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CLASSROOM CONTACT PROGRAMME
(ACADEMIC SESSION 2014-2015)

TARGET : JEE (Main + Advanced) 2015
ALLEN JEE (Advanced) TEST

ENTHUSIAST COURSE : SCORE-II
DATE : 03 - 02 - 2015

ANSWER KEY

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,C,D	A	A,C	A,C	A,B,C,D	A,C,D	A,D	A,B,C,D	B,C,D	A,B,C,D
	Q.	11	12	13	14						
	A.	B	B	C	A						
SECTION-IV	Q.	1	2	3	4	5	6				
	A.	4	8	4	3	8	2				

PART-2 : CHEMISTRY

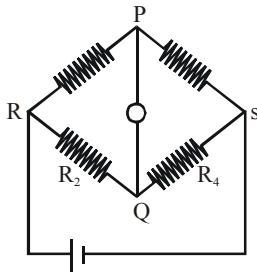
SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,B	B,C,D	B,C,D	A,D	B,C,D	A,C	A,D	B,C	A,B,C,D	A
	Q.	11	12	13	14						
	A.	B	C	D	C						
SECTION-IV	Q.	1	2	3	4	5	6				
	A.	1	8	4	5	8	3				

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,B	C,D	A,B,C,D	B,D	B,D	A,B,C	A,C,D	C,D	B,C	C,D
	Q.	11	12	13	14						
	A.	D	A	D	A						
SECTION-IV	Q.	1	2	3	4	5	6				
	A.	7	1	3	6	4	9				

SECTION-I

1. Ans. (A, C, D)



Sol.

equivalent diagram is as shown

is P is moved 2cm right then $R_1 = 12, R_3 = 3$

$$\frac{R_1}{R_3} = \frac{R_2}{R_4} \text{ (Hence wheat stone will be balanced)}$$

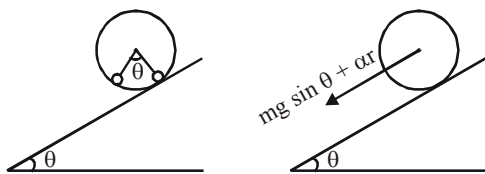
If s is moved left $\frac{5}{3}$ cm then $R_3 = \frac{10}{3}$ and

$$R_4 = \frac{20}{3} \text{ hence } \frac{R_1}{R_3} = \frac{R_2}{R_4} \text{ (hence wheatstone will be balanced)}$$

2. Ans. (A)

Sol. Acceleration slowly increases from zero to α , so small sphere will be at rest w.r.t. ground when maximum acceleration α is to be reached in frame of cylinder, small sphere will have αr downward acceleration

Applying condition of rolling



$$\alpha r = \frac{g \sin \theta + \alpha r}{1 + \frac{2}{5}}$$

$$\frac{7}{5} \alpha r - \alpha r = g \sin \theta$$

$$\theta = \sin^{-1} \left(\frac{2\alpha r}{5g} \right)$$

3. Ans. (A, C)

Sol. R is independent of freq.

(A) is true.

Current becomes maximum at resonance

(B) cannot be true

$$X_L = L = 2\pi fL$$

(C) is true

$$Y = \frac{1}{C} = \frac{1}{2\pi fC}$$

(D) can not be true.

