PART (C) : MATHEMATICS

SECTION-I : (SINGLE ANSWER CORRECT TYPE)

This section contains **08 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

41. If the line $\frac{x+6}{5}=\frac{y+10}{3}=\frac{z+14}{8}$ is the hypotenuse of an isosceles right angled whose opposite vertex is $(7,2,4)$. Then, which of the following is not the side of the triangle?

(A) $\frac{x-7}{2}=\frac{y-2}{-3}=\frac{z-4}{6}$

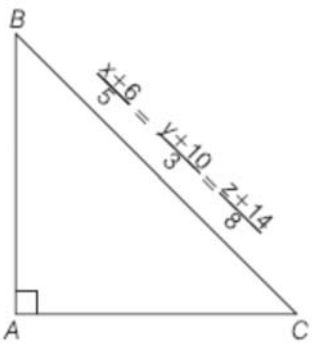
(B) $\frac{x-7}{3}=\frac{y-2}{6}=\frac{z-4}{2}$

(C) $\frac{x-7}{3}=\frac{y-2}{5}=\frac{z-4}{-1}$

(D) None of the above

41. **(C)**

Let ABC be the given isosceles right angled triangle with vertex $A(7,2,4)$ and hypotenuse BC, whose equation is $\frac{x+6}{5}=\frac{y+10}{3}=\frac{z+14}{8}$.



Clearly, the coordinates of B is given by

$(5\lambda-6,3\lambda-10,8\lambda-14)$, for some $\lambda\in R$

Now DR's of line AB are $(5\lambda-13,3\lambda-12,8\lambda-18)$

Since, DR's of line BC are 5,3,8 and angle between AB and BC is $\frac{\pi}{4}$.

42. Let C_1 and C_2 be concentric circles of radii 1 and 8/3 respectively, having centre at (3, 0) on the argand plane. If the complex number z satisfies the inequality $\log_{1/3}\left(\frac{|z-3|^2+2}{11|z-3|-2}\right)>1$, then

(A) z lies outside C_1 but inside C_2

(B) z lies inside of both C_1 and C_2

(C) z lies outside of both C_1 and C_2

(D) None of the above

42. **(A)**

We have, $\log_{1/3}\left(\frac{|z-3|^2+2}{11|z-3|-2}\right)>1$

$\Rightarrow \quad \dfrac{|z-3|^2+2}{11|z-3|-2} < \dfrac{1}{3}$

$\Rightarrow \quad 3|z-3|^2+6 < 11|z-3|-2$

$\Rightarrow \quad 3x^2-11x+8<0$, where $x=|z-3|$

$\Rightarrow \quad (3x-8)(x-1)<0$

$\Rightarrow \quad 1<x<\dfrac{8}{3}$

$\Rightarrow \quad 1<|z-3|<\dfrac{8}{3}$

Hence, z lies between the two concentric circles.

43. If a, b, c are any three non – coplanar vectors, then the equation
$[b\times c \;\; c\times a \;\; a\times b]x^2+[a+b \;\; b+c \;\; c+a]x+1+[b-c \;\; c-a \;\; a-b]=0$ has

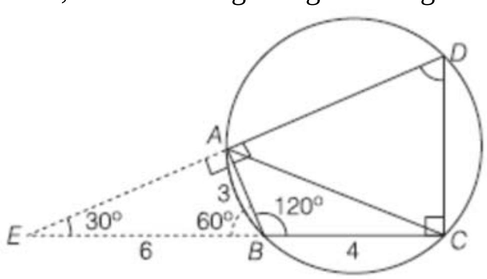
(A) Real and distinct roots (B) Imaginary roots
(C) Equal roots (D) None

43. **(C)**
The given equation reduces to
$[a \;\; b \;\; c]^2 x^2+2[a \;\; b \;\; c]x+1=0$
Now, $D=\left(2[a \;\; b \;\; c]\right)^2-4[a \;\; b \;\; c]^2=0$
$\Rightarrow$ The given equation has equal roots.

