. (B)

Let $S_n=\sum_{r=1}^{n}T_r=\frac{n(n+1)(n+2)(n+3)}{8}$

$\therefore T_r=S_r-S_{r-1}$

$$=\frac{r(r+1)(r+2)(r+3)}{8}-\frac{(r-1)r(r+1)(r+2)}{8}$$

$$=\frac{r(r+1)(r+2)}{2}$$

$$\therefore \quad \frac{1}{T_r}=\frac{2}{r(r+1)(r+2)}$$

$$\Rightarrow \quad \frac{1}{T_r}=\left(\frac{1}{r}-\frac{1}{r+1}\right)-\left(\frac{1}{r+1}-\frac{1}{r+2}\right)$$

$$\Rightarrow \quad \sum_{r=1}^{n}\frac{1}{T_r}=\sum_{r=1}^{n}\left\{\left(\frac{1}{r}-\frac{1}{r+1}\right)-\left(\frac{1}{r+1}-\frac{1}{r+2}\right)\right\}$$

$$\Rightarrow \quad \sum_{r=1}^{n}\frac{1}{T_r}=\left(1-\frac{1}{n+1}\right)-\left(\frac{1}{2}-\frac{1}{n+2}\right)$$

$$\Rightarrow \quad \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{T_r}=\lim_{n\to\infty}\left(1-\frac{1}{n+1}-\frac{1}{2}+\frac{1}{n+2}\right)$$

$$=1-\frac{1}{2}=\frac{1}{2}$$

44. If the coefficient of x^3 and x^4 in the expansion of $(1+ax+bx^2)(1-2x)^{18}$ in powers of x are both zero, then (a, b) is equal to

(A) $\left(14,\frac{272}{3}\right)$ (B) $\left(16,\frac{272}{3}\right)$ (C) $\left(16,\frac{251}{3}\right)$ (D) $\left(14,\frac{251}{3}\right)$

44. **(B)**

We have,

$\left(1+ax+bx^2\right)\left(1-2x\right)^{18}$

$=\left(1+ax+bx^2\right)\sum_{r=0}^{18} {}^{18}C_r\left(-1\right)^r 2^r x^r$

$=\sum_{r=0}^{18} {}^{18}C_r\left(-1\right)^r 2^r x^r + a\sum_{r=0}^{18} {}^{18}C_r\left(-1\right)^r 2^r x^{r+1} + b\sum_{r=0}^{18} {}^{18}C_r\left(-1\right)^r 2^r x^{r+2}$

The coefficient of x^3 and x^4 in the expansion of $\left(1+ax+bx^2\right)\left(1-2x\right)^{18}$ are both zero.

$\therefore {}^{18}C_3\left(-1\right)^3 2^3 + a.{}^{18}C_2\left(-1\right)^2 2^2 + b.{}^{18}C_1\left(-1\right)^1 2^1 = 0$

And ${}^{18}C_4\left(-1\right)^4 2^4 + a.{}^{18}C_3\left(-1\right)^3 2^3 + b.{}^{18}C_2\left(-1\right)^2 2^2 = 0$

$\Rightarrow \quad 51a - 3b = 544$

And $32a - 3b = 240$

$\Rightarrow \quad a = 16, b = \frac{272}{3}$

SECTION-II : (COMPREHENSIONS TYPE)

This section contains **06** questions. Based on each paragraph, there are **TWO** questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out which **ONLY ONE** is correct.

PARAGRAPH FOR QUESTIONS NO. 45 & 46

A bag contains p red and q blue balls. Two players P and Q alternatively draw a ball from the bag, replacing the ball each time after the draw till one of them draws a red ball and wins the game. If P begins the game and the probability of P winning the game is three times that of R.

45. The probability of Q wins the game is

(A) $\frac{p}{p+2q}$ (B) $\frac{q}{2p+q}$ (C) $\frac{q}{p+2q}$ (D) $\frac{p}{2p+q}$

45. **(C)**

Bag contains p red and q blue balls. Let E_i be the event of drawing a red ball in the ith draw.

Then, $E_i^{'}$: event of drawing a blue ball in the ith draw.

Let E : event that P wins the game.

Then, E' : event that Q wins the game.

Since, after each draw, the ball drawn is replaced.