4. Ans. (A, C)

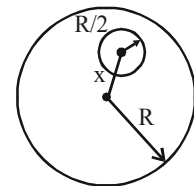
Sol. Asteroid-I

Asteroid-II

$$\text{Field } E = \frac{GM}{R^3} x$$

$$\text{Field } E_{II} = \frac{GM}{R^3} \left(\frac{R}{2} \right)$$

$$\frac{F_I}{F_{II}} = \frac{2x}{R}$$



$$T_I = \frac{2\pi}{4} \sqrt{\frac{R^3}{GM}}$$

$$R = \frac{1}{2} \frac{GM}{R^3} \left(\frac{R}{2} \right) T_{II}^2$$

$$T_{II} = 2 \sqrt{\frac{R^3}{GM}}$$

5. Ans. (A, B, C, D)

Sol. $\vec{E} = (10 - 5x) \hat{i}$

$$m = 5\text{kg}$$

$$\vec{F} = q\vec{E}$$

$$\vec{F} = (10 - 5x) \hat{i}$$

(a) $F \propto -x$

so motion of charge particle is oscillatory

(b) $F = 0$ at $x = 2$

oscillation about $x = 2$

so maximum displacement is 4

$$F = 10 - 5x$$

$$a = \frac{10 - 5x}{2}$$

$$a = 2 - x$$

$$a = -(x - 2)$$

$$\text{so } w^2 = 1$$

$$w = 1$$

$$(c) v_{\max} = Aw$$

$$A = 2$$

$$w = 1$$

$$v_{\max} = 2$$

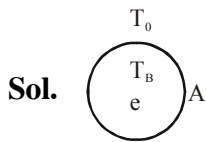
(d) velocity at mean position

at $x = 2$

6. Ans. (A, C, D)

7. Ans. (A, D)

8. Ans. (A, B, C, D)



emissivity of body e

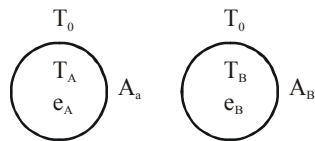
Area of body A

Temperature of body T_B

Temperature of environment T_0

Heat energy radiated by body = $e\sigma AT_B^4$

absorbed by body = $e\sigma AT_0^4$



$$e_A \neq e_B$$

Then heat emitted by heat emitted by a body is not equal to B.

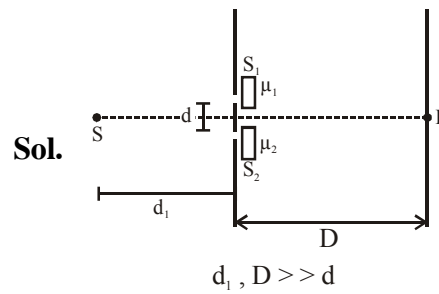
9. Ans. (B, C, D)

Sol. *the probability of decay in 1 sec is the decay constant λ , which remains constant

*after average life 37% of nuclei remain undecayed

*Part of energy of disintegration is taken by the recoiling daughter nucleus.

10. Ans. (A, B, C, D)



Optical path difference due to

$$S_1 \text{ is } t(\mu_1 - 1) = 9\lambda \left(\frac{4}{3} - 1 \right)$$

$$= 3\lambda$$

Optical path differ due to S_2 is

$$= t(\mu_2 - 1) = 2\lambda \left(\frac{3}{2} - 1 \right) = \lambda$$

Path diff. at point P = $3\lambda - \lambda = 2\lambda$

$$\text{Phase diff.} = \frac{2\pi}{\lambda} \times 2\lambda = 4\pi$$

So net intensity at point P is

$$I_{\text{net}} = I_0 + I_0 + 2I_0 \cos 4\pi$$

$$I_{\text{net}} = 4I_0$$

Optical path is conserved by S_1 is more so fringe will shift at upper side.

11. Ans. (B)

Sol. $\frac{1}{2} I_x \omega^2 + \frac{1}{2} I_y \omega^2 = \frac{f}{2} kT$ { $f_{\text{rotational}} = 2$ }

$$I_x = I_y \Rightarrow \mu r^2 = \frac{m}{2} a^2 \quad \left\{ \mu = \frac{m_1 m_2}{m_1 + m_2} \right\}$$

$$\frac{m a^2}{2} \omega^2 = \frac{2}{2} kT \quad \mu = \frac{m}{2}$$

$$\omega = \sqrt{\frac{2kT}{m a^2}}$$

12. Ans. (B)

13. Ans. (C)

Sol. A = 27 P = 12

z = 12 n = 15

n > p

By graph 'β' decay is possible

14. Ans. (A)

Sol. ${}_{88}^{178}X \rightarrow P + 88$

$n = 178 - 88 = 90$

$n > p$ & $z > 83$ α decay will take place

SECTION-IV

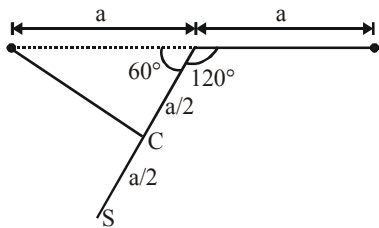
1. Ans. 4

Sol. After moving the loop about side PQ through 120° , the normal on the side PS from the wire will be passing through midpoint C and the net magnetic flux through the loop in the new position will be zero.

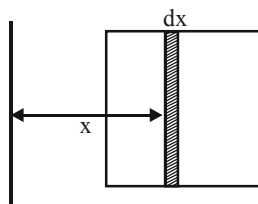
\therefore Change in flux $\Delta\phi =$ Flux through the loop in the initial position.

