BANSALCLASSES

TARGET IIT JEE 2007

MATHEMATICS

NUCLEUS

QUESTION BANK ON

DEFINITE & INDEFINITE INTEGRATION

Time Limit : 5 Sitting Each of 100 Minutes duration approx.

Question bank on Definite & Indefinite Integration

There are 168 questions in this question bank.

<u>Select the correct alternative</u> : (Only one is correct)

Q.1 The value of the definite integral, $\int_1^{\infty} (e^{x+1} + e^{3-x})^{-1} dx$ is

(A) $\frac{\pi}{4e^2}$ (B) $\frac{\pi}{4e}$ (C) $\frac{1}{e^2}\left(\frac{\pi}{2} - \tan^{-1}\frac{1}{e}\right)$ (D) $\frac{\pi}{2e^2}$

Q.2 The value of the definite integral, $\int_0^{\sqrt{ln\left(\frac{\pi}{2}\right)}} \cos\left(e^{x^2}\right)\cdot 2x\, e^{x^2} dx$ is

(A) 1 (B) 1 + (sin 1) (C) 1 – (sin 1) (D) (sin 1) – 1

Q.3 Value of the definite integral $\int_{-1/2}^{1/2} (\sin^{-1}(3x - 4x^3) - \cos^{-1}(4x^3 - 3x))\, dx$

(A) 0 (B) $-\frac{\pi}{2}$ (C) $\frac{7\pi}{2}$ (D) $\frac{\pi}{2}$

Q.4 Let $f(x) = \int_2^x \frac{dt}{\sqrt{1+t^4}}$ and g be the inverse of f. Then the value of g'(0) is

(A) 1 (B) 17 (C) $\sqrt{17}$ (D) none of these

Q.5 $\int \frac{\cot^{-1}(e^x)}{e^x} dx$ is equal to :

(A) $\frac{1}{2} ln (e^{2x} + 1) - \frac{\cot^{-1}(e^x)}{e^x} + x + c$ (B) $\frac{1}{2} ln (e^{2x} + 1) + \frac{\cot^{-1}(e^x)}{e^x} + x + c$

(C) $\frac{1}{2} ln (e^{2x} + 1) - \frac{\cot^{-1}(e^x)}{e^x} - x + c$ (D) $\frac{1}{2} ln (e^{2x} + 1) + \frac{\cot^{-1}(e^x)}{e^x} - x + c$

Q.6 $\lim_{k\to 0} \frac{1}{k}\int_0^k (1+\sin 2x)^{\frac{1}{x}} dx$

(A) 2 (B) 1 (C) e^2 (D) non existent

Q.7 $\int_0^{ln 5} \frac{e^x\sqrt{e^x - 1}}{e^x + 3} dx =$

(A) $4 - \pi$ (B) $6 - \pi$ (C) $5 - \pi$ (D) None

Q.8 If x satisfies the equation $\left(\int_0^1 \frac{dt}{t^2+2t\cos\alpha+1}\right)x^2 - \left(\int_{-3}^{3} \frac{t^2 \sin 2t}{t^2+1}dt\right)x - 2 = 0$ $(0 < \alpha < \pi)$, then the value x is

(A) $\pm\sqrt{\frac{\alpha}{2\sin\alpha}}$ (B) $\pm\sqrt{\frac{2\sin\alpha}{\alpha}}$ (C) $\pm\sqrt{\frac{\alpha}{\sin\alpha}}$ (D) $\pm 2\sqrt{\frac{\sin\alpha}{\alpha}}$

Q.9 If $f(x) = e^{g(x)}$ and $g(x) = \int_2^x \frac{t\ dt}{1+t^4}$ then $f'(2)$ has the value equal to :

(A) 2/17 (B) 0 (C) 1 (D) cannot be determined

Q.10 $\int e^{\tan\theta}(\sec\theta - \sin\theta)\,d\theta$ equals :

(A) $-e^{\tan\theta}\sin\theta + c$ (B) $e^{\tan\theta}\sin\theta + c$ (C) $e^{\tan\theta}\sec\theta + c$ (D) $e^{\tan\theta}\cos\theta + c$

Q.11 $\int_0^{\pi} (x \cdot \sin^2 x \cdot \cos x)\,dx =$

(A) 0 (B) 2/9 (C) – 2/9 (D) – 4/9

Q.12 The value of $\lim_{n\to\infty} \sum_{r=1}^{r=4n} \frac{\sqrt{n}}{\sqrt{r}\left(3\sqrt{r}+4\sqrt{n}\right)^2}$ is equal to

(A) $\frac{1}{35}$ (B) $\frac{1}{14}$ (C) $\frac{1}{10}$ (D) $\frac{1}{5}$

Q.13 $\int_{a-c}^{b-c} f(x+c)\,dx =$

(A) $\int_a^b f(x)dx$ (B) $\int_a^b f(x+c)dx$ (C) $\int_{a-2c}^{b-2c} f(x)dx$ (D) $\int_a^b f(x+2c)dx$

Q.14 Let $I_1 = \int_0^{\pi/2} \frac{\sin x - \cos x}{1+\sin x.\cos x}dx$; $I_2 = \int_0^{2\pi} (\cos^6 x)dx$; $I_3 = \int_{-\pi/2}^{\pi/2} (\sin^3 x)dx$ & $I_4 = \int_0^1 ln\left(\frac{1}{x}-1\right)dx$ then

(A) $I_1 = I_2 = I_3 = I_4 = 0$ (B) $I_1 = I_2 = I_3 = 0$ but $I_4 \neq 0$
(C) $I_1 = I_3 = I_4 = 0$ but $I_2 \neq 0$ (D) $I_1 = I_2 = I_4 = 0$ but $I_3 \neq 0$

Q.15 $\int \frac{1-x^7}{x(1+x^7)}\,dx$ equals :

(A) $ln\,x + \frac{2}{7}\,ln\,(1 + x^7) + c$ (B) $ln\,x - \frac{2}{7}\,ln\,(1 - x^7) + c$

(C) $ln\,x - \frac{2}{7}\,ln\,(1 + x^7) + c$ (D) $ln\,x + \frac{2}{7}\,ln\,(1 - x^7) + c$

Q.16 $\int_0^{\pi/2n} \frac{dx}{1+\tan^n nx} =$

(A) 0 (B) $\frac{\pi}{4n}$ (C) $\frac{n\pi}{4}$ (D) $\frac{\pi}{2n}$

Q.17 $f(x) = \int_0^x t(t-1)(t-2)\,dt$ takes on its minimum value when:

(A) $x = 0, 1$ (B) $x = 1, 2$ (C) $x = 0, 2$ (D) $x = \frac{3+\sqrt{3}}{3}$

Q.18 $\int_{-a}^{a} f(x)\,dx =$

(A) $\int_0^a [f(x)+f(-x)]dx$ (B) $\int_0^a [f(x)-f(-x)]dx$ (C) $2\int_0^a f(x)\,dx$ (D) Zero

Q.19 Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and g be the function satisfying $f(x) + g(x) = x^2$. The value of the integral $\int_0^1 f(x)g(x)\,dx$ is

(A) $e - \frac{1}{2}e^2 - \frac{5}{2}$ (B) $e - e^2 - 3$ (C) $\frac{1}{2}(e-3)$ (D) $e - \frac{1}{2}e^2 - \frac{3}{2}$

Q.20 $\int \frac{\ln|x|}{x\sqrt{1+\ln|x|}}\,dx$ equals :

(A) $\frac{2}{3}\sqrt{1+\ln|x|}\,(\ln|x| - 2) + c$ (B) $\frac{2}{3}\sqrt{1+\ln|x|}\,(\ln|x| + 2) + c$

(C) $\frac{1}{3}\sqrt{1+\ln|x|}\,(\ln|x| - 2) + c$ (D) $2\sqrt{1+\ln|x|}\,(3\ln|x| - 2) + c$

Q.21 $\int_{\frac{1}{2}}^{3\frac{1}{2}} \left\{\frac{1}{2}(|x-3|+|1-x|-4)\right\} dx$ equals:

(A) $-\frac{3}{2}$ (B) $\frac{9}{8}$ (C) $\frac{1}{4}$ (D) $\frac{3}{2}$

Where {*} denotes the fractional part function.