$\therefore \quad P\left(E_i\right) = \frac{p}{p+q}$ and $P\left(E_i^{'}\right) = \frac{q}{p+q}$

Now, $P\left(E\right) = P\left(E_1\right) + P\left(E_1^{'}E_2^{'}E_3\right) + P\left(E_1^{'}E_2^{'}E_3^{'}E_4^{'}E_5\right) + ...$

$= P\left(E_1\right) + P\left(E_1^{'}\right)P\left(E_2^{'}\right)P\left(E_3\right) + P\left(E_1^{'}\right)P\left(E_2^{'}\right)P\left(E_3^{'}\right)P\left(E_4^{'}\right)P\left(E_5\right) + ...$

$= \frac{p}{p+q} + \frac{q^2}{\left(p+q\right)^2}.\frac{p}{\left(p+q\right)} + \frac{q^4}{\left(p+q\right)^4}.\frac{p}{p+q} + ...$

$$=\frac{p}{(p+q)}\left[1+\left(\frac{q}{p+q}\right)^2+\left(\frac{q}{p+q}\right)^4+.....\right]$$

$$=\left(\frac{p}{p+q}\right).\frac{1}{\left[1-\left(\frac{q}{p+q}\right)^2\right]}=\frac{p(p+q)}{p^2+2pq}=\frac{p+q}{p+2q}$$

$\therefore \quad P(E')=1-P(E)=1-\frac{p+q}{p+2q}=\frac{q}{p+2q}$

46. The ratio of p and q is

(A) 1 : 2 (B) 2 : 1 (C) 1 : 3 (D) 3 : 1

46. **(B)**

Since, $P(E)=3P(E')$

$\therefore \quad \frac{p+q}{p+2q}=\frac{3q}{p+2q}$

$\Rightarrow \quad p+q=3q\Rightarrow p=2q$

$\Rightarrow \quad p:q=2:1$

PARAGRAPH FOR QUESTIONS NO. 47 & 48

A curve $y=f(x)$ satisfies the differential equation $\frac{dy}{dx}+\left(\frac{2x}{1+x^2}\right)y=\frac{4x^2}{1+x^2}$ and passes through the origin.

47. The function $y=f(x)$

(A) Has no inflection point
(B) Is strictly decreasing, $\forall x\in R$
(C) Is strictly increasing, $\forall x\in R$
(D) Is such that it has a maxima but no minima

47. **(C)**

We have, $\frac{dy}{dx}+\left(\frac{2x}{1+x^2}\right)y=\frac{4x^2}{1+x^2}$

$IF=e^{\int\frac{2x}{1+x^2}dx}=e^{\log(1+x^2)}=1+x^2$

$\therefore \quad y(1+x^2)=\int(1+x^2)\left(\frac{4x^2}{1+x^2}\right)dx+C$

$\Rightarrow \quad y(1+x^2)=\int 4x^2dx+C=\frac{4x^3}{3}+C$

Since, curve passing through origin.

$\therefore \quad c=0$

$\therefore \quad y=\frac{4x^3}{3(1+x^2)}$

$\Rightarrow \quad \frac{dy}{dx}=\frac{4}{3}\left[\frac{(1+x^2)(3x^2)-x^3(2x)}{(1+x^2)^2}\right]$

$\Rightarrow \quad \frac{dy}{dx} = \frac{4(3+x^2)}{3(1+x^2)} \Rightarrow \frac{dy}{dx} > 0, \forall x \neq 0$

$\Rightarrow \quad f(x)$ is strictly increasing for all $x \in R$,

$\frac{dy}{dx} = 0$ at $x = 0$ and it does not change sign.

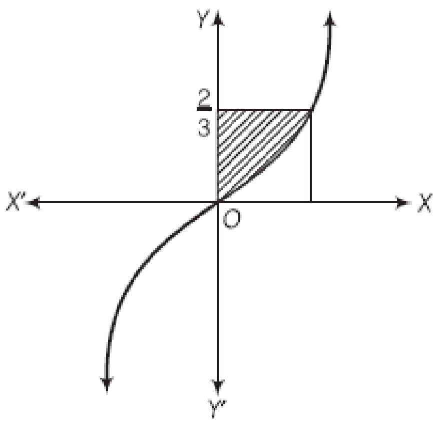
$\Rightarrow \quad x = 0$ is the point of inflection.

