HINTS & SOLUTIONS

PART-A : PHYSICS

1. Figure shows ....

Sol. For maximum x, $v_f = 0$

Total area under curve = 0

$$\frac{1}{2}\times(4+2)\times 2-\frac{1}{2}(x-6)^2=0$$

$(x-6)^2 = 12$

$x = 6 + 2\sqrt{3}$

2. A cylinder with ....

Sol. Electric field outside wire is

$$E = \frac{2k\lambda}{r}$$

where r = 2R

$\lambda = \rho\pi R^2$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{\rho\pi R^2}{2\pi\varepsilon_0 2R} = \frac{\rho R}{4\varepsilon_0}$$

3. Light from a ....

Sol. Fringes are obtained from YDSE.

4. Determine the ....

Sol. For input circuit

$80 \times 10^3 \times I_B + 0.8 - 5 = 0$

or, $I_B = 0.0525$mA

For output circuit

$5 \times 10^3 \times I_C + V_{CE} = 12V$

Or $I_C = 2.36$mA

$$\beta = \frac{I_C}{I_B} = \frac{2.36mA}{0.0525mA} \approx 45$$

or $\beta = 45$

5. A mass is ....

Sol. In this problem, the object starts with just gravitational potential energy and ends with all the energy in elastic potential energy in the spring.

$$mgy_i = \frac{1}{2}kx^2$$

Solving for m,

$$m = \frac{kx^2}{2gy_i}$$

The only tricky thing is using the correct y_i. Remember, the box fall 4 meters before hitting the spring and then moves down an additional 1.25 m while the spring is compressing. The total distance down is therefore 5.25m.

$$m = \frac{kx^2}{2gy_i} = \frac{\left(120\frac{N}{m}\right)(1.25m)^2}{2\left(9.8\frac{m}{s^2}\right)(5.25m)} = 1.82kg$$

6. A traction device ....

Sol. T = mg

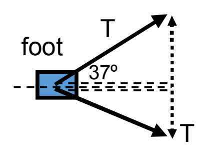
In the y-direction

$$\sum F_y = T\sin(37^\circ) - T\sin(37^\circ) = 0$$

In the x-direction, positive to right

$$\sum F_x = T\cos(37^\circ) + T\cos(37^\circ)$$

$160 = 2T\cos(37^\circ)$



$$T = \frac{160}{2\cos 37^\circ} = 100N$$

m = 10kg

7. A metal bracket ....

Sol. M = mass of complete plate

m_1 = mass of small plate

m_2 = mass of remaining plate

$6 \times M = \vec{y}\times m_2 + 9\times m_1$

$$\vec{y} = \frac{6\times M - 9\times m_1}{m_2}$$

$$\Rightarrow \vec{y} = \frac{6\times 144 - 9\times 16}{128} = \frac{45}{8} = 5.625$$

8. A proton makes ....

Sol.

Since collision is elastic

$u - 0 = v + \frac{2u}{3}$

$\Rightarrow v = \frac{u}{3}$

By conservation of linear momentum

$m_1\, u + 0 = -m_1 \frac{2u}{3} + m_2 \frac{u}{3} \Rightarrow \frac{m_1}{m_2} = \frac{1}{5}$

9. An object moves ....

Sol. $V_{max} = A\omega = \frac{2\pi A}{T}$

10. The figure ....

Sol. $v = f\lambda$

$= 100 \times 0.4$

$= 40$ m/s

11. An ultrasonic ....

Sol. $f' = f_s \frac{v + v_b}{v - v_b} \Rightarrow f' = f\left(1 + \frac{2v_b}{v}\right) \Rightarrow v_b = 0.19$ m/s

12. Consider a ....

Sol. $E = \frac{\sigma}{\varepsilon_0} = J \times \rho$

13. Consider a ....

Sol. Check dimensions both sides and θ must be proportional to τ.