Q.22 $\int_0^{4/\pi} \left(3x^2.\sin\frac{1}{x} - x.\cos\frac{1}{x}\right) dx$ has the value :

(A) $\frac{8\sqrt{2}}{\pi^3}$ (B) $\frac{24\sqrt{2}}{\pi^3}$ (C) $\frac{32\sqrt{2}}{\pi^3}$ (D) None

Q.23 $\lim_{n\to\infty} \frac{\pi}{6n}\left[\sec^2\left(\frac{\pi}{6n}\right) + \sec^2\left(2\cdot\frac{\pi}{6n}\right) + ... + \sec^2(n-1)\frac{\pi}{6n} + \frac{4}{3}\right]$ has the value equal to

(A) $\frac{\sqrt{3}}{3}$ (B) $\sqrt{3}$ (C) 2 (D) $\frac{2}{\sqrt{3}}$

Q.24 Suppose that F (x) is an antiderivative of $f(x)=\frac{\sin x}{x}$, x > 0 then $\int_1^3 \frac{\sin 2x}{x}dx$ can be expressed as

(A) F (6) – F (2) (B) $\frac{1}{2}$(F (6) – F (2)) (C) $\frac{1}{2}$(F (3) – F (1)) (D) 2(F (6) – F (2))

Q.25 Primitive of $\frac{3x^4-1}{(x^4+x+1)^2}$ w.r.t. x is :

(A) $\frac{x}{x^4+x+1}+c$ (B) $-\frac{x}{x^4+x+1}+c$ (C) $\frac{x+1}{x^4+x+1}+c$ (D) $-\frac{x+1}{x^4+x+1}+c$

Q.26 $\lim_{n\to\infty}\frac{\pi}{2n}\left(1+\cos\frac{\pi}{2n}+\cos\frac{2\pi}{2n}+.....+\cos\frac{(n-1)\pi}{2n}\right)$ equal to

(A) 1 (B) $\frac{1}{2}$ (C) 2 (D) none

Q.27 $\int_2^4\left[\log_x 2-\frac{(\log_x 2)^2}{\ell n 2}\right]dx=$

(A) 0 (B) 1 (C) 2 (D) 4

Q.28 If *m* & *n* are integers such that (m – n) is an odd integer then the value of the definite integral $\int_0^\pi \cos mx \cdot \sin nx\, dx$ =

(A) 0 (B) $\frac{2n}{n^2-m^2}$ (C) $\frac{2m}{n^2-m^2}$ (D) none

Q.29 Let $y=\{x\}^{[x]}$ where {x} denotes the fractional part of x & [x] denotes greatest integer ≤ x, then $\int_0^3 y\,dx$ =

(A) 5/6 (B) 2/3 (C) 1 (D) 11/6

Q.30 If $\int\frac{x^4+1}{x(x^2+1)^2}dx = A\ln|x|+\frac{B}{1+x^2}+c$, where c is the constant of integration then :

(A) A = 1 ; B = – 1 (B) A = – 1 ; B = 1 (C) A = 1 ; B = 1 (D) A = – 1 ; B = – 1

Q.31 $\int_{\pi/2}^{\pi}\frac{1-\sin x}{1-\cos x}dx=$

(A) $1-\ln 2$ (B) $\ln 2$ (C) $1+\ln 2$ (D) none

Q.32 Let f : R → R be a differentiable function & f(1) = 4, then the value of ; $\lim_{x\to 1}\int_4^{f(x)}\frac{2t\,dt}{x-1}$ is :

(A) $f'(1)$ (B) $4f'(1)$ (C) $2f'(1)$ (D) $8f'(1)$

Q.33 If $\int_0^{f(x)} t^2\,dt = x\cos \pi x$, then $f'(9)$

(A) is equal to $-\frac{1}{9}$ (B) is equal to $-\frac{1}{3}$ (C) is equal to $\frac{1}{3}$ (D) is non existent

Q.34 $\int_0^{(\pi/2)^{1/3}} x^5 \cdot \sin x^3\,dx =$

(A) 1 (B) 1/2 (C) 2 (D) 1/3

Q.35 Integral of $\sqrt{1+2\cot x(\cot x+\operatorname{cosec} x)}$ w.r.t. x is :

(A) $2\,ln\cos\frac{x}{2}+c$ (B) $2\,ln\sin\frac{x}{2}+c$

(C) $\frac{1}{2}\,ln\cos\frac{x}{2}+c$ (D) $ln\sin x - ln(\operatorname{cosec} x-\cot x)+c$

Q.36 If $f(x)=|x|+|x-1|+|x-2|$, $x\in R$ then $\int_0^3 f(x)\,dx =$

(A) 9/2 (B) 15/2 (C) 19/2 (D) none

Q.37 Number of values of x satisfying the equation $\int_{-1}^{x}\left(8t^2+\frac{28}{3}t+4\right)dt = \frac{\left(\frac{3}{2}\right)x+1}{\log_{(x+1)}\sqrt{x+1}}$, is

(A) 0 (B) 1 (C) 2 (D) 3

Q.38 $\int_0^1 \frac{\tan^{-1}x}{x}\,dx =$

(A) $\int_0^{\pi/4}\frac{\sin x}{x}\,dx$ (B) $\int_0^{\pi/2}\frac{x}{\sin x}\,dx$ (C) $\frac{1}{2}\int_0^{\pi/2}\frac{x}{\sin x}\,dx$ (D) $\frac{1}{2}\int_0^{\pi/4}\frac{x}{\sin x}\,dx$

Q.39 Domain of definition of the function $f(x)=\int_0^x \frac{dt}{\sqrt{x^2+t^2}}$ is

(A) R (B) R^+ (C) $R^+\cup\{0\}$ (D) $R-\{0\}$

Q.40 If $\int e^{3x}\cos 4x\,dx = e^{3x}(A\sin 4x+B\cos 4x)+c$ then :

(A) 4A = 3B (B) 2A = 3B (C) 3A = 4B (D) 4B + 3A = 1

Q.41 If $f(a+b-x)=f(x)$, then $\int_a^b x.f(a+b-x)\,dx =$

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{a+b}{2}\int_a^b f(x)\,dx$ (D) $\frac{a-b}{2}\int_a^b f(x)\,dx$

Q.42 The set of values of 'a' which satisfy the equation $\int_0^2 (t - \log_2 a)\, dt = \log_2\left(\frac{4}{a^2}\right)$ is

(A) $a \in R$ (B) $a \in R^+$ (C) $a < 2$ (D) $a > 2$

Q.43 The value of the definite integral $\int_2^3 \left[\sqrt{2x - \sqrt{5(4x-5)}} + \sqrt{2x + \sqrt{5(4x-5)}}\right] dx =$

(A) $\frac{7\sqrt{3} + 3\sqrt{5}}{3\sqrt{2}}$ (B) $4\sqrt{2}$ (C) $4\sqrt{3} + \frac{4}{3}$ (D) $\frac{7\sqrt{7} - 2\sqrt{5}}{3\sqrt{2}}$

Q.44 Number of ordered pair(s) of (a, b) satisfying simultaneously the system of equation

$\int_a^b x^3 dx = 0$ and $\int_a^b x^2 dx = \frac{2}{3}$ is

(A) 0 (B) 1 (C) 2 (D) 4

Q.45 $\int \frac{\tan^{-1}x - \cot^{-1}x}{\tan^{-1}x + \cot^{-1}x}\, dx$ is equal to :

(A) $\frac{4}{\pi}\, x \tan^{-1}x + \frac{2}{\pi}\, \ln(1 + x^2) - x + c$ (B) $\frac{4}{\pi}\, x \tan^{-1}x - \frac{2}{\pi}\, \ln(1 + x^2) + x + c$