48. The area enclosed by $y = f(x)$, $y -$ axis and $y = 2/3$, is

(A) $\log 2$ (B) $\frac{4}{3}\log 2$ (C) $\frac{2}{3}\log 2$ (D) $\frac{1}{3}\log 2$

48. **(C)**

$x \to \infty, y \to \infty$ and $x \to -\infty, y \to -\infty$



Area enclosed by $y = f(x)$, X-axis and ordinate at $x = \frac{2}{3}$.

$A = \frac{2}{3} - \frac{4}{3}\int_0^1 \frac{x^3}{1+x^2} dx$

Put $1 + x^2 = t$

$\Rightarrow \quad 2x dx = dt$

$\therefore \quad A = \frac{2}{3} - \frac{2}{3}\int_1^2 \frac{(t-1)}{t} dt = \frac{2}{3} - \frac{2}{3}\int_1^2 \left(1 - \frac{1}{t}\right) dt$

$= \frac{2}{3} - \frac{2}{3}\left[t - \log t\right]_1^2$

$= \frac{2}{3} - \frac{2}{3}\left[(2 - \log 2) - (1 - \log 1)\right]$

$= \frac{2}{3} - \frac{2}{3}\left[1 - \log 2\right]$

$= \frac{2}{3}\log 2$

PARAGRAPH FOR QUESTIONS NO. 49 & 50

Let $f(x)=(1-x)^2\sin^2 x+x^2, \forall x\in R$ and $g(x)=\int_1^x\left(\frac{2(t-1)}{t+1}-\ln t\right)f(t)dt, \forall x\in(1,\infty)$

49. Consider the statements

P: There exists some $x\in R$ such that,
$f(x)+2x=2(1+x^2)$.

Q: There exists some $x\in R$ such that,
$2f(x)+1=2x(1+x)$.

Then,

(A) Both P and Q are true
(B) P is true and Q is false
(C) P is false and Q is true
(D) Both P and Q are false

49. **(C)**

Here , $f(x)+2x=(1-x)^2.\sin^2 x+x^2+2x$

Where, $P:f(x)+2x=2(1+x)^2$

$\therefore \quad 2(1+x^2)=(1-x)^2\sin^2 x+x^2+2x$

$\Rightarrow \quad (1-x)^2\sin^2 x=x^2-2x+2$

$\Rightarrow \quad (1-x)^2\sin^2 x=(1-x)^2+1$

$\Rightarrow \quad (1-x)^2\cos^2 x=-1$

Which is never possible.

$\therefore$ P is false.

Again, let $Q:h(x)=2f(x)+1-2x(1+x)$

Where, $h(0)=2f(0)+1-0=1$

$h(1)=2f(1)+1-4=-3$

As, $h(0)h(1)<0$

$\Rightarrow$ $h(x)$ must have a solution.

$\therefore$ Q is true.

50. Which of the following is true?

(A) g is decreasing on $(1,\infty)$

(B) g is decreasing on $(1,\infty)$

(C) g is increasing on (1, 2) and decreasing on $(2,\infty)$

(D) g is decreasing on (1, 2) and increasing on $(2,\infty)$

50. **(B)**

Here, $f(x)=(1-x)^2.\sin^2 x+x^2\geq 0, \forall x$

And $g(x)=\int_1^x\left(\frac{2(t-1)}{(t+1)}-\log t\right).f(t)dt$

$\Rightarrow \quad g'(x) = \left(\dfrac{2(x-1)}{(x+1)} - \log x\right).\underbrace{f(x)}_{+ve}$ …… (i)

For g'(x) to be increasing or decreasing, let

$\phi(x) = \dfrac{2(x-1)}{(x+1)} - \log x$

$\Rightarrow \quad \phi'(x) = \dfrac{4}{(x+1)^2} - \dfrac{1}{x} = \dfrac{-(x-1)^2}{x(x+1)^2}$

$\phi'(x) < 0$, for $x > 1 \Rightarrow \phi(x) < \phi(x) < \phi(1) \Rightarrow \phi(x) < 0$ ……….. (ii)

From Eqs. (i) and (ii), we get

$g'(x) < 0$ for $x \in (1, \infty)$

$\therefore \quad g(x)$ is decreasing for $x \in (1, \infty)$.

SECTION-III : (MULTIPLE CORRECT ANSWER(S) TYPE)

This section contains **06 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE than one is/are correct.**

51. Consider the family of all circles whose centres lie on the straight line $y = x$. If this family of circles is represented by the differential equation

$Py'' + Qy' + 1 = 0$, where P, Q are functions of x, y and $y'\left(\text{here } y' = \dfrac{dy}{dx}, y'' = \dfrac{d^2y}{dx^2}\right)$, then which of the following statement(s) is (are) true?