44. In $\Delta ABC, \angle ABC=120^o, AB=3$ and $BC=4$. If perpendiculars constructed to the side AB at A and to the side BC at C meets at D, then CD is equal to

(A) 3 (B) $\dfrac{8\sqrt{3}}{3}$ (C) 5 (D) $\dfrac{10\sqrt{3}}{3}$

44. **(D)**
Let us draw a circle passing through A, B, C and D.
Now, extend CB and DA to meet at E.
Then, ΔECD is a right angled triangle as shown in the figure.



Clearly, $\angle ABE=60^o$ and $\angle BAE=90^o$

$\therefore \quad \angle AEB=180^o-90^o-60^o=30^o$

Now, in ΔAEB

$\sin 30^o=\dfrac{3}{EB}$

$\Rightarrow \quad EB = \frac{3}{\sin 30^o} = 6$

Now, in ΔECD

$\tan 30^o = \frac{CD}{EC} = \frac{CD}{10}$

$\Rightarrow \quad CD = \frac{10}{\sqrt{3}} = \frac{10\sqrt{3}}{3}$

45. If x_1, x_2, x_3 as well as y_1, y_2, y_3 are in GP with same common ratio, then the points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3)

(A) Lie on a straight line
(B) Lie on an ellipse
(C) Lie on a circle
(D) Are vertices of a triangle

45. (A)

Let $x_2 = x_1 r, x_3 = x_1 r^2$ and so $y_2 = y_1 r, y_3 = y_1 y^2$,

Where, r is common ratio.

Consider, $\Delta = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} x_1 & y_1 & 1 \\ rx_1 & ry_1 & 1 \\ r^2x_1 & r^2y_1 & 1 \end{vmatrix}$

$= x_1 \times y_1 \begin{vmatrix} 1 & 1 & 1 \\ r & r & 1 \\ r^2 & r^2 & 1 \end{vmatrix} = x_1 \times y_1 \times 0 = 0$

$\Rightarrow$ The given points are collinear.

46. If $\prod_{k=1}^{n} \cos \frac{x}{2^k} = \frac{\sin x}{2^n \sin\left(\frac{x}{2^n}\right)}$ and $f(x) = \begin{cases} \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{2^k} \tan\left(\frac{x}{2^k}\right), & x \in (0, \pi) - \left\{\frac{\pi}{2}\right\} \\ \frac{2}{\pi}, & x = \frac{\pi}{2} \end{cases}$

Then, which one of the following is true?

(A) $f(x)$ has non – removable discontinuity of finite type at $x = \frac{\pi}{2}$

(B) f(x) has removable discontinuity at $x = \frac{\pi}{2}$

(C) f(x) is continuous at $x = \frac{\pi}{2}$

(D) f(x) has non – removable discontinuity of infinite type at $x = \frac{\pi}{2}$

46. (C)

We have, $\cos\frac{x}{2} \cos\frac{x}{2^2} \ldots\ldots \cos\frac{x}{2^n} = \frac{\sin x}{2^n \sin\left(\frac{x}{2^n}\right)}$

On taking log both sides, we get

$$\sum_{k=1}^{n} \log\cos\frac{x}{2^k} = \log\sin x - \log 2^n - \log\sin\left(\frac{x}{2^n}\right)$$

On differentiating both sides w.r.t. x, we get

$$-\left(\sum_{k=1}^{n}\left(\tan\left(\frac{x}{2^k}\right).\frac{1}{2^k}\right)\right)\cot x - \cot\left(\frac{x}{2^n}\right).\frac{1}{2^n}$$

$$\Rightarrow \quad \sum_{k=1}^{n}\left(\tan\left(\frac{x}{2^k}\right).\frac{1}{2^k}\right) = \frac{1}{2^n}.\cot\left(\frac{x}{2^n}\right) - \cot x$$