(C) $\frac{4}{\pi}\, x \tan^{-1}x + \frac{2}{\pi}\, \ln(1 + x^2) + x + c$ (D) $\frac{4}{\pi}\, x \tan^{-1}x - \frac{2}{\pi}\, \ln(1 + x^2) - x + c$

Q.46 Variable x and y are related by equation $x = \int_0^y \frac{dt}{\sqrt{1+t^2}}$. The value of $\frac{d^2y}{dx^2}$ is equal to

(A) $\frac{y}{\sqrt{1+y^2}}$ (B) y (C) $\frac{2y}{\sqrt{1+y^2}}$ (D) 4y

Q.47 Let $f(x) = \lim_{h \to 0} \frac{1}{h} \int_x^{x+h} \frac{dt}{t + \sqrt{1+t^2}}$, then $\lim_{x \to -\infty} x \cdot f(x)$ is

(A) equal to 0 (B) equal to $\frac{1}{2}$ (C) equal to 1 (D) non existent

Q.48 If the primitive of $f(x) = \pi \sin \pi x + 2x - 4$, has the value 3 for x = 1, then the set of x for which the primitive of f (x) vanishes is :

(A) {1, 2, 3} (B) (2, 3) (C) {2} (D) {1, 2, 3, 4}

Q.49 If f & g are continuous functions in [0, a] satisfying f (x) = f (a – x) & g (x) + g (a – x) = 4 then $\int_0^a f(x).g(x)dx =$

(A) $\frac{1}{2}\int_0^a f(x)dx$ (B) $2\int_0^a f(x)dx$ (C) $\int_0^a f(x)dx$ (D) $4\int_0^a f(x)dx$

Q.50 $\int x \cdot \frac{\ln\left(x+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}} dx$ equals :

(A) $\sqrt{1+x^2}\ \ln\left(x+\sqrt{1+x^2}\right) - x + c$
(B) $\frac{x}{2} \cdot \ln^2\left(x+\sqrt{1+x^2}\right) - \frac{x}{\sqrt{1+x^2}} + c$
(C) $\frac{x}{2} \cdot \ln^2\left(x+\sqrt{1+x^2}\right) + \frac{x}{\sqrt{1+x^2}} + c$
(D) $\sqrt{1+x^2}\ \ln\left(x+\sqrt{1+x^2}\right) + x + c$

Q.51 If $f(x) = \begin{cases} \sqrt{1-x} & 0 \le x \le 1 \\ (7x-6)^{-1/3} & 1 < x \le 2 \end{cases}$, then $\int_0^2 f(x)\,dx$ is equal to

(A) $\frac{31}{6}$ (B) $\frac{32}{21}$ (C) $\frac{1}{42}$ (D) $\frac{55}{42}$

Q.52 The value of the definite integral $\int_0^1 e^{e^x}(1+x\cdot e^x)dx$ is equal to

(A) e^e (B) $e^e - e$ (C) $e^e - 1$ (D) e

Q.53 $\int_{1/2}^{2} \frac{1}{x}\sin\left(x-\frac{1}{x}\right) dx$ has the value equal to

(A) 0 (B) $\frac{3}{4}$ (C) $\frac{5}{4}$ (D) 2

Q.54 The value of the integral $\int_0^{\infty} e^{-2x}(\sin 2x + \cos 2x)\,dx =$

(A) 1 (B) –2 (C) 1/2 (D) zero

Q.55 The value of definite integral $\int_{\infty}^{0} \frac{z\,e^{-z}}{\sqrt{1-e^{-2z}}} dz$.

(A) $-\frac{\pi}{2}\ln 2$ (B) $\frac{\pi}{2}\ln 2$ (C) $-\pi \ln 2$ (D) $\pi \ln 2$

Q.56 A differentiable function satisfies
$3f^2(x)\,f'(x) = 2x$. Given $f(2) = 1$ then the value of $f(3)$ is

(A) $\sqrt[3]{24}$ (B) $\sqrt[3]{6}$ (C) 6 (D) 2

Q.57 For $I_n = \int_1^e (\ln x)^n dx$, $n \in N$; which of the following holds good?

(A) $I_n + (n+1)\,I_{n+1} = e$
(B) $I_{n+1} + n\,I_n = e$
(C) $I_{n+1} + (n+1)\,I_n = e$
(D) $I_{n+1} + (n-1)\,I_n = e$

Q.58 Let f be a continuous functions satisfying $f'(\ln x) = \begin{cases} 1 & \text{for } 0 < x \le 1 \\ x & \text{for } x > 1 \end{cases}$ and $f(0) = 0$ then $f(x)$ can be defined as

(A) $f(x) = \begin{cases} 1 & \text{if } x \le 0 \\ 1 - e^x & \text{if } x > 0 \end{cases}$ (B) $f(x) = \begin{cases} 1 & \text{if } x \le 0 \\ e^x - 1 & \text{if } x > 0 \end{cases}$

(C) $f(x) = \begin{cases} x & \text{if } x < 0 \\ e^x & \text{if } x > 0 \end{cases}$ (D) $f(x) = \begin{cases} x & \text{if } x \le 0 \\ e^x - 1 & \text{if } x > 0 \end{cases}$

Q.59 Let $f : R \to R$ be a differentiable function such that $f(2) = 2$. Then the value of $\underset{x \to 2}{\text{Limit}} \int_2^{f(x)} \frac{4t^3}{x-2}\, dt$ is

(A) $6\, f'(2)$ (B) $12\, f'(2)$ (C) $32\, f'(2)$ (D) none

Q.60 $\int_0^{\pi/2} \frac{dx}{1 + a^2 \sin^2 x}$ has the value :

(A) $\frac{\pi}{2\sqrt{1+a^2}}$ (B) $\frac{\pi}{\sqrt{1+a^2}}$ (C) $\frac{2\pi}{\sqrt{1+a^2}}$ (D) none

Q.61 Let $f(x) = \frac{1}{x} \ln\left(\frac{x}{e^x}\right)$ then its primitive w.r.t. x is

(A) $\frac{1}{2} e^x - \ln x + C$ (B) $\frac{1}{2} \ln x - e^x + C$ (C) $\frac{1}{2} \ln^2 x - x + C$ (D) $\frac{e^x}{2x} + C$

Q.62 $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{n}{n^2 + k^2 x^2}$, $x > 0$ is equal to

(A) $x \tan^{-1}(x)$ (B) $\tan^{-1}(x)$ (C) $\frac{\tan^{-1}(x)}{x}$ (D) $\frac{\tan^{-1}(x)}{x^2}$

Q.63 Let $f(x) = \begin{vmatrix} 2\cos^2 x & \sin(2x) & -\sin x \\ \sin 2x & 2\sin^2 x & \cos x \\ \sin x & -\cos x & 0 \end{vmatrix}$ then $\int_0^{\pi/2} [f(x) + f'(x)]\, dx =$

(A) π (B) $\pi/2$ (C) 2π (D) zero

Q.64 The absolute value of $\int_{10}^{19} \frac{\sin x}{1 + x^8}$ is less than :

(A) 10^{-10} (B) 10^{-11} (C) 10^{-7} (D) 10^{-9}

Q.65 The value of the integral $\int_{-\pi}^{\pi} (\cos px - \sin qx)^2\, dx$ where p, q are integers, is equal to :

(A) $-\pi$ (B) 0 (C) π (D) 2π

Q.66 Primitive of $f(x) = x \cdot 2^{ln(x^2+1)}$ w.r.t. x is

(A) $\dfrac{2^{ln(x^2+1)}}{2(x^2+1)} + C$ (B) $\dfrac{(x^2+1)2^{ln(x^2+1)}}{ln 2+1} + C$

(C) $\dfrac{(x^2+1)^{ln 2+1}}{2(ln 2+1)} + C$ (D) $\dfrac{(x^2+1)^{ln 2}}{2(ln 2+1)} + C$

Q.67 $\lim\limits_{n\to\infty} \int\limits_0^2 \left(1+\dfrac{t}{n+1}\right)^n dt$ is equal to