(A) $P = y + x$ (B) $P = y - x$

(C) $P + Q = 1 - x + y + y' + (y')^2$ (D) $P - Q = x + y - y' - (y')^2$

51. (BC)

The equation of the family of circles whose centres lie on the straight line $y = x$ is $x^2 + y^2 - 2ax - 2ay + c = 0$,

Whose a and c are parameters.

On differentiating w.r.t. x, we get

$2x + 2yy' - 2a - 2ay' = 0 \Rightarrow a = \dfrac{x + yy'}{1 + y'}$

Again, differentiating this w.r.t. x, we get

$$\frac{(1+y')\left\{1 + (y')^2 + yy''\right\} - (x + yy')y''}{(1+y')^2} = 0$$

$\Rightarrow \quad (1+y')\left\{1 + (y')^2 + yy''\right\} - (x + yy')y'' = 0$

$\Rightarrow \quad 1 + y'\left\{(y')^2 + y' + 1\right\} + y''(y - x) = 0$

Comparing this equation with $Py'' + Qy' + 1 = 0$,

We get

$P = y - x$ and $Q = (y')^2 + y' + 1$

$\Rightarrow \quad P = y - x$ and $P + Q = 1 - x + y + y' + (y')^2$

52. Let $a \in R$ and $f : R \to R$ be given by $f(x) = x^5 - 5x + a$, then

(A) $f(x)$ has three real roots, if $a > 4$

(B) $f(x)$ has only one real root, if $a > 4$

(C) $f(x)$ has three real roots, if $a < -4$

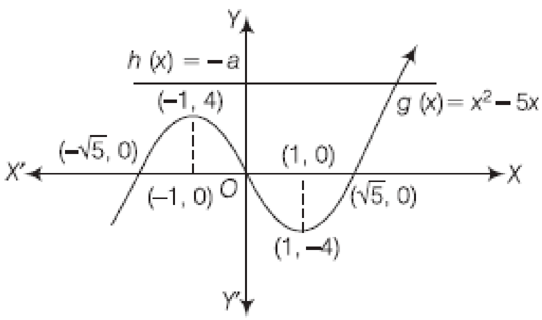
(D) $f(x)$ has three real roots, if $-4 < a < 4$

52. **(BD)**

Let $g(x) = x^5 - 5x$ and $h(x) = -a$. Then, the number of real rotos of $f(x) = 0$ is the number of points of intersection of the curves $y = g(x)$ and $y = h(x)$.

Clearly, these two curves intersection at three points, if

So, $f(x)$ has three real roots, if $-4 < a < 4$.



The two curves $y = g(x)$ and $y = h(x)$ intersect at exactly one point, if $a > 4$ or < -4.

So, $f(x)$ has only one real root, if $a < -4$ or $a > 4$.

53. If two distinct chords of a parabola $y^2 = 4ax$ passing through $(a, 2a)$ are bisected on the line $x + y = 1$, then the length of the latusrectum can be

(A) 2 (B) 1 (C) 4 (D) 3

53. **(ABD)**

Any point on $x + y = 1$ can be taken as $(t, 1 - t)$.

The equation of chord with this as mid – point is

$y(1-t) - 2a(x+t) = (1-t^2) - 4at$

It passes through $(a, 2a)$.

So, $t^2 - 2t + 2a^2 - 2a + 1 = 0$

This should have two distinct real roots, So,

$D > 0$

$\Rightarrow \quad a^2 - a < 0$

$\Rightarrow \quad 0 < a < 1$

So, length of latusrectum < 4 and $0 < a < 1$.

54. Given an real – valued function f such that

$$f(x)=\begin{cases}\dfrac{\tan^2\{x\}}{\left(x^2-[x]^2\right)}, & \text{for } x>0\\ 1, & \text{for } x=0\\ \sqrt{\{x\}\cot\{x\}}, & \text{for } x<0\end{cases}$$

Where, [x] is the integral part and {x} is the fractional part of x, then

(A) $\lim_{x\to0^+} f(x)=1$ (B) $\lim_{x\to0^-} f(x)=\cot 1$

(C) $\cot^{-1}\left(\lim_{x\to0^-} f(x)\right)^2=1$ (D) $\tan^{-1}\left(\lim_{x\to0^+} f(x)\right)=\frac{\pi}{4}$