Flux through area element dA

$$d\phi = \vec{B} \cdot d\vec{A} = \frac{\mu_0 I}{2\pi x} a \cdot dx$$



$$\phi = \int_a^{2a} \frac{\mu_0 I a dx}{2\pi x} = \frac{\mu_0 I a}{2\pi} \ln(2)$$



\therefore Charge flown through the square

$$loop = \frac{\Delta\phi}{R} = \frac{\mu_0 I a}{2\pi R} \ln(2)$$

2. Ans. 8

Sol. Height of water column = $\frac{20}{5} = 4$ cm

mass of water = 20 gm

By work energy theorem

$$W_{ext} = \Delta U_{piston} + \Delta U_{water} + KE_{water} = 8 \times 10^{-2} \text{ Joule}$$

3. Ans. 4

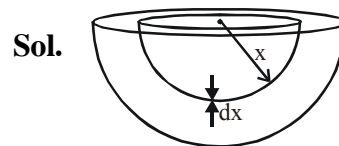
Sol. $W_g + W_{mg} + W_{friction} = \Delta KE$

$$W_{friction} = 0 - \frac{1}{2} m \left[\frac{qBR}{m} \right]^2$$

$$W_{friction} = -\frac{1}{2} \frac{q^2 B^2 R^2}{m} = \frac{-Kq^2 B^2 R^2}{8m}$$

$K = 4$

4. Ans. 3



$$V = \int_a^b \frac{K(2\pi x^2 dx) \rho}{x}$$

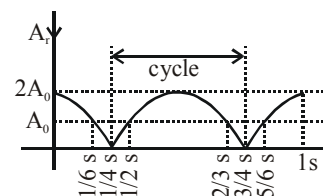
$$V = 2\pi k \rho \left[\frac{b^2 - a^2}{2} \right] = \pi k \rho (b^2 - a^2)$$

$b = 2a$

$v = 3\pi k \rho b^2$

5. Ans. 8

Sol. $y_1 = A \sin \omega_1 t$



$y_2 = A \sin \omega_2 t$

$$y_1 = 2A \cos \left\{ \frac{(\omega_2 - \omega_1)}{2} t \right\} \left\{ \sin \frac{(\omega_2 + \omega_1)}{2} t \right\}$$

Resultant amplitude $A_r = 2A_0 |\cos(\Delta\omega)t/2|$

$$(\Delta\omega) \frac{t}{2} = \frac{\pi}{2} \Rightarrow t = \frac{1}{4} s$$

$$(\Delta\omega) \frac{t}{2} = \frac{\pi}{3} \Rightarrow t = \frac{1}{6} s$$

In one cycle of intensity of $1/2s$, the detector remain idle for

$$2\left(\frac{1}{4} - \frac{1}{6}\right)s = \frac{1}{6}\text{sec}$$

$$\therefore \text{In } \frac{1}{2} \text{ sec cycle, active time is } \left(\frac{1}{2} - \frac{1}{6}\right)$$

$$= \frac{1}{3} \text{ sec}$$

$$\therefore \text{In 12 sec interval, active time is}$$

$$12 \times \frac{(1/3)}{(1/2)} = 8 \text{ sec}$$

6. Ans. 2

Sol. Applying conservation of energy

$$mgs(\sin\alpha - \sin\beta) = \frac{1}{2}I\omega^2 + 2\left(\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2\right)$$

$$\text{where } \omega = v/r \text{ \& } I = mr^2/2$$

Putting values & solving

$$mgs(\sin\alpha - \sin\beta) = \frac{7}{4}mv^2$$

$$\therefore v = 2\sqrt{\frac{1}{7}gs(\sin\alpha - \sin\beta)}$$

$$= 2\sqrt{\frac{10 \times 3.5}{7} \left(\frac{4}{5} - \frac{3}{5}\right)} = 2 \text{ m/s.}$$