(A) 0 (B) e^2 (C) $e^2 - 1$ (D) does not exist

Q.68 $\text{Limit}\limits_{h\to 0} \dfrac{\int\limits_a^{x+h} \ell n^2 t \; dt - \int\limits_a^{x} \ell n^2 t \; dt}{h} =$

(A) 0 (B) $\ln^2 x$ (C) $\dfrac{2\ell n x}{x}$ (D) does not exist

Q.69 Let a, b, c be non–zero real numbers such that ;

$\int\limits_0^1 (1 + \cos^8 x)(ax^2 + bx + c)\,dx = \int\limits_0^2 (1 + \cos^8 x)(ax^2 + bx + c)\,dx$, then the quadratic equation $ax^2 + bx + c = 0$ has :

(A) no root in (0, 2) (B) atleast one root in (0, 2)

(C) a double root in (0, 2) (D) none

Q.70 Let $I_n = \int\limits_0^{\pi/4} \tan^n x\,dx$, then $\dfrac{1}{I_2+I_4}, \dfrac{1}{I_3+I_5}, \dfrac{1}{I_4+I_6}$,..... are in :

(A) A.P. (B) G.P. (C) H.P. (D) none

Q.71 Let $g(x)$ be an antiderivative for $f(x)$. Then $ln\left(1+(g(x))^2\right)$ is an antiderivative for

(A) $\dfrac{2f(x)g(x)}{1+(f(x))^2}$ (B) $\dfrac{2f(x)g(x)}{1+(g(x))^2}$ (C) $\dfrac{2f(x)}{1+(f(x))^2}$ (D) none

Q.72 $\int\limits_0^{\pi/4} (\cos 2x)^{3/2} . \cos x\,dx =$

(A) $\dfrac{3\pi}{16}$ (B) $\dfrac{3\pi}{32}$ (C) $\dfrac{3\pi}{16\sqrt{2}}$ (D) $\dfrac{3\pi\sqrt{2}}{16}$

Q.73 The value of the definite integral $\int\limits_0^{1/\sqrt{2}} \dfrac{x^2 dx}{\sqrt{1-x^2}\,(1+\sqrt{1-x^2})}$ is

(A) $\dfrac{\pi}{4}$ (B) $\dfrac{\pi}{4}+\dfrac{1}{\sqrt{2}}$ (C) $\dfrac{\pi}{4}-\dfrac{1}{\sqrt{2}}$ (D) none

Q.74 The value of the definite integral $\int_{19}^{37}\left(\{x\}^2 + 3(\sin 2\pi x)\right)dx$ where { x } denotes the fractional part function.

(A) 0 (B) 6 (C) 9 (D) can not be determined

Q.75 The value of the definite integral $\int_{0}^{\pi/2}\sqrt{\tan x}\,dx$, is

(A) $\sqrt{2}\,\pi$ (B) $\frac{\pi}{\sqrt{2}}$ (C) $2\sqrt{2}\,\pi$ (D) $\frac{\pi}{2\sqrt{2}}$

Q.76 Evaluate the integral : $\int\frac{ln(6x^2)}{x}dx$

(A) $\frac{1}{8}[ln(6x^2)]^3 + C$ (B) $\frac{1}{4}[ln^2(6x^2)] + C$

(C) $\frac{1}{2}[ln(6x^2)] + C$ (D) $\frac{1}{16}[ln(6x^2)]^4 + C$

Q.77 $\int_{\pi/6}^{5\pi/6}\left(\frac{1}{2}(3\sin\theta)^2 - \frac{1}{2}(1+\sin\theta)^2\right)d\theta$

(A) $\pi - \sqrt{3}$ (B) π (C) $\pi - 2\sqrt{3}$ (D) $\pi + \sqrt{3}$

Q.78 Let $l = \lim_{x\to\infty}\int_{x}^{2x}\frac{dt}{t}$ and $m = \lim_{x\to\infty}\frac{1}{x\,ln\,x}\int_{1}^{x}ln\,t\,dt$ then the correct statement is

(A) $l\,m = l$ (B) $l\,m = m$ (C) $l = m$ (D) $l > m$

Q.79 If f (x) = $e^{-x} + 2\,e^{-2x} + 3\,e^{-3x} + ... + \infty$, then $\int_{ln 2}^{ln 3} f(x)\,dx =$

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $ln\,2$

Q.80 If $I = \int_{0}^{\pi/2}\ell n(\sin x)\,dx$ then $\int_{-\pi/4}^{\pi/4}\ell n(\sin x + \cos x)\,dx =$

(A) $\frac{I}{2}$ (B) $\frac{I}{4}$ (C) $\frac{I}{\sqrt{2}}$ (D) I

Q.81 The value of $\int_{0}^{1}\left(\prod_{r=1}^{n}(x+r)\right)\left(\sum_{k=1}^{n}\frac{1}{x+k}\right)dx$ equals

(A) n (B) n ! (C) (n + 1) ! (D) n · n !

Q.82 $\int\frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x}\,dx$

(A) $\sin x - 6\tan^{-1}(\sin x) + c$ (B) $\sin x - 2\sin^{-1}x + c$
(C) $\sin x - 2(\sin x)^{-1} - 6\tan^{-1}(\sin x) + c$ (D) $\sin x - 2(\sin x)^{-1} + 5\tan^{-1}(\sin x) + c$

Q.83 $\int_0^3 \left(\frac{1}{\sqrt{x^2+4x+4}} + \sqrt{x^2-4x+4}\right) dx =$

(A) $\ln\frac{5}{2}-\frac{3}{2}$ (B) $\ln\frac{5}{2}+\frac{3}{2}$ (C) $\ln\frac{5}{2}+\frac{5}{2}$ (D) none

Q.84 The value of the function $f(x) = 1 + x + \int_1^x (\ln^2 t + 2\ln t)\, dt$ where $f'(x)$ vanishes is :

(A) e^{-1} (B) 0 (C) $2e^{-1}$ (D) $1 + 2e^{-1}$

Q.85 $\underset{n\to\infty}{\text{Limit}} \frac{1}{n}\left[1 + \sqrt{\frac{n}{n+1}} + \sqrt{\frac{n}{n+2}} + \sqrt{\frac{n}{n+3}} + \ldots\ldots + \sqrt{\frac{n}{n+3(n-1)}}\right]$ has the value equal to

(A) $2\sqrt{2}$ (B) $2\sqrt{2}-1$ (C) 2 (D) 4

Q.86 Let a function $h(x)$ be defined as $h(x) = 0$, for all $x \neq 0$. Also $\int_{-\infty}^{\infty} h(x)\cdot f(x)\,dx = f(0)$, for every function $f(x)$. Then the value of the definite integral $\int_{-\infty}^{\infty} h'(x)\cdot \sin x\, dx$, is

(A) equal to zero (B) equal to 1 (C) equal to – 1 (D) non existent

Q.87 $\int_0^{\pi/4} (\tan^n x + \tan^{n-2} x)\, d(x-[x])$ is : ([·] denotes greatest integer function)

(A) $\frac{1}{n-1}$ (B) $\frac{1}{n+2}$ (C) $\frac{2}{n-1}$ (D) none of these

Q.88 $\lim_{\lambda\to 0}\left(\int_0^1 (1+x)^{\lambda}\, dx\right)^{1/\lambda}$ is equal to

(A) $2\ln 2$ (B) $\frac{4}{e}$ (C) $\ln\frac{4}{e}$ (D) 4

Q.89 Which one of the following is TRUE.

(A) $x.\int\frac{dx}{x} = x\ln|x| + C$ (B) $x.\int\frac{dx}{x} = x\ln|x| + Cx$

(C) $\frac{1}{\cos x}.\int \cos x\ dx = \tan x + C$ (D) $\frac{1}{\cos x}.\int \cos x\ dx = x + C$

Q.90 $\int_0^\infty x^{2n+1} \cdot e^{-x^2}\, dx$ is equal to $(n \in N)$.