54. (ACD)

We have, $\lim_{x\to0^+} f(x)=\lim_{x\to0^+}\frac{\tan^2\{x\}}{\left(x^2-[x]^2\right)}$

$=\lim_{x\to0^+}\frac{\tan^2 x}{x^2}=1\left[\because x\to0^+,[x]=0\Rightarrow\{x\}=x\right]$

Also, $\lim_{x\to0^-} f(x)=\lim_{x\to0^-}\sqrt{\{x\}\cot\{x\}}=\sqrt{\cot 1}$

$\left[\because x\to0^-,[x]=-1\Rightarrow\{x\}=x+1\Rightarrow\{x\}\to1\right]$

Also, $\cot^{-1}\left(\lim_{x\to0^-} f(x)\right)^2=\cot^{-1}(\cot 1)=1$

Also, $\tan^{-1}\left(\lim_{x\to0^+} f(x)\right)=\tan^{-1}1=\frac{\pi}{4}$

55. Let $\omega=\frac{\sqrt{3}+i}{2}$ and $P=\{\omega^n : n=1,2,3,.....\}$.

Further $H_1=\left\{z\in C:\operatorname{Re}(z)>\frac{1}{2}\right\}$ and

$H_2=\left\{z\in C:\operatorname{Re}(z)<-\frac{1}{2}\right\}$, where C is the set of all complex numbers.

If $z_1\in P\cap H_1, z_2\in P\cap H_2$ and O represents the origin, then $\angle z_1Oz_2$ is equal to

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{6}$ (C) $\frac{2\pi}{3}$ (D) $\frac{5\pi}{6}$

55. (CD)

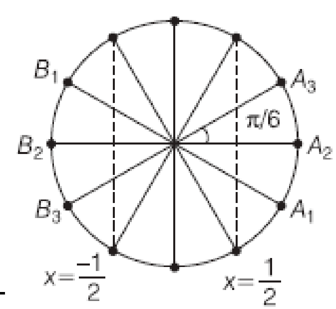
$\omega=\frac{\sqrt{3}+i}{2}=e^{i\pi/6}$, So $\omega^n=e^{i\left(\frac{n\pi}{6}\right)}, n=0,1,2,.....,12$

Now, for $z_1, \cos\frac{n\pi}{6}>\frac{1}{2}$ and

For $z_2, \cos\frac{n\pi}{6}<\frac{-1}{2}$ possible

position of z_1 are A_1, A_2, A_3 whereas of z_2 are B_1, B_2, B_3

(as shown in figure) so possible value of



$\angle z_1Oz_2$ according to the given options is $\frac{2\pi}{3}, \frac{5\pi}{6}$.

56. The circles $x^2+y^2+2x+4y-20=0$ and $x^2+y^2+6x-8y+10=0$ are

(A) Such that the numbers of common tangents on them is 2

(B) Orthogonal

(C) Such that the length of their common tangent is $5(12/5)^{1/4}$

(D) Such that the length of their common chord is $5\sqrt{3/2}$

56. **(ABCD)**

$r_1 = 5, r_2 = \sqrt{15}, C_1C_2 = \sqrt{40}$

$\therefore \quad r_1 + r_2 > C_1C_2 > r_1 - r_2$

Hence, the circles intersect at two distinct points.

There are two common tangents.

Also, $2g_1g_2 + 2f_1f_2 = 2(1)(3) + 2(2)(-4) = -10$

And $C_1 + C_2 = -20 + 10 = -10$

Thus, the two circles are orthogonal.

Length of common chord $= \frac{2r_1r_2}{\sqrt{r_1^2+r_2^2}} = 5\sqrt{\frac{3}{2}}$

Length of common tangent $= \sqrt{C_1C_2^2 - (r_1-r_2)^2} = 5\left(\frac{12}{5}\right)^{1/4}$

SECTION-IV : (INTEGER ANSWER TYPE)

This section contains **04** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. 6.25, 7.00, 0.33, 30.27, 127.30)

57. Let x be the solution set of the equation $A^x = I$, where $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ and I is the corresponding unit matrix and $x \subseteq N$, then the minimum value of $\Sigma(\cos^x\theta + \sin^x\theta), \theta \in R - \frac{n\pi}{2}$ is…………