14. A straight wire ....

Sol. Gravitational force on the loop

$F_g = mg \sin\theta$

Magnetic force on the loop

$F_m = (B\cos\theta)\, i\ell = B\cos\theta\ \frac{\xi\ell}{R}$

$= \frac{B\cos\theta\,\ell}{2R}\ vB\cos\theta.\ \ell = \frac{vB^2\ell^2\cos^2\theta}{2R}$

For steady speed, $F_{net} = F_m - F_g = 0$

$\therefore \frac{vB^2\ell^2\cos^2\theta}{2R} - mg\sin\theta = 0$

or $v = \frac{2mgR\sin\theta}{B^2\ell^2\cos^2\theta}$

15. A rigid body ....

Sol. $mgh = \frac{1}{2}mv_0^2 + \frac{1}{2}mK^2\left(\frac{v_0}{R}\right)^2$ ....(i)

$mgh = \frac{1}{2}m\left(\frac{5v_0}{4}\right)^2$ ....(ii)

$K = \frac{3R}{4}$

16. An ideas ....

Sol. Process AB $\quad Q_{in} = \frac{3}{2}(2P_0.2V_0 - P_0V_0) + \frac{1}{2}(2P_0 + 2P_0) \times V_0$

$= \frac{3}{2}(3P_0V_0) + \frac{3P_0V_0}{2}$

$= 6P_0V_0$

Process BC $\quad Q_{out} = \frac{3}{2}(2P_0)(V_0)$

$= 3P_0V_0$

Process CA $\quad Q_{out} = \frac{5}{2}(P_0)(V_0)$

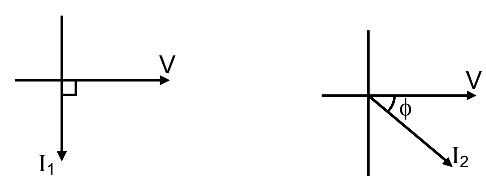
$= \frac{5}{2}P_0V_0$

Total $\quad Q_{out} = 3P_0V_0 + \frac{5}{2}P_0V_0 = \frac{11}{2}P_0V_0$

$\frac{Q_{in}}{Q_{out}} = \frac{6P_0V_0}{\frac{11}{2}P_0V_0} = \frac{12}{11} = 1.09$

17. Pure inductor is ....

Sol.



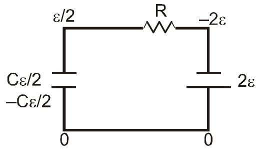
$\tan 30° = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{1}{\sqrt{3}}$

$L = \frac{R}{\sqrt{3}\omega} = \frac{10\sqrt{3}}{\sqrt{3} \times 100\pi}$

L = 0.1H

18. In the circuit ……………………….

Sol.

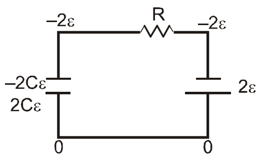


t = 0

$t = \frac{\varepsilon/2-(-2\varepsilon)}{R}$

$= \frac{5\varepsilon}{2R}$

After a long time :



Total charge flown by battery $\Delta q = 2C\varepsilon - (-C\varepsilon/2) = \frac{5C\varepsilon}{2}$

Work done by battery = $\frac{5C\varepsilon}{2} \times 2\varepsilon = 5C\varepsilon^2$

Since intially (t = 0) charge on capacitor is Cε/2 it becomes zero after some time and finally becomes 2Cε so energy first decreases then increases.

19. For an amplitude ……………………….

Sol. $A_{max} = A_m + A_c = 10$

$A_{min} = A_c - A_m = 2$

$A_c = 6$ V

$A_m = 4$ V

$m = \frac{A_m}{A_c} = \frac{4}{6} = \frac{2}{3}$

20. Two radioactive ……………………….

Sol. $N_1 = 2N_2$

$N_0e^{-\lambda_1 t} = 2N_0e^{-\lambda_2 t}$

$\Rightarrow \frac{1}{t} = \frac{1}{T_2} - \frac{1}{T_1}$

21. A rod of ……………………….

Sol. The maximum emf will be at mean position of oscillation

$\therefore \frac{mg\ell}{2}(1-\cos\alpha) = \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2$

and $\varepsilon = \frac{1}{2}B\omega\ell^2$

22. A steel ball ……………………….

Sol. $D_S - D_H = 0.05$ mm

Where D_S is diameter of ball and D_H is diameter of ball hole.