Now, for $x \in (0,\pi) - \left\{\frac{\pi}{2}\right\}$,

$$f(x) = \lim_{n\to\infty}\left(\frac{1}{2^n}\cot\left(\frac{x}{2^n}\right) - \cot x\right)$$

$$= \lim_{n\to\infty}\left(\frac{1}{2^n}\frac{\frac{x}{2^n}}{\tan\left(\frac{x}{2^n}\right)}.\frac{2^n}{x}\right) - \cot x$$

$$= \frac{1}{x}\lim_{n\to\infty}\left(\frac{\frac{x}{2^n}}{\tan\left(\frac{x}{2^n}\right)}\right) - \cot x = \frac{1}{x} - \cot x$$

$$\therefore \quad f(x) = \begin{cases} \frac{1}{x} - \cot x, & x \in (0,\pi) - \left\{\frac{\pi}{2}\right\} \\ \frac{2}{\pi}, & x = \frac{\pi}{2} \end{cases}$$

Clearly, $\lim_{x\to\frac{\pi}{2}} f(x) = \lim_{x\to\frac{\pi}{2}} \frac{1}{x} - \cot x$

$$= \frac{2}{\pi} = f\left(\frac{\pi}{2}\right)$$

$\therefore f(x)$ is continuous at $x = \frac{\pi}{2}$.

47. If n is selected from the set {1, 2, 3, ….., 100} and the number $2^n + 3^n + 5^n$ is formed, then the total number of ways of selecting n so that the formed number is divisible by 4, is

(A) 50 (B) 49 (C) 48 (D) None of these

47. **(B)**

If n is odd, then

$3^n = 4\lambda_1 - 1$ and $5^n = 4\lambda_2 + 1$

$\Rightarrow \quad 2^n + 3^n + 5^n$ is divisible by 4, if $n \geq 2$.

Thus, n = 3, 5, 7, 9, ……, 99, i.e. n can take 49 different values.

If n is even, then $3^n = 4\lambda_1 + 1$ and $5^n = 4\lambda_2 + 1$

$\Rightarrow 2^n + 3^n + 5^n$ is not divisible by 4 as $2^n + 3^n + 5^n$ will be in the form of $4\lambda + 2$.

Thus, the total number of ways of selecting n is equal to 49.

48. The product of all values of x satisfying the equation

$$\sin^{-1}\cos\left(\frac{2x^2+10|x|+4}{x^2+5|x|+3}\right)$$

$$=\cot\left\{\cot^{-1}\left(\frac{2-18|x|}{9|x|}\right)\right\}+\frac{\pi}{2}\text{ is}$$

(A) 9 (B) – 9 (C) – 3 (D) – 1

48. **(A)**

Given equation can be written as

$$\frac{\pi}{2}-\cos^{-1}\cos\left(\frac{2x^2+10|x|+4}{x^2+5|x|+3}\right)=\cot\left\{\cot^{-1}\left(\frac{2-18|x|}{9|x|}\right)\right\}+\frac{\pi}{2}$$

$$\Rightarrow \quad \frac{\pi}{2}-\frac{2\left(x^2+5|x|+3\right)-2}{x^2+5|x|+3}=\frac{2-18|x|}{9|x|}+\frac{\pi}{2}$$

$$\Rightarrow \quad \frac{\pi}{2}-2+\frac{2}{x^2+5|x|+3}=\frac{2}{9|x|}-2+\frac{\pi}{2}$$

$$\Rightarrow \quad x^2+5|x|+3=9|x|$$

$$\Rightarrow \quad |x|=1,3$$

$$\Rightarrow \quad x=\pm1,\pm3$$

SECTION-II : (MULTIPLE CORRECT ANSWER(S) TYPE)

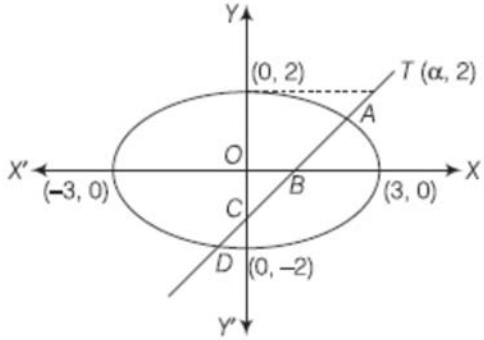
This section contains **06 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE than one is/are correct.**

49. Let the straight line passing through $T(\alpha,2)$ meets the ellipse $4x^2+9y^2=36$ at A and D and meets the coordinate axes at B and C such that TA, TB, TC and TD (take in that order) are in GP, then the possible value of α can be

(A) – 7 (B) 5 (C) 8 (D) 10

49. **(ACD)**

We have, $4x^2+9x^2=36\Rightarrow\frac{x^2}{9}+\frac{y^2}{4}=1$



Let inclination of line be ‘ θ ’.