PART-2 : CHEMISTRY

SOLUTION

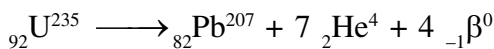
SECTION-I

1. Ans. (A,B)

2. Ans. (B,C,D)

3. Ans. (B,C,D)

4. Ans. (A, D)



5. Ans. (B, C, D)

6. Ans. (A, C)

7. Ans. (A,D)

8. Ans. (B, C)

9. Ans. (A, B, C, D)

10. Ans. (A)

11. Ans. (B)

$$P = K_H \cdot \frac{M}{1000} \times M_{\text{solvent}}$$

$$1.8 = 10^4 \times \frac{M}{1000} \times 18$$

$$M = 10^{-2} \text{ mol/L}$$

$$= 0.044 \% \text{ (w/v)}$$

12. Ans. (C)

$$[\text{H}^+] = \sqrt{K_a C} = 10^{-4}$$

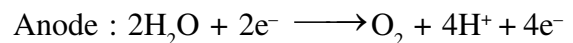
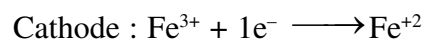
$$\text{pH} = 4$$

13. Ans. (D)

14. Ans. (C)

SECTION-IV

1. Ans. 1



milliequivalent of $\text{Fe}^{2+} = \text{KMnO}_4$ in acidic medium

$$= 0.01 \times 30 \times 5$$

millimoles of Fe^{2+} formed = 1.5

$$\text{moles of Fe}^{2+} \times 1 = \frac{I \times t}{96000} = \frac{40 \times 10^{-3} \times t}{96000}$$

$$t = 3600 \text{ sec.} = 1 \text{ hr.}$$

2. Ans. 8

3. Ans. 4

4. Ans. 5

5. Ans. 8

6. Ans. 3

PART-3 : MATHEMATICS

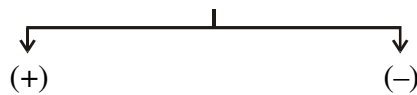
SOLUTION

SECTION-I

1. **Ans. (A,B)**

$$\cos(\cos x - \sin x) = \cos\left(\frac{\pi}{2} - (\sin x + \cos x)\right)$$

$$\Rightarrow \cos x - \sin x = 2n\pi \pm \left(\frac{\pi}{2} - (\sin x + \cos x)\right)$$



$$\Rightarrow \cos x = n\pi + \frac{\pi}{4} \quad \sin x = -n\pi + \frac{\pi}{4}$$

Clearly, $n = 0$ clearly $n = 0$

$$\Rightarrow \cos x = \frac{\pi}{4} \quad \Rightarrow \sin x = \frac{\pi}{4}$$

$$\Rightarrow \sin x = \pm \frac{\sqrt{16 - \pi^2}}{4}$$

2. **Ans. (C,D)**

$$\because z^2 - z = |z|^2 - \frac{64}{|z|^5} = \text{a purely real number}$$

$$\Rightarrow z^2 - z = \bar{z}^2 - \bar{z} \Rightarrow (z^2 - \bar{z}^2) - (z - \bar{z}) = 0$$

$$\text{if } z - \bar{z} = 0 \Rightarrow z = \bar{z}$$

$\Rightarrow z$ is purely real number

So put $z = x$ in given equation.

$$x^2 - x - |x|^2 + \frac{64}{|x|^5} = 0$$

$$\Rightarrow x^2 - x - |x|^2 + \frac{64}{x^5} = 0 \quad (\text{if } x > 0)$$

($\because x < 0$ not possible)

$$\Rightarrow x^6 = 64 \Rightarrow x = 2 \quad \therefore |z| = 2$$

3. **Ans. (A,B,C,D)**

Check each option.

4. **Ans. (B,D)**

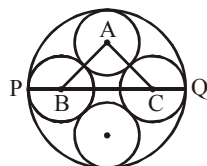
Clearly ABC is a right angled triangle with right angle at A and $AB = 2r = AC$,

$$PB = QC = r$$

$$\therefore BC^2 = 4r^2 + 4r^2 \Rightarrow BC = 2\sqrt{2}r$$

$$\therefore PQ = 2\sqrt{2}r + 2r = 2r(1 + \sqrt{2})$$

$$\Rightarrow 2R = 2r(1 + \sqrt{2}) \Rightarrow \frac{R}{r} = 1 + \sqrt{2}$$



5. **Ans. (B,D)**

Without loss of generality, we take equation of circle is $x^2 + y^2 = 1$ and fixed point is $(a, 0)$ where $a \geq 0$. Let $P(x, y)$ is a point which is equidistant to both, then

$$\left(\sqrt{x^2 + y^2} - 1\right)^2 = (x - a)^2 + y^2$$

$$\Rightarrow 2\sqrt{x^2 + y^2} = 2ax + (1 - a^2) \quad \dots\dots(i)$$

\downarrow squaring

$$4(1 - a^2)x^2 - 4a(1 - a^2)x + 4y^2 = (1 - a^2)^2$$

if $a < 1$, then locus is an ellipse and if $a > 1$, then locus is hyperbola but we get only one branch of hyperbola because if $a < 1$, then equation (i) $\Rightarrow x$ is (+)ve.