(A) $n!$ (B) $2(n!)$ (C) $\frac{n!}{2}$ (D) $\frac{(n+1)!}{2}$

Q.91 The true set of values of 'a' for which the inequality $\int_a^0 (3^{-2x} - 2.\,3^{-x})\, dx \geq 0$ is true is:

(A) $[0, 1]$ (B) $(-\infty, -1]$ (C) $[0, \infty)$ (D) $(-\infty, -1] \cup [0, \infty)$

Q.92 If $\alpha \in (2, 3)$ then number of solution of the equation $\int_0^\alpha \cos(x + \alpha^2)\, dx = \sin\alpha$ is :

(A) 1 (B) 2 (C) 3 (D) 4.

Q.93 If $x \cdot \sin \pi x = \int_0^{x^2} f(t)\, dt$ where f is continuous functions then the value of $f(4)$ is

(A) $\frac{\pi}{2}$ (B) 1 (C) $\frac{1}{2}$ (D) can not be determined

Q.94 $\int \frac{(2x+1)}{(x^2+4x+1)^{3/2}}\, dx$

(A) $\frac{x^3}{(x^2+4x+1)^{1/2}} + C$ (B) $\frac{x}{(x^2+4x+1)^{1/2}} + C$

(C) $\frac{x^2}{(x^2+4x+1)^{1/2}} + C$ (D) $\frac{1}{(x^2+4x+1)^{1/2}} + C$

Q.95 If the value of the integral $\int_1^2 e^{x^2}\, dx$ is α, then the value of $\int_e^{e^4} \sqrt{\ell n\, x}\, dx$ is :

(A) $e^4 - e - \alpha$ (B) $2e^4 - e - \alpha$ (C) $2(e^4 - e) - \alpha$ (D) $2e^4 - 1 - \alpha$

Q.96 $\int_0^{\sqrt{3}} \frac{1}{2} \frac{d}{dx}\left(\tan^{-1} \frac{2x}{1-x^2}\right)$ equals

(A) $\frac{\pi}{3}$ (B) $-\frac{\pi}{6}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

Q.97 Let $A = \int_0^1 \frac{e^t\, dt}{1+t}$ then $\int_{a-1}^{a} \frac{e^{-t}\, dt}{t-a-1}$ has the value

(A) Ae^{-a} (B) $-Ae^{-a}$ (C) $-ae^{-a}$ (D) Ae^{a}

Q.98 $\int_0^{\pi/2} \sqrt{\sin 2\theta}\ \sin\theta\, d\theta$ is equal to :

(A) 0 (B) $\pi/4$ (C) $\pi/2$ (D) π

Q.99 $\int \frac{x^2+2}{x^4+4}\,dx$ is equal to

(A) $\frac{1}{2}\ \tan^{-1}\frac{x^2+2}{2x} + C$

(B) $\frac{1}{2}\ \tan^{-1}(x^2+2) + C$

(C) $\frac{1}{2}\ \tan^{-1}\frac{2x}{x^2-2} + C$

(D) $\frac{1}{2}\ \tan^{-1}\frac{x^2-2}{2x} + C$

Q.100 If $\beta + 2\int_0^1 x^2\, e^{-x^2}\,dx = \int_0^1 e^{-x^2}\,dx$ then the value of β is

(A) e^{-1} (B) e (C) 1/2e (D) can not be determined

Q.101 A quadratic polynomial P(x) satisfies the conditions, P(0) = P(1) = 0 & $\int_0^1 P(x)\,dx = 1$. The leading coefficient of the quadratic polynomial is :

(A) 6 (B) – 6 (C) 2 (D) 3

Q.102 Which one of the following functions is not continuous on $(0,\pi)$?

(A) f(x)= cotx

(B) $g(x) = \int_0^x t\sin\frac{1}{t}\,dt$

(C) $h(x) = \begin{cases} 1 & 0 < x \le \frac{3\pi}{4} \\ 2\sin\frac{2}{9}x & \frac{3\pi}{4} < x < \pi \end{cases}$

(D) $l(x) = \begin{cases} x\sin x\ , & 0 < x \le \frac{\pi}{2} \\ \frac{\pi}{2}\sin(x+\pi)\ , & \frac{\pi}{2} < x < \pi \end{cases}$

Q.103 If $f(x) = \int_0^{\pi} \frac{t\sin t\,dt}{\sqrt{1+\tan^2 x\ \sin^2 t}}$ for $0 < x < \frac{\pi}{2}$

(A) $f(0^+) = -\pi$

(B) $f\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8}$

(C) f is continuous and differentiable in $\left(0, \frac{\pi}{2}\right)$

(D) f is continuous but not differentiable in $\left(0, \frac{\pi}{2}\right)$

Q.104 Consider $f(x) = \frac{x^2}{1+x^3}$; $g(t) = \int f(t)\,dt$. If g(1) = 0 then g(x) equals

(A) $\frac{1}{3}\ell n(1+x^3)$ (B) $\frac{1}{3}\ell n\left(\frac{1+x^3}{2}\right)$ (C) $\frac{1}{2}\ell n\left(\frac{1+x^3}{3}\right)$ (D) $\frac{1}{3}\ell n\left(\frac{1+x^3}{3}\right)$

Q.105 The value of the definite integral, $\int_0^{100} \frac{x}{e^{x^2}}\,dx$ is equal to

(A) $\frac{1}{2}(1-e^{-10})$ (B) $2(1-e^{-10})$ (C) $\frac{1}{2}(e^{-10}-1)$ (D) $\frac{1}{2}\left(1-e^{-10^4}\right)$

Q.106 $\int_0^{\infty} [2\,e^{-x}]\,dx$ where [x] denotes the greatest integer function is

(A) 0 (B) $\ln 2$ (C) e^2 (D) 2/e

Q.107 The value of $\int_{-1}^{1} \frac{dx}{\sqrt{|x|}}$ is

(A) $\frac{1}{2}$ (B) 2 (C) 4 (D) undefined

Q.108 $\int_0^1 x\ln\left(1+\frac{x}{2}\right)dx$ =

(A) $\frac{3}{4}\left(1-2\ln\frac{3}{2}\right)$ (B) $\frac{3}{2}-\frac{7}{2}\ln\frac{3}{2}$ (C) $\frac{3}{4}+\frac{1}{2}\ln\frac{1}{54}$ (D) $\frac{1}{2}\ln\frac{27}{2}-\frac{3}{4}$

Q.109 The evaluation of $\int \frac{p\,x^{p+2q-1} - q\,x^{q-1}}{x^{2p+2q} + 2x^{p+q} + 1}\,dx$ is

(A) $-\frac{x^p}{x^{p+q}+1}+C$ (B) $\frac{x^q}{x^{p+q}+1}+C$ (C) $-\frac{x^q}{x^{p+q}+1}+C$ (D) $\frac{x^p}{x^{p+q}+1}+C$

Q.110 $\int_{-1}^{1} \frac{x^3+|x|+1}{x^2+2|x|+1}\,dx = a\ln 2 + b$ then :

(A) a = 2 ; b = 1 (B) a = 2 ; b = 0 (C) a = 3 ; b = – 2 (D) a = 4 ; b = – 1

Q.111 $\int_a^b [x]\,dx + \int_a^b [-x]\,dx$ where [.] denotes greatest integer function is equal to :

(A) a + b (B) b – a (C) a – b (D) $\frac{a+b}{2}$

Q.112 If $\int_0^2 375\,x^5(1+x^2)^{-4}\,dx = 2^n$ then the value of n is :

(A) 4 (B) 5 (C) 6 (D) 7

Q.113 $\int_0^{1/2} \frac{1}{1-x^2}\ell n\frac{1+x}{1-x}\,dx$ is equal to :

(A) $\frac{1}{4}\ell n^2\frac{1}{3}$ (B) $\frac{1}{2}\ln^2 3$ (C) $-\frac{1}{4}\ln^2 3$ (D) cannot be evaluated.