Let $TA = r_1, TB = r_2, TC = r_3$ and $TD = r_4$

A and D are $(\alpha + r\cos\theta, 2 + r\sin\theta)$ with r_1 and r_4 satisfy

Ellipse $4x^2 + 9y^2 = 36$

$r^2(4\cos^2\theta + 9\sin^2\theta) + r(8\alpha\cos\theta + 36\sin\theta) + 4\alpha^2 = 0$

Have roots r_1 and r_4

$r_1 r_4 = \dfrac{4\alpha^2}{4\cos^2\theta + 9\sin^2\theta} = \dfrac{4\alpha^2}{4 + 5\sin^2\theta}$

$B = (\alpha + r_2\cos\theta, 2 + r_2\sin\theta) = (x_0, 0)$

$C = (\alpha + r_3\cos\theta, 2 + r_3\sin\theta) = (0, y_0)$

Hence, $r_2 = \dfrac{-\alpha}{\cos\theta}, r_3 = \dfrac{-2}{\sin\theta}$

r_1, r_2, r_3, r_4 are GP $\Rightarrow r_1 r_4 = r_2 r_3$

$\Rightarrow \quad 2\alpha = 4\cot\theta + 9\tan\theta \Rightarrow |\alpha| \ge 6$

$\therefore \quad \alpha = -7, 8, 10$

50. Let $f(x) = [x]^2 + [x+1] - 3$, where $[x]$ is G. I. F then

(A) f(x) is many – one

(B) $f(x) = 0$ for infinite number of values of x.

(C) $f(x) = 0$ for only two real values

(D) None of the above

50. **(AB)**

We have, $f(x) = [x]^2 + [x+1] - 3$

$\Rightarrow \quad f(x) = [x]^2 + [x] + 1 - 3$

$\Rightarrow \quad f(x) = ([x]+2)([x]-1)$

$\Rightarrow \quad f(x) = 0$, then $[x] = -2$

And $[x] = 1$

$\because \quad -2 \le x < -1$ and $1 \le x < 2$

Hence, $f(x)$ is many one and into and f(x) = 0 for infinite values of x.

51. If $A(z_1), B(z_2), C(z_3)$ and $D(z_4)$ lies on $|z| = 4$ (taken in order), where $z_1 + z_2 + z_3 + z_4 = 0$, then

(A) Maximum area of quadrilateral, ABCD = 32

(B) The ΔABC is right angled

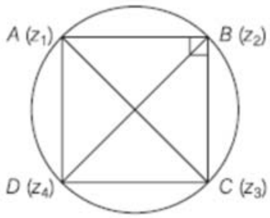
(C) The quadrilateral ABCD is rectangle

(D) Maximum area of quadrilateral ABCD = 16

51. **(ABC)**

We have,

$z_1 + z_2 + z_3 + z_4 = 0$



$\Rightarrow \quad z_1 + z_3 = -z_2 - z_4$

$\Rightarrow \quad \dfrac{z_1 + z_3}{2} = \dfrac{-z_2 - z_4}{2}$

Quadrilateral with vertices $z_1, z_3, -z_2, -z_4$ is a parallelogram but it is inscribed in circle, so it must be rectangle.

If ABCD is a rectangle, then ΔABC is right angled.

$\therefore$ Maximum area of quadrilateral ABCD

$= \dfrac{1}{2}(2r)^2 = 2r^2 = 2\times 16 = 32 \ [\because r = 4]$

52. Let P_1 denotes the equation of a plane to which the vector $\hat{i} + \hat{j}$ is normal and which contains the line L whose equation is $r = \hat{i} + \hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} - \hat{k})$ and P_2 denotes the equation of the plane containing the line L and a point with position vector $\hat{j}$. Which of the following holds good?