6. **Ans. (A,B,C)**

$$\because f(n+1) = \frac{f(n)-1}{f(n)+1} = \frac{\frac{f(n-1)-1}{f(n-1)+1} - 1}{\frac{f(n-1)-1}{f(n-1)+1} + 1} = \frac{-1}{f(n-1)}$$

$$\therefore f(n+1) = -\frac{1}{f(n-1)} \quad \dots\dots(i)$$

$$\text{In a similar way } f(n-1) = -\frac{1}{f(n-3)} \quad \dots\dots(ii)$$

from (i) and (ii)

$$f(n+1) = -\frac{1}{-1} = f(n-3)$$

or $f(n+4) = f(n) \quad \forall n \in \mathbb{N}$

hence $f(n)$ is a periodic sequence with period 4 using (A) put $n = 1, 2, 3$

$$\text{we get } f(2) = \frac{1}{3}, f(3) = -\frac{1}{2}, f(4) = -3$$

$$\text{Now } f(2012) = f(4) = -3$$

$$f(2013) = f(1) = 2$$

$$f(2015) = f(3) = -\frac{1}{2}$$

$$f(1001) = f(1) = 2$$

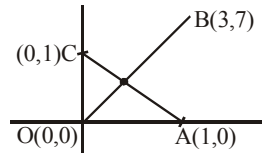
7. **Ans. (A,C,D)**

Let $O(0, 0)$, $A = (1, 0)$, $B = (3, 4)$, $C = (0, 1)$

Now for $f(x, y)$ to be minimum, (x, y) must be the point of intesection of OB and AC

$\therefore f(x, y)_{\min} = OB + AC = 5 + \sqrt{2}$
which occurs at

$$x = \frac{3}{7}, y = \frac{4}{7}$$



8. Ans. (C,D)

(A) False statement because A^{-1} exist only if $\det. A \neq 0$

(B) False statement, as $\det(AB^{-1})$

$$= \det(A) \cdot \det(B^{-1}) = \frac{|A|}{|B|} = -\frac{6}{2} = -3.$$

(C) True Statement, $\therefore A^2 = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} = 3A$

$$\Rightarrow A^3 = 3A^2 = 3(3A) \Rightarrow A^3 = 9A$$

(D) Given $A^2 = A$ and $B = I - A$.

$$\begin{aligned} \text{Now } AB + BA + I - (I - A)^2 \\ = AB + BA + I - (I + A^2 - 2A) = AB + BA + A \\ \therefore \text{ True statement} \end{aligned}$$

9. Ans. (B,C)

Disjoint \Rightarrow mutually exclusive.

(A) Two mutually exclusive events E & F with $P(E)$ and $P(F) > 0$ cannot be independent

(B) $P(E) + P(\bar{E}F) + P(\overline{E \cap F})$

$$= P(E) + P(F) - P(E \cap F) + P(\bar{E} \cap \bar{F}) = 1$$

\Rightarrow exhaustive

(C) $\therefore P(E/F) > P(E)$

$$\Rightarrow \frac{P(E \cap F)}{P(F)} > P(E)$$

$$\Rightarrow \frac{P(E \cap F)}{P(E)} > P(F) \text{ or } P(F/E) > P(F)$$

(D) Every element of set of positive integer can be of the form $6n, 6n + 1, 6n + 2, \dots, 6n + 5$.
But favourable are $6n + 1, 6n + 5$.

$$\text{So, required probability} = \frac{2}{6} = \frac{1}{3}.$$

10. Ans. (C,D)

$\therefore AA^T = I \Rightarrow \vec{a}, \vec{b}, \vec{c}$ are orthogonal unit vectors

$$\therefore \vec{c} = \pm(\vec{a} \times \vec{b}) = \pm \frac{1}{49} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 6 & 2 & -3 \end{vmatrix} \Rightarrow c = \pm \frac{(-3\hat{i} + 6\hat{j} - 2\hat{k})}{7}$$

$$\therefore |\vec{a} \times \vec{b}| = 1$$

11 Ans. (D)

$$\text{Given } \int_0^x f(t) dt + \int_0^x (x-t) \cdot f(t) dt = e^{-x} - 1$$

$$\text{or, } \int_0^x f(t) dt + x \int_0^x f(t) dt - \int_0^x t f(t) dt = e^{-x} - 1$$