Q.114 If $\int (x^3-2x^2+5)e^{3x}\,dx = e^{3x}(Ax^3+Bx^2+Cx+D)$ then the statement which is incorrect is

(A) C + 3D = 5 (B) A + B + 2/3 = 0

(C) C + 2B = 0 (D) A + B + C = 0

Q.115 Given $\int_0^{\pi/2} \frac{dx}{1+\sin x+\cos x} = \ln 2$, then the value of the def. integral. $\int_0^{\pi/2} \frac{\sin x}{1+\sin x+\cos x}\,dx$ is equal to

(A) $\frac{1}{2}\ln 2$ (B) $\frac{\pi}{2}-\ln 2$ (C) $\frac{\pi}{4}-\frac{1}{2}\ln 2$ (D) $\frac{\pi}{2}+\ln 2$

Q.116 A function f satisfying $f'(\sin x) = \cos^2 x$ for all x and f(1) = 1 is :

(A) $f(x) = x+\frac{x^3}{3}-\frac{1}{3}$ (B) $f(x) = \frac{x^3}{3}+\frac{2}{3}$

(C) $f(x) = x-\frac{x^3}{3}+\frac{1}{3}$ (D) $f(x) = \sqrt{x}-\frac{x^3}{3}+\frac{1}{3}$

Q.117 For $0 < x < \frac{\pi}{2}$, $\int_{1/2}^{\sqrt{3}/2} \ln(e^{\cos x}).\,d(\sin x)$ is equal to :

(A) $\frac{\pi}{12}$ (B) $\frac{\pi}{6}$

(C) $\frac{1}{4}\left[\left(\sqrt{3}-1\right)+\left(\sin\sqrt{3}-\sin 1\right)\right]$ (D) $\frac{1}{4}\left[\left(\sqrt{3}-1\right)-\left(\sin\sqrt{3}-\sin 1\right)\right]$

Q.118 $\int_0^{\pi} \frac{x\cos x}{(1+\sin x)^2}\,dx$ is equal to :

(A) $\pi - 2$ (B) $-(2+\pi)$ (C) zero (D) $2-\pi$

Q.119 $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\left(x+\sqrt{x}\right)dx$

(A) $2e^{\sqrt{x}}\left[x-\sqrt{x}+1\right]+C$ (B) $e^{\sqrt{x}}\left[x-2\sqrt{x}+1\right]$

(C) $e^{\sqrt{x}}\left(x+\sqrt{x}\right)+C$ (D) $e^{\sqrt{x}}\left(x+\sqrt{x}+1\right)+C$

Q.120 $\int_0^{\pi/2} \frac{dx}{\cos^6 x+\sin^6 x}$ is equal to :

(A) zero (B) π (C) $\pi/2$ (D) 2π

Q.121 The true solution set of the inequality, $\sqrt{5x-6-x^2}+\frac{\pi}{2}\int\limits_0^x dz > x\int\limits_0^{\pi} \sin^2 x\, dx$ is :

(A) R (B) (1, 6) (C) (– 6, 1) (D) (2, 3)

Q.122 If $\int\limits_0^1 \frac{\ell nx}{\sqrt{1-x^2}}\, dx = k\int\limits_0^{\pi} \ln(1+\cos x)\, dx$ then the value of k is :

(A) 2 (B) 1/2 (C) – 2 (D) – 1/2

Q.123 Let a, b and c be positive constants. The value of 'a' in terms of 'c' if the value of integral $\int\limits_0^1 (acx^{b+1}+a^3bx^{3b+5})\, dx$ is independent of b equals

(A) $\sqrt{\frac{3c}{2}}$ (B) $\sqrt{\frac{2c}{3}}$ (C) $\sqrt{\frac{c}{3}}$ (D) $\sqrt{\frac{3}{2c}}$

Q.124 $\int \sec^2\theta\,(\sec\theta+\tan\theta)^2\, d\theta$

(A) $\frac{(\sec\theta+\tan\theta)}{2}[2+\tan\theta(\sec\theta+\tan\theta)] + C$

(B) $\frac{(\sec\theta+\tan\theta)}{3}[2+4\tan\theta(\sec\theta+\tan\theta)] + C$

(C) $\frac{(\sec\theta+\tan\theta)}{3}[2+\tan\theta(\sec\theta+\tan\theta)] + C$

(D) $\frac{3(\sec\theta+\tan\theta)}{2}[2+\tan\theta(\sec\theta+\tan\theta)] + C$

Q.125 $\int\limits_1^{\sqrt{2}} \frac{x^2+1}{x^4+1}\, dx$ is equal to:

(A) $\frac{1}{2}\tan^{-1}\sqrt{2}$ (B) $\frac{1}{\sqrt{2}}\cot^{-1}2$ (C) $\frac{1}{2}\tan^{-1}\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}\tan^{-1}2$

Q.126 $\underset{x\to x_1}{\text{Limit}}\ \frac{x}{x-x_1}\int\limits_{x_1}^{x} f(t)\, dt$ is equal to :

(A) $\frac{f(x_1)}{x_1}$ (B) $x_1 f(x_1)$ (C) $f(x_1)$ (D) does not exist

Q.127 Which of the following statements could be true if, $f''(x) = x^{1/3}$.

I	II	III	IV
$f(x) = \frac{9}{28}x^{7/3} + 9$	$f'(x) = \frac{9}{28}x^{7/3} - 2$	$f'(x) = \frac{3}{4}x^{4/3} + 6$	$f(x) = \frac{3}{4}x^{4/3} - 4$

(A) I only (B) III only (C) II & IV only (D) I & III only

Q.128 The value of the definite integral $\int_0^{\pi/2} \sin x \sin 2x \sin 3x\, dx$ is equal to :

(A) $\frac{1}{3}$ (B) $-\frac{2}{3}$ (C) $-\frac{1}{3}$ (D) $\frac{1}{6}$

Q.129 $\int \frac{e^{\tan^{-1}x}}{(1+x^2)}\left[\left(\sec^{-1}\sqrt{1+x^2}\right)^2 + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right] dx \quad (x > 0)$

(A) $e^{\tan^{-1}x}.\tan^{-1}x + C$

(B) $\frac{e^{\tan^{-1}x}.\left(\tan^{-1}x\right)^2}{2} + C$

(C) $e^{\tan^{-1}x}.\left(\sec^{-1}\left(\sqrt{1+x^2}\right)\right)^2 + C$

(D) $e^{\tan^{-1}x}.\left(\operatorname{cosec}^{-1}\left(\sqrt{1+x^2}\right)\right)^2 + C$

Q.130 Number of positive solution of the equation, $\int_0^x \left(t - \{t\}\right)^2 dt = 2(x-1)$ where { } denotes the fractional part function is :

(A) one (B) two (C) three (D) more than three

Q.131 If $f(x) = \cos(\tan^{-1}x)$ then the value of the integral $\int_0^1 x\, f''(x)\, dx$ is

(A) $\frac{3-\sqrt{2}}{2}$ (B) $\frac{3+\sqrt{2}}{2}$ (C) 1 (D) $1-\frac{3}{2\sqrt{2}}$

Q.132 If $\int \sqrt{1+\sin\frac{x}{2}}\, dx = A \sin\left(\frac{x}{4}-\frac{\pi}{4}\right)$ then value of A is:

(A) $2\sqrt{2}$ (B) $\sqrt{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $4\sqrt{2}$

Q.133 For $U_n = \int_0^1 x^n (2-x)^n dx$; $V_n = \int_0^1 x^n (1-x)^n dx$ $n \in N$, which of the following statement(s) is/are ture?

(A) $U_n = 2^n V_n$ (B) $U_n = 2^{-n} V_n$ (C) $U_n = 2^{2n} V_n$ (D) $U_n = 2^{-2n} V_n$

Q.134 $\int \frac{(x^2-1)\, dx}{(x^4+3x^2+1)\ \tan^{-1}\left(\frac{x^2+1}{x}\right)} = \ln|f(x)| + C$ then $f(x)$ is

(A) $\ln\left(x+\frac{1}{x}\right)$ (B) $\tan^{-1}\left(x+\frac{1}{x}\right)$ (C) $\cot^{-1}\left(x+\frac{1}{x}\right)$ (D) $\ln\left(\tan^{-1}\left(x+\frac{1}{x}\right)\right)$

Q.135 Let $f(x)$ be integrable over (a, b), $b > a > 0$. If $I_1 = \int_{\pi/6}^{\pi/3} f(\tan\theta + \cot\theta).\sec^2\theta\, d\theta$ &

$I_2 = \int_{\pi/6}^{\pi/3} f(\tan\theta + \cot\theta).\text{cosec}^2\theta\, d\theta$, then the ratio $\frac{I_1}{I_2}$:

(A) is a positive integer
(B) is a negative integer
(C) is an irrational number
(D) cannot be determined.