(A) The equation of P_1 is $x + y = 2$

(B) The equation of P_2 is $r.(\hat{i} - 2\hat{j} + \hat{k}) = 2$

(C) The acute angle between P_1 and P_2 is $\cot^{-1}(\sqrt{3})$

(D) The angle between the plane P_2 and the line L is $\tan^{-1}(\sqrt{3})$

52. (AC)

Let P_1 contains the line $r = \hat{i} + \hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} - \hat{k})$, hence contains the point $\hat{i} + \hat{j} + \hat{k}$ and is normal to vector $(\hat{i} + \hat{j})$.

Hence, equation of plane is

$\left(r - (\hat{i} + \hat{j} + \hat{k})\right).(\hat{i} + \hat{j}) = 0$

or $x + y = 2$

Plane P_2 contains the line

$r = \hat{i} + \hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} - \hat{k})$ and $\hat{j}$

Hence, the equation of plane is

$$\begin{vmatrix} x-0 & y-1 & z-0 \\ 1-0 & 1-1 & 1-0 \\ 1 & -1 & -1 \end{vmatrix} = 0 \text{ or } x + 2y - z = 2$$

If θ is the acute angle between P_1 and P_2, then

$$\cos\theta = \frac{n_1.n_2}{|n_1||n_2|} = \frac{(\hat{i}+\hat{j}).(\hat{i}+2\hat{j}-\hat{k})}{\sqrt{2}\sqrt{6}} = \frac{3}{\sqrt{2}\sqrt{6}} = \frac{\sqrt{3}}{2}$$

$\therefore \quad \theta = \cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}$

As, L is contained in $P_2 \Rightarrow \theta = 0^o$

53. Let $f:(0,\infty)\to R$ be given by $f(x)=\int_{1/x}^{x} e^{-\left(t+\frac{1}{t}\right)}\frac{dt}{t}$ then

(A) $f(x)$ is monotonically increasing on $[1,\infty)$

(B) $f(x)$ is monotonically decreasing on $[0,1)$

(C) $f(x)+f\left(\frac{1}{x}\right)=0$, for all $x\in(0,\infty)$

(D) $f(2^x)$ is an odd function of x on R

53. **(ACD)**

We have, $f(x)=\int_{1/x}^{x}\frac{e^{-\left(t+\frac{1}{t}\right)}}{t}dt$

$$f'(x)=\frac{e^{-\left(x+\frac{1}{x}\right)}}{x}-\frac{e^{-\left(\frac{1}{x}+x\right)}}{\frac{1}{x}}\left(-\frac{1}{x^2}\right)$$

$$=\frac{e^{-\left(x+\frac{1}{x}\right)}}{x}+\frac{e^{-\left(x+\frac{1}{x}\right)}}{x}=\frac{2e^{-\left(x+\frac{1}{x}\right)}}{x}$$

As, $f'(x)>0\ \forall\ x\in(0,\infty)$

$\therefore f(x)$ is monotonically increasing on $(0,\infty)$.

Now, $f(x)+f\left(\frac{1}{x}\right)=\int_{1/x}^{x}\frac{e^{-\left(t+\frac{1}{t}\right)}}{t}dt+\int_{x}^{1/x}\frac{e^{-\left(t+\frac{1}{t}\right)}}{t}dt$

$=0, \forall x\in(0,\infty)$

Now, let $g(x)=f(2^x)=\int_{2^{-x}}^{2^x}\frac{e^{-\left(t+\frac{1}{t}\right)}}{t}dt$

$$g(-x)=f(2^{-x})=\int_{2^x}^{2^{-x}}\frac{e^{-\left(t+\frac{1}{t}\right)}}{t}dt=-g(x)$$

$\therefore f(2^x)$ is an odd function.