Diff. both sides with respect to x , we get.

$$f(x) + x \cdot f(x) + \int_0^x f(t) dt - x f(x) = -e^{-x}$$

$$\Rightarrow f(x) + \int_0^x f(t) dt = -e^{-x} \quad \dots(1)$$

again diff. w.r.to $x \Rightarrow e^x(f'(x) + f(x)) = 1 \dots(2)$

$$e^x \cdot f(x) = x + c, \text{ from (1), } f(0) = -1.$$

$$\therefore f(x) = (x-1)e^{-x}$$

$$\therefore \int_0^1 f(x) dx = \int_0^1 (x-1)e^{-x} dx = -(x-1)e^{-x} \Big|_0^1 + \int_0^1 e^{-x} dx$$

$$= -1 - (e^{-x})_0^1 = -1 - \left(\frac{1}{e} - 1\right) = -\frac{1}{e}$$

12. Ans. (A)

Put $x = 0$ in (2)

$$f'(0) + f(0) = 1 \Rightarrow f'(0) = 2$$

Paragraph for Question 13 & 14

$$g''(x) = Ax$$

$$\Rightarrow g'(x) = \frac{Ax^2}{2} + B \therefore g'\left(-\frac{1}{2\sqrt{2}}\right) = 0$$

$$\Rightarrow \frac{A}{16} + B = 0 \Rightarrow B = -\frac{A}{16}$$

$$\Rightarrow g'(x) = \frac{Ax^2}{2} - \frac{A}{16}$$

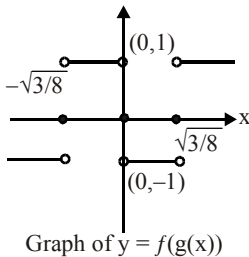
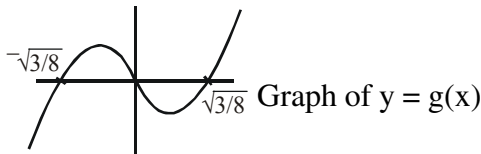
$$\Rightarrow g(x) = \frac{Ax^3}{6} - \frac{Ax}{16} + C$$

$$\therefore g(0) = 0 \text{ and } g(1) = 5 \Rightarrow c = 0, A = 48$$

$$g(x) = 8x^3 - 3x$$

$$\text{so, } f(g(x)) = \text{sgn}(8x^3 - 3x)$$

$$= \begin{cases} -1 & \text{if } x \in \left(-\infty, -\sqrt{\frac{3}{8}}\right) \\ 1 & \text{if } x \in \left(-\sqrt{\frac{3}{8}}, 0\right) \\ -1 & \text{if } x \in \left(0, \sqrt{\frac{3}{8}}\right) \\ 1 & \text{if } x \in \left(\sqrt{\frac{3}{8}}, \infty\right) \end{cases}$$



clearly $f(g(x))$ is not a periodic function
area enclosed between ordinates $x = -\alpha$ to $x = \alpha$

$$\text{is} = 2 \int_0^{\alpha} dx = 2\alpha$$

13. Ans. (D)

14. Ans. (A)

$$\frac{dy}{dx} + g'(x) \cdot y = (g(x) + 1) \cdot g'(x)$$

\therefore Integrating factor = $e^{g(x)}$

\therefore solution is

$$y \cdot e^{g(x)} = \int e^{g(x)} \cdot (g(x) + 1) \cdot g'(x) dx$$

$$\text{Put } g(x) = t \Rightarrow g'(x) dx = dt$$

$$\Rightarrow y \cdot e^{g(x)} = \int e^t (t+1) dt = te^t + c$$

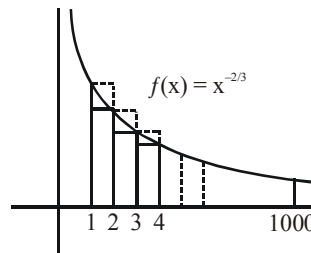
$$\Rightarrow h(0) = 1 \Rightarrow 1 = 0 + c \Rightarrow c = 1$$