57. **(2)**

We, have $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$

$\Rightarrow \quad A^2 = A.A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix} . \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$

$$=\begin{bmatrix} 0+4-3 & 0-3+3 & 0+4-4 \\ 0-12+12 & 4+9-12 & -4-12+16 \\ 0-12+12 & 3+9-12 & -3-12+16 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow \quad A^2 = A^4 = A^6 = ... = I$

Now, $A^x = I \Rightarrow x = 2, 4, 6, 8, ...$

$\therefore \quad \Sigma\left(\cos^x\theta + \sin^x\theta\right)$

$=\left(\cos^2\theta+\sin^2\theta\right)+\left(\cos^4\theta+\sin^4\theta\right)+\left(\cos^6\theta+\sin^6\theta\right)+......$

$=\left(\cos^2\theta+\cos^4\theta+\cos^6\theta+.....\right)+\left(\sin^2\theta+\sin^4\theta+\sin^6\theta+....\right)$

$=\frac{\cos^2\theta}{1-\cos^2\theta}+\frac{\sin^2\theta}{1-\sin^2\theta}=\cot^2\theta+\tan^2\theta$

Which has minimum value of 2.

58. The largest value of the non – negative integer a for which

$\lim_{x\to1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}}=\frac{1}{4}$ is ……….

58. (0)

$$\lim_{x\to1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}}=\frac{1}{4}$$

$$\Rightarrow \quad \lim_{x\to1}\left(\frac{\frac{\sin(x-1)}{(x-1)}-a}{\frac{\sin(x-1)}{(x-1)}+1}\right)^{1+\sqrt{x}}=\frac{1}{4}$$

$$\Rightarrow \quad \left(\frac{1-a}{2}\right)^2=\frac{1}{4}\Rightarrow a=0, a=2$$

59. Let $f: R \to R$ be a function defined by $f(x)=\begin{cases}[x], & x\le 2\\ 0, & x>2\end{cases}$, where [x] is the greatest integer less than or equal to x. If $I=\int_{-1}^{2}\frac{xf(x^2)}{2+f(x+1)}dx$, then the value of $(4I-1)$ is ……….

59. (0)

We have, $f(x)=\begin{cases}[x], & x\le 2\\ 0, & x>2\end{cases}$

$$\therefore \quad I=\int_{-1}^{2}\frac{xf(x^2)}{2+f(x+1)}dx$$

$$\Rightarrow \quad I=\int_{-1}^{0}\frac{x\times0}{2+0}dx+\int_{0}^{1}\frac{x\times0}{2+1}dx+\int_{1}^{\sqrt{2}}\frac{x\times1}{2+0}dx+\int_{\sqrt{2}}^{2}\frac{x\times0}{2+f(x+1)}dx$$

$\Rightarrow \quad I = \frac{1}{2}\left[\frac{x^2}{2}\right]_1^{\sqrt{2}} = \frac{1}{2}\left(1 - \frac{1}{2}\right) = \frac{1}{4}$

$\therefore \quad 4I - 1 = 4 \times \frac{1}{4} - 1 = 1 - 1 = 0$

60. The least positive integral value of x for which the angle between vectors $a = x\hat{i} - 3\hat{j} - \hat{k}$ and $b = 2x\hat{i} + x\hat{j} - \hat{k}$ is acute, is ………

60. **(2)**

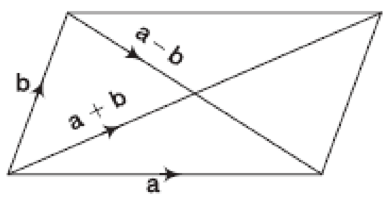
Let $a = x\hat{i} - 3\hat{j} - \hat{k}$ and $b = 2x\hat{i} + x\hat{i} + x\hat{j} - \hat{k}$ be the adjacent sides of the parallelogram.

Now, angle between a and b is acute, i.e.

$|a + b| > |a - b|$

$\Rightarrow \quad \left|3x\hat{i} + (x-3)\hat{j} - 2\hat{k}\right|^2 > \left|-x\hat{i} - (x+3)\hat{j}\right|^2$

$\Rightarrow \quad 9x^2 + (x-3)^2 + 4 > x^2 + (x+3)^2$



$\Rightarrow \quad 8x^2 - 12x + 4 > 0 \qquad \Rightarrow 2x^2 - 3x + 1 > 0$

$\Rightarrow \quad (2x-1)(x-1) > 0 \Rightarrow x < 1/2 \text{ or } x > 1$

Hence, the least positive integral value is 2.