54. If point M is moved on the circle $(x-4)^2+(y-8)^2=20$, then it broke away from it and moving along a tangent to the circle cut the X-axis at point $(-2,0)$. The coordinates of the point on the circle at which the moving point broke away, is

(A) $\left(\frac{42}{5},\frac{36}{5}\right)$ (B) $\left(\frac{-2}{5},\frac{44}{5}\right)$ (C) (6, 4) (D) (2, 4)

54. **(BC)**

We have, $x^2+y^2-8x-16y+60=0$ ………. (i)

Equation of chord of contact from $(-2,0)$ is

$-2x-4(x-2)-8y+60=0$

$\Rightarrow \quad 3x+4y-34=0$ ………. (ii)

On solving Eqs. (i) and (ii), we get the points are (6, 4) and $\left(\frac{-2}{5},\frac{44}{5}\right)$

SECTION-III : (MATRIX-MATCH TYPE)

This section contains **02 Matrix Match**. Each question has matching lists. Each question has four choice (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

55. Match the following columns.

	Column – I		**Column – II**
P.	If $I=\int_{-2}^{2}\left(\alpha x^3+\beta x+\gamma\right)dx$, then I is	**1.**	Independent of α
Q.	Let α,β be the distinct positive roots of the equation $\tan x=2x$. Then, $\gamma\int_0^1(\sin\alpha x.\sin\beta x)dx$ (where $\gamma\neq 0$) is	**2.**	Independent of β
R.	If $f(x+\alpha)+f(x)=0$, where $\alpha>0$, then $\int_{\beta}^{\beta+2\gamma\alpha}f(x)dx$, where $\gamma\in N$ is	**3.**	Independent of γ
S.	$\gamma\int_0^{\alpha}[\sin x]dx$ is, where $\gamma\neq 0$, $\alpha\in\left[(2\beta+1)\pi,(2\beta+2)\pi\right],\beta\in N$, and where [.] denotes the greatest integer function.	**4.**	Depends on α

Codes:

	P	Q	R	S
(A)	1,2	2,3	1,2,3	3
(B)	1,3	2,4	3,4	4
(C)	1,2	1,2,3	2,3	3
(D)	2,4	1,3	1,4	3

55. **(C)**

(P) $I=\int_{-2}^{2}\left(\alpha x^3+\beta x+\gamma\right)dx$

$\alpha x^3+\beta x$ is an odd function.

$I=0+2\int_0^2\gamma dx=2\times 2\gamma=4\gamma$

(Q) $I=\frac{1}{2}\int_0^2 2\sin\alpha x\sin\beta x\ dx$

$I=\frac{1}{2}\int_0^1\left[\cos(\alpha-\beta)x-\cos(\alpha+\beta)x\right]dx$

$$=\frac{1}{2}\left[\frac{\sin(\alpha-\beta)x}{\alpha-\beta}-\frac{\sin(\alpha+\beta)x}{\alpha+\beta}\right]_0^1$$

$$=\frac{1}{2}\left[\frac{\sin(\alpha-\beta)}{\alpha-\beta}-\frac{\sin(\alpha+\beta)}{\alpha+\beta}\right]$$

Also, $2\alpha=\tan\alpha$ and $2\beta=\tan\beta$

$\therefore \quad 2(\alpha-\beta)=\tan\alpha-\tan\beta$

And $2(\alpha+\beta)=\tan\alpha+\tan\beta$

$$2(\alpha-\beta)=\frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta} \text{ and } 2(\alpha+\beta)=\frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta}$$

Substituting these values, we get

$I=(\cos\alpha\cos\beta)-(\cos\alpha\cos\beta)=0$

(R) $f(x+\alpha)+f(x)=0$ or $f(x+2\alpha)+f(x+\alpha)=0$

or $f(x+2\alpha)=f(x)$

Thus, $f(x)$ is periodic with period 2α.

Hence,

$$\int_{\beta}^{\beta+2\gamma\alpha}\left(\alpha x^3+\beta x+\gamma\right)dx=\gamma\int_0^{2\alpha}f(x)\,dx$$

(S) Let $I=\int_0^{\alpha}[\sin x]\,dx, \alpha\in[(2\beta+1)\pi,(2\beta+2)], \beta\in N$

[where [.] denotes the greatest integer function]

$$=\int_0^{2\beta\pi}[\sin x]\,dx+\int_{2\beta x}^{(2\beta+1)\pi}[\sin x]\,dx+\int_{(2\beta+1)\pi}^{\alpha}[\sin x]\,dx$$

$$=\beta\int_0^{2\pi}[\sin x]\,dx+0+\int_{(2\beta+1)\pi}^{\alpha}(-1)\,dx$$

$$=-\beta\pi+(2\beta+1)\pi-\alpha=(\beta+1)\pi-\alpha$$

Thus, $\gamma\int_0^{\alpha}[\sin x]\,dx$ depends on α,β and γ.