$$\therefore h(x) = g(x) + e^{-g(x)}$$

$$\text{Hence } h(1) = 5 + e^{-5}$$

SECTION - IV

1. Ans. 7



$$\therefore \sum_{n=1}^{1000} n^{-2/3} > \int_1^{1000} x^{-2/3} dx = 3(x^{1/3})^{1000} = 27 \dots(1)$$

$$\text{and } \sum_{n=1}^{1000} n^{-2/3} < \int_1^{1000} x^{-2/3} dx$$

$$\begin{aligned} & \{(1^{-2/3} - 2^{-2/3}) + (2^{-2/3} - 3^{-2/3}) + (3^{-2/3} - 4^{-2/3}) \\ & + \dots + (999^{-2/3} - 1000^{-2/3})\} \\ & = 27 + \underbrace{\{1^{-2/3} - 1000^{-2/3}\}}_{\text{No. is less than 1}} \dots(2) \end{aligned}$$

$$\therefore \text{From (1) and (2)} \left[\sum_{n=1}^{1000} n^{-2/3} \right] = 27$$

$$\Rightarrow m - 20 = 7$$

2. Ans. 1

$$\therefore y = x^4 - 6x^3 + 12x^2 + cx + 1$$

$$\Rightarrow \frac{dy}{dx} = 4x^3 - 18x^2 + 24x + c = g(x)$$

$$\text{and } \frac{d^2y}{dx^2} = 12x^2 - 36x + 24 = 12(x-1)(x-2)$$

Necessary condition, $g(1) \cdot g(2) < 0$

$$(8+c) \cdot (10+c) < 0 \Rightarrow -10 < c < -8$$

\therefore Number of integers = 1.

3. Ans. 3

$$\text{Let } f(x) = x^2 - 2ax + a^2 - 4$$

$$\therefore A \cap B = \{x : -2 < x \leq 1\}$$

$$\therefore \text{vertex} \geq 1 \dots(1)$$

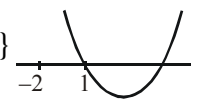
$$f(1) \geq 0 \dots(2)$$

$$(1) \Rightarrow a \geq 1$$

$$(2) \Rightarrow a^2 - 2a - 3 \geq 0$$

$$\Rightarrow (a-3)(a+1) \geq 0 \Rightarrow a \leq -1 \text{ and } a \geq 3.$$

so smallest positive integral value of a is 3.



4. Ans. 6

$$L_1 : \frac{x-0}{1} = \frac{y+1}{1} = \frac{z-0}{1} = \lambda;$$

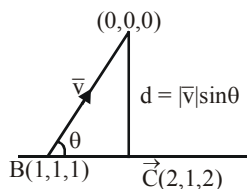
$$L_2 : \frac{x+1}{2} = \frac{y-0}{1} = \frac{z-0}{1} = \mu$$

Hence any point on L_1 and L_2 can be $(\lambda, \lambda-1, 1)$ and $(2\mu-1, \mu, \mu)$

$$\therefore \frac{2\mu-1-\lambda}{2} = \frac{\mu-\lambda+1}{1} = \frac{\mu-\lambda}{2}$$

on solving $\mu = 1, \lambda = 3.$

$\therefore A = (3, 2, 3)$ & $B = (1, 1, 1).$



Hence equation of AB is $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-1}{2}$

$$d = \frac{|\vec{v} \times \vec{c}|}{|\vec{c}|} = \frac{\sqrt{2}}{3} \Rightarrow 27d^2 = 27 \cdot \frac{2}{9} = 6$$

5. Ans. 4

$$\sin^3\theta + \cos^3\theta + 1 = 3\sin\theta \cos\theta$$

$$\Rightarrow \sin\theta + \cos\theta + 1 = 0 \text{ or } \sin\theta = \cos\theta = 1$$

$$\Rightarrow \sin\theta + \cos\theta + 1 = 0$$

$$\Rightarrow \theta = \pi, \frac{3\pi}{2}, 3\pi, \frac{7\pi}{2} \text{ in } \theta \in (0, 4\pi)$$

6. Ans. 9

Req. Probability

$$= \frac{{}^3C_1 \cdot {}^3C_2}{{}^7C_4} = \frac{3 \times 3}{7 \times 6 \times 5} \times 1 \times 2 \times 3 = \frac{9}{35} = p$$

$$\Rightarrow 35P = 9$$