Q.136 $f(x) = \int_{\cos x}^{\sin x} (1 - t + 2t^3)\, dt$ has in $[0, 2\pi]$

(A) a maximum at $\frac{\pi}{4}$ & a minimum at $\frac{3\pi}{4}$
(B) a maximum at $\frac{3\pi}{4}$ & a minimum at $\frac{7\pi}{4}$
(C) a maximum at $\frac{5\pi}{4}$ & a minimum at $\frac{7\pi}{4}$
(D) neither a maxima nor minima

Q.137 Let $S(x) = \int_{x^2}^{x^3} \ln t\, dt$ $(x > 0)$ and $H(x) = \frac{S'(x)}{x}$. Then $H(x)$ is :

(A) continuous but not derivable in its domain
(B) derivable and continuous in its domain
(C) neither derivable nor continuous in its domain
(D) derivable but not continuous in its domain.

Q.138 Number of solution of the equation $\frac{d}{dx}\int_{\cos x}^{\sin x} \frac{dt}{1-t^2} = 2\sqrt{2}$ in $[0, \pi]$ is

(A) 4
(B) 3
(C) 2
(D) 0

Q.139 Let $f(x) = \frac{2\sin^2 x - 1}{\cos x} + \frac{\cos x(2\sin x + 1)}{1 + \sin x}$ then

$\int e^x\left(f(x) + f'(x)\right)dx$ (where c is the constant of integeration)

(A) $e^x \tan x + c$
(B) $e^x \cot x + c$
(C) $e^x\, \text{cosec}^2 x + c$
(D) $e^x \sec^2 x + c$

Q.140 The value of x that maximises the value of the integral $\int_x^{x+3} t(5-t)\,dt$ is

(A) 2
(B) 0
(C) 1
(D) none

Q.141 For a sufficiently large value of n the sum of the square roots of the first n positive integers i.e. $\sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots\ldots\ldots\ldots\ldots\ldots + \sqrt{n}$ is approximately equal to

(A) $\frac{1}{3}n^{3/2}$
(B) $\frac{2}{3}n^{3/2}$
(C) $\frac{1}{3}n^{1/3}$
(D) $\frac{2}{3}n^{1/3}$

Q.142 The value of $\int_0^2 \frac{dx}{(1-x)^2}$ is

(A) –2
(B) 0
(C) 15
(D) indeterminate

Q.143 If $\int_0^a \frac{dx}{\sqrt{x+a}+\sqrt{x}} = \int_0^{\pi/8} \frac{2\tan\theta}{\sin 2\theta} d\theta$, then the value of 'a' is equal to (a > 0)

(A) $\frac{3}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{3\pi}{4}$ (D) $\frac{9}{16}$

Q.144 The value of the integral $\int \frac{\sin(\ln(2+2x))}{x+1} dx$ is

(A) $-\cos \ln(2x+2) + C$ (B) $\ln\left(\sin\frac{2}{x+1}\right) + C$

(C) $\cos\left(\frac{2}{x+1}\right) + C$ (D) $\sin\left(\frac{2}{x+1}\right) + C$

Q.145 If $f(x) = A\sin\left(\frac{\pi x}{2}\right) + B$, $f'\left(\frac{1}{2}\right) = \sqrt{2}$ and $\int_0^1 f(x)\,dx = \frac{2A}{\pi}$, Then the constants A and B are respectively.

(A) $\frac{\pi}{2} \& \frac{\pi}{2}$ (B) $\frac{2}{\pi} \& \frac{3}{\pi}$ (C) $0 \& -\frac{4}{\pi}$ (D) $\frac{4}{\pi} \& 0$

Q.146 Let $I_1 = \int_0^{\pi/2} e^{-x^2}\sin(x)dx$; $I_2 = \int_0^{\pi/2} e^{-x^2}dx$; $I_3 = \int_0^{\pi/2} e^{-x^2}(1+x)dx$

and consider the statements

I $I_1 < I_2$ **II** $I_2 < I_3$ **III** $I_1 = I_3$

Which of the following is(are) true?

(A) I only (B) II only

(C) Neither I nor II nor III (D) Both I and II

Q.147 Let $f(x) = \frac{\sin x}{x}$, then $\int_0^{\pi/2} f(x)\, f\left(\frac{\pi}{2}-x\right)dx =$

(A) $\frac{2}{\pi}\int_0^{\pi} f(x)\,dx$ (B) $\int_0^{\pi} f(x)\,dx$ (C) $\pi\int_0^{\pi} f(x)\,dx$ (D) $\frac{1}{\pi}\int_0^{\pi} f(x)\,dx$

Q.148 Let $u = \int_0^1 \frac{\ln(x+1)}{x^2+1}dx$ and $v = \int_0^{\pi/2} \ln(\sin 2x)\,dx$ then

(A) u = 4v (B) 4u + v = 0 (C) u + 4v = 0 (D) 2u + v = 0

Q.149 If $f(x) = \int_{\pi^2/16}^{x^2} \frac{\sin x \cdot \sin\sqrt{\theta}}{1+\cos^2\sqrt{\theta}}.d\theta$ then the value of $f'\left(\frac{\pi}{2}\right)$, is

(A) π (B) $-\pi$ (C) 2π (D) 0

Q.150 The value of the definite integral, $\int_0^{\pi/2} \frac{\sin 5x}{\sin x} dx$ is

(A) 0 (B) $\frac{\pi}{2}$ (C) π (D) 2π

<u>Select the correct alternatives</u> : (More than one are correct)

Q.151 $\int_a^b \text{sgn } x \, dx =$ (where a, b $\in$ R)

(A) $|b| - |a|$ (B) $(b-a)\,\text{sgn}(b-a)$ (C) $b\,\text{sgn} b - a\,\text{sgn} a$ (D) $|a| - |b|$

Q.152 $\int \frac{dx}{5+4\cos x} = \lambda \tan^{-1}\left(m \tan\frac{x}{2}\right) + C$ then :

(A) $\lambda = 2/3$ (B) $m = 3$ (C) $\lambda = 1/3$ (D) $m = 2/3$

Q.153 Which of the following are true ?

(A) $\int_a^{\pi-a} x \,.\, f(\sin x)\, dx = \frac{\pi}{2} . \int_a^{\pi-a} f(\sin x)\, dx$ (B) $\int_{-a}^{a} f(x)^2\, dx = 2. \int_0^a f(x)^2\, dx$

(C) $\int_0^{n\pi} f\left(\cos^2 x\right) dx = n. \int_0^{\pi} f\left(\cos^2 x\right) dx$ (D) $\int_0^{b-c} f(x+c)\, dx = \int_c^b f(x)\, dx$

Q.154 The value of $\int_0^1 \frac{2x^2+3x+3}{(x+1)\left(x^2+2x+2\right)} dx$ is :

(A) $\frac{\pi}{4} + 2\ln 2 - \tan^{-1} 2$ (B) $\frac{\pi}{4} + 2\ln 2 - \tan^{-1}\frac{1}{3}$

(C) $2\ln 2 - \cot^{-1} 3$ (D) $-\frac{\pi}{4} + \ln 4 + \cot^{-1} 2$

Q.155 $\int \frac{x^2 + \cos^2 x}{1 + x^2} \text{cosec}^2 x \, dx$ is equal to :

(A) $\cot x - \cot^{-1} x + c$ (B) $c - \cot x + \cot^{-1} x$

(C) $-\tan^{-1} x - \frac{\text{cosec}\, x}{\sec x} + c$ (D) $-e^{\ell n \tan^{-1} x} - \cot x + c$

where 'c' is constant of integration .