56. Match the following columns.

	Column – I		**Column – II**
P.	If a, b, c are (a, b, c are distinct) in GP, then $\log_a 10, \log_b 10, \log_c 10$ are in	**1.**	AP
Q.	If $\frac{a+be^x}{a-be^x}=\frac{b+ce^x}{b-ce^x}=\frac{c+de^x}{c-de^x}$, then (a, b, c, d are distinct in)	**2.**	HP
R.	If a, b, c are in AP; a, x, b are in GP and b, y, c are in GP, then x^2, b^2, y^2 are in (a, b, c are distinct)	**3.**	GP
S.	If x, y, z are in GP and $a^x=b^y=c^z$, then log a, log b, log c are in (x, y, z are distinct)	**4.**	None of these

Codes:

	P	**Q**	**R**	**S**		**P**	**Q**	**R**	**S**
(A)	2	3	1	3	(B)	1	4	2	3

(C) 3 2 4 1 (D) 1 2 3 4

56. **(A)**

(P) Since, a, b, c are in GP.

Hence, $b^2 = ac$

$\Rightarrow \quad 2\log_{10} b = \log_{10} a + \log_{10} c$

$\Rightarrow \quad \dfrac{2}{\log_b 10} = \dfrac{1}{\log_a 10} + \dfrac{1}{\log_c 10}$

$\Rightarrow \quad \dfrac{2}{y} = \dfrac{1}{x} + \dfrac{1}{z}$

Hence, x, y, z are in HP.

(Q) $\dfrac{a+be^x}{a-be^x} = \dfrac{b+ce^x}{b-ce^x} = \dfrac{c+de^x}{c-de^x}$

$\Rightarrow \quad \dfrac{2a}{a-be^x} - 1 = \dfrac{2b}{b-ce^x} - 1 = \dfrac{2c}{c-de^x} - 1$

$\Rightarrow \quad \dfrac{a-be^x}{a} = \dfrac{b-ce^x}{b} = \dfrac{c-de^x}{c}$

$\Rightarrow \quad 1 - \dfrac{b}{a}e^x = 1 - \dfrac{c}{b}e^x = 1 - \dfrac{d}{c}e^x$

$\Rightarrow \quad \dfrac{b}{a} = \dfrac{c}{b} = \dfrac{d}{c}$

Hence, a, b, c, d are in GP.

(R) Given, $2b = a + c, x^2 = ab, y^2 = bc$

Now, $x^2 + y^2 = b(a+c)$

$= b(2b) = 2b^2$

$\Rightarrow \quad x^2 + y^2 = 2b^2$

Hence, x^2, b^2, y^2 are in AP.

(S) $x\log a = y\log b = z\log c = k(\text{say})$

Also, $y^2 = xz$

$\Rightarrow \quad \dfrac{k^2}{(\log b)^2} = \dfrac{k^2}{\log a \log c}$

Hence, log a, log b, log c are in GP.

SECTION-IV : (INTEGER ANSWER TYPE)

This section contains **04** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**; e.g. 6.25, 7.00, 0.33, 30.27, 127.30)

57. If $F(x) = \dfrac{1}{x^2}\int_4^x \left[4t^2 - 2F'(t)\right]dt$, then $(9F'(4))/4$ is ……….

57. **(8)**

Here, $\frac{d}{dx}\int_4^x\left[4t^2-2F'(t)\right]dt=\left[4x^2-2F'(x)\right].1-0$

$\Rightarrow \quad F'(x)=\frac{1}{x^2}\left[4x^2-2F'(x)\right]+\frac{(-2)}{x^3}\int_4^x\left[4t^2-2F'(t)\right]dt$