Q.156 Let $f(x) = \int_0^x \frac{\sin t}{t} dt \ (x > 0)$ then f (x) has :

(A) Maxima if $x = n\pi$ where n = 1, 3, 5,.....
(B) Minima if $x = n\pi$ where n = 2, 4, 6,......
(C) Maxima if $x = n\pi$ where n = 2, 4, 6,......
(D) The function is monotonic

Q.157 If $I_n = \int_0^1 \frac{dx}{(1+x^2)^n}$; $n \in N$, then which of the following statements hold good ?

(A) $2n\, I_{n+1} = 2^{-n} + (2n-1)\, I_n$
(B) $I_2 = \frac{\pi}{8} + \frac{1}{4}$
(C) $I_2 = \frac{\pi}{8} - \frac{1}{4}$
(D) $I_3 = \frac{\pi}{16} - \frac{5}{48}$

Q.158 $\int \frac{1}{x^2-1} \; \ell n \frac{x-1}{x+1}\, dx$ equals :

(A) $\frac{1}{2}\, ln^2 \frac{x-1}{x+1} + c$ (B) $\frac{1}{4}\, ln^2 \frac{x-1}{x+1} + c$ (C) $\frac{1}{2}\, ln^2 \frac{x+1}{x-1} + c$ (D) $\frac{1}{4}\, ln^2 \frac{x+1}{x-1} + c$

Q.159 If $A_n = \int_0^{\pi/2} \frac{\sin(2n-1)x}{\sin x}\, dx$; $B_n = \int_0^{\pi/2} \left(\frac{\sin nx}{\sin x}\right)^2 dx$; for $n \in N$, then :

(A) $A_{n+1} = A_n$
(B) $B_{n+1} = B_n$
(C) $A_{n+1} - A_n = B_{n+1}$
(D) $B_{n+1} - B_n = A_{n+1}$

Q.160 $\int_0^{\infty} \frac{x}{(1+x)(1+x^2)}\, dx$:

(A) $\frac{\pi}{4}$
(B) $\frac{\pi}{2}$
(C) is same as $\int_0^{\infty} \frac{dx}{(1+x)(1+x^2)}$
(D) cannot be evaluated

Q.161 $\int \sqrt{1+\csc x}\; dx$ equals

(A) $2\sin^{-1}\sqrt{\sin x} + c$
(B) $\sqrt{2}\cos^{-1}\sqrt{\cos x} + c$
(C) $c - 2\sin^{-1}(1-2\sin x)$
(D) $\cos^{-1}(1-2\sin x) + c$

Q.162 If $f(x) = \int_0^{\pi/2} \frac{\ell n(1 + x\sin^2\theta)}{\sin^2\theta}\, d\theta$, $x \ge 0$ then :

(A) $f(t) = \pi\left(\sqrt{t+1} - 1\right)$
(B) $f'(t) = \frac{\pi}{2\sqrt{t+1}}$
(C) $f(x)$ cannot be determined
(D) none of these.

Q.163 If a, b, c $\in$ R and satisfy $3a + 5b + 15c = 0$, the equation $ax^4 + bx^2 + c = 0$ has :

(A) atleast one root in $(-1, 0)$
(B) atleast one root in $(0, 1)$
(C) atleast two roots in $(-1, 1)$
(D) no root in $(-1, 1)$

Q.164 Let $u = \int_0^{\infty} \frac{dx}{x^4+7x^2+1}$ & $v = \int_0^{\infty} \frac{x^2\,dx}{x^4+7x^2+1}$ then :

(A) $v > u$ (B) $6v = \pi$ (C) $3u + 2v = 5\pi/6$ (D) $u + v = \pi/3$

Q.165 If $\int e^u . \sin 2x \, dx$ can be found in terms of known functions of x then u can be :

(A) x (B) sin x (C) cos x (D) cos 2x

Q.166 If $f(x) = \int_1^x \frac{\ell n\, t}{1+t} dt$ where x > 0 then the value(s) of x satisfying the equation,

$f(x) + f(1/x) = 2$ is :

(A) 2 (B) e (C) e^{-2} (D) e^2

Q.167 A polynomial function f(x) satisfying the conditions $f(x) = [f'(x)]^2$ & $\int_0^1 f(x)\,dx = \frac{19}{12}$ can be:

(A) $\frac{x^2}{4} + \frac{3}{2}x + \frac{9}{4}$ (B) $\frac{x^2}{4} - \frac{3}{2}x + \frac{9}{4}$ (C) $\frac{x^2}{4} - x + 1$ (D) $\frac{x^2}{4} + x + 1$

Q.168 A continuous and differentiable function 'f' satisfies the condition ,

$$\int_0^x f(t)\,dt = f^2(x) - 1 \text{ for all real 'x'. Then :}$$

(A) 'f' is monotonic increasing $\forall\ x \in R$
(B) 'f' is monotonic decreasing $\forall\ x \in R$
(C) 'f' is non monotonic
(D) the graph of y = f(x) is a straight line.

ANSWER KEY

Q.1	A	Q.2	C	Q.3	B	Q.4	C	Q.5	C
Q.6	C	Q.7	A	Q.8	D	Q.9	A	Q.10	D
Q.11	D	Q.12	C	Q.13	A	Q.14	C	Q.15	C
Q.16	B	Q.17	C	Q.18	A	Q.19	D	Q.20	A
Q.21	C	Q.22	C	Q.23	A	Q.24	A	Q.25	B
Q.26	A	Q.27	A	Q.28	B	Q.29	D	Q.30	C
Q.31	A	Q.32	D	Q.33	A	Q.34	D	Q.35	B
Q.36	C	Q.37	B	Q.38	C	Q.39	D	Q.40	C
Q.41	C	Q.42	B	Q.43	D	Q.44	B	Q.45	D
Q.46	B	Q.47	D	Q.48	C	Q.49	B	Q.50	A
Q.51	D	Q.52	A	Q.53	A	Q.54	C	Q.55	A
Q.56	B	Q.57	C	Q.58	D	Q.59	C	Q.60	A
Q.61	C	Q.62	C	Q.63	A	Q.64	C	Q.65	D
Q.66	C	Q.67	C	Q.68	B	Q.69	B	Q.70	A
Q.71	B	Q.72	C	Q.73	C	Q.74	B	Q.75	B
Q.76	B	Q.77	B	Q.78	A	Q.79	B	Q.80	A
Q.81	D	Q.82	C	Q.83	C	Q.84	D	Q.85	C
Q.86	C	Q.87	A	Q.88	B	Q.89	B	Q.90	C
Q.91	D	Q.92	B	Q.93	A	Q.94	B	Q.95	B
Q.96	A	Q.97	B	Q.98	B	Q.99	D	Q.100	A
Q.101	B	Q.102	D	Q.103	C	Q.104	B	Q.105	D
Q.106	B	Q.107	C	Q.108	A	Q.109	C	Q.110	B
Q.111	C	Q.112	B	Q.113	A	Q.114	C	Q.115	C
Q.116	C	Q.117	A	Q.118	D	Q.119	A	Q.120	B
Q.121	D	Q.122	B	Q.123	A	Q.124	C	Q.125	B
Q.126	B	Q.127	D	Q.128	D	Q.129	C	Q.130	B
Q.131	D	Q.132	D	Q.133	C	Q.134	B	Q.135	A
Q.136	B	Q.137	B	Q.138	C	Q.139	A	Q.140	C
Q.141	B	Q.142	D	Q.143	D	Q.144	A	Q.145	D
Q.146	D	Q.147	A	Q.148	B	Q.149	A	Q.150	B
Q.151	A,C	Q.152	A,B	Q.153	A,B,C,D	Q.154	A,C,D		
Q.155	B,C,D	Q.156	A,B	Q.157	A,B	Q.158	B,D		
Q.159	A,D	Q.160	A,C	Q.161	A,D	Q.162	A,B		
Q.163	A,B,C	Q.164	B,C,D	Q.165	A,B,C,D	Q.166	C,D		
Q.167	B,D								
Q.168	A,D								