$\Rightarrow \quad F'(4)=\frac{1}{16}\left[64-2F'(4)\right]-\frac{1}{32}\int_4^4\left[4t^2-2F'(t)\right]dt$

$\Rightarrow \quad F'(4)=\frac{1}{16}\left[64-2F'(4)\right]$

$\Rightarrow \quad \left(1+\frac{1}{8}\right)F'(4)=4\Rightarrow F'(4)=\frac{32}{9}$

Hence, $\frac{9F'(4)}{4}=8$

58. Let $a=3^{\frac{1}{233}}+1$ and for all $n\ge 3$, let $f(n)={}^nC_0.a^{n-1}-{}^nC_1.a^{n-2}+{}^nC_2.a^{n-3}-.....+(-1)^{n-1}.{}^nC_{n-1}.a^0$. If the value of $f(2007)+f(2008)=3^k$, where $k\in N$, then the values of k is ………

58. **(9)**

We have, $f(n)={}^nC_0a^{n-1}-{}^nC_1a^{n-2}+{}^nC_2a^{n-3}-.....+(-1)^{n-1}.{}^nC_{n-1}a^o$

$=\frac{1}{a}\left({}^nC_0a^n-{}^nC_1a^{n-1}+{}^nC_2a^{n-2}-.....+(-1)^{n-1}C_{n-1}a\right)$

$=\frac{1}{a}\left[(a-1)^n-(-1)^n\,{}^nC_n\right]=\frac{1}{a}\left\{\left(\frac{1}{3^{223}}-(-1)^n\right)\right\}$

$\therefore f(x)=\frac{3^{\frac{n}{223}}-(-1)^n}{\left(3^{\frac{1}{223}}+1\right)}\Rightarrow f(2007)=\frac{3^{\frac{2007}{223}}+1}{3^{\frac{1}{223}}+1}$

And $\quad f(2008)=\frac{3^{\frac{2008}{223}}-1}{3^{\frac{1}{223}}+1}$

$\Rightarrow \quad f(2007)+f(2008)=\frac{3^{\frac{2007}{223}}+3^{\frac{2008}{223}}}{3^{\frac{1}{223}+1}}$

$=\frac{3^9+3^{9+\frac{1}{223}}}{3^{\frac{1}{223}}+1}=3^9.\frac{\left(1+3^{\frac{1}{223}}\right)}{\left(1+3^{\frac{1}{223}}\right)}=3^9$

$\Rightarrow \quad 3^9=3^k$

Hence, $k=9$

59. Let $I=\int_2^3\frac{dx}{\sqrt{x^3-3x^2+5}}$, then the value of $\left[I+\sqrt{3}\right]$, Where [.] represents greatest integral function, is ……..

59. **(2)**

Let $f(x) = x^3 - 3x^2 + 5$

$f'(x) = 3x^2 - 6x$

$f'(x) = 3x(x-2) > 0$, as $x \in [2,3]$

$\Rightarrow \quad \int_2^3 \frac{1}{\sqrt{5}}\,dx < \int_2^3 \frac{dx}{\sqrt{x^3 - 3x^2 + 5}} < \int_2^3 dx$

$\Rightarrow \quad \frac{1}{\sqrt{5}} < I < 1$

$\Rightarrow \quad \frac{1}{\sqrt{5}} + \sqrt{3} < I + \sqrt{3} < 1 + \sqrt{3} \Rightarrow \left[I + \sqrt{3}\right] = 2$

60. If $k \sin 17^\circ \sin 43^\circ \sin 77^\circ = \cos 39^\circ$, then the value of k is …

60. **(4)**

Now, $\sin 17^\circ \sin 43^\circ \sin 77^\circ$

$= \sin 17^\circ \sin(60^\circ - 17^\circ) \sin(60^\circ + 17^\circ)$

$= \frac{1}{4} \sin 51^\circ = \frac{1}{4} \cos 39^\circ$

$\Rightarrow \quad 4 \sin 17^\circ \sin 43^\circ \sin 77^\circ = \cos 39^\circ$

$\therefore \quad k = 4$