As we increase temperature

$D_H > D_S$ for ball to enter hole

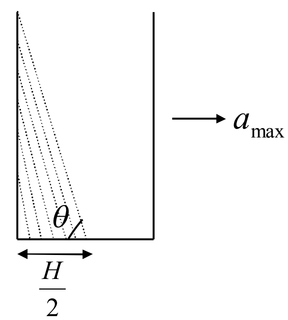
$D_H(1 + 12 \times 10^{-6}\Delta T) - D_S(1 + \frac{3.2}{3} \times 10^{-6}\Delta T) > 0$

$(12 \times 10^{-4} - \frac{3.2}{3} \times 10^{-4})\Delta T > 0.05$

Gives $\Delta T > 46°$

23. An open cubical ……………………….

Sol. Volume of liquid inside container is $\frac{H^3}{4}$.when container is moved with maximum possible acceleration.



$\tan\theta = \frac{a}{g} = 2 \Rightarrow a = 2g$

Force on front wall is $\int_0^{H/2} \rho \times 2g \times \frac{x}{2} \times (2xdx) = \frac{\rho g H^3}{12}$

$= \frac{mg}{3}$

24. Two spherical ……………………….

Sol. Pressure in left bubble = $\frac{4s}{r_1}$

Volume of left bubble = $\frac{4}{3}\pi r_1^3$

Pressure in right bubble = $\frac{4s}{r} = \frac{4}{3}\pi r_2^3$

$n_1 + n_2 = 2n$

$r_1^2 + r_2^2 = 2R^2$

$A_1 + A_2$ and $n_1 + n_2$ are constant

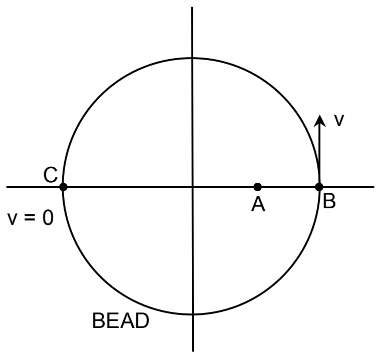
$$\frac{n_1}{n_2}=\frac{r_1^2}{r_2^2}=\frac{4}{1}\Rightarrow r_2^2=\frac{2R^2}{5}$$

$$r_2=\sqrt{\frac{2}{5}}R$$

$$r_1=\sqrt{\frac{8}{5}}R$$

25. A bead of mass ……………………….

Sol. If v is the assumed minimum speed in nearest position B, corresponding speed in furthest position C is zero.



By conservation of energy between B & C, Loss in K.E. = Gain in P.E.

$$\frac{1}{2}(1)v^2=\int\frac{3}{r^2}dr$$

$$=\left[-\frac{3}{r}\right]_1^3$$

= (–1) – (–3) = 2

$\Rightarrow$ v = 2 m/s

26. One mole ……………………….

Sol. As process is isochoric, let temperature at any instant of time t be T.

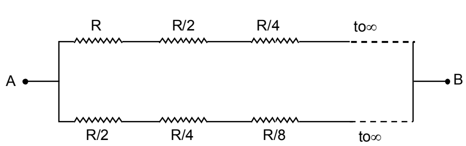
$$\frac{dQ}{dt}=\frac{KA.(2T_0-T)}{d}=\frac{5}{2}\frac{RdT}{dt}$$

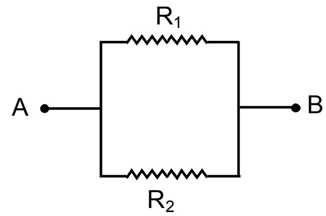
$$\Rightarrow\int_{T_0}^{T}\frac{dT}{2T_0-T}=\int_0^t\frac{2}{5}\frac{KA}{dR}dt$$

$$\Rightarrow T=T_0\left(2-e^{-\frac{2KAt}{5dR}}\right)$$

27. In the circuit ……………………….

Sol. By the principle balanced Wheatstone bridge, the given circuit can be redrawn as





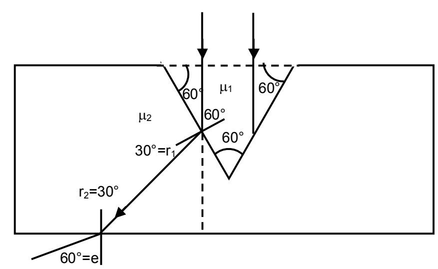
$$R_1=R\left(\frac{1}{1-\frac{1}{2}}\right)=2R$$

$$R_2=\frac{R}{2}\left(\frac{1}{1-\frac{1}{2}}\right)=R$$

$$\therefore\quad R_{AB}=\frac{2R\times R}{2R+R}=\frac{2R}{3}$$

28. Consider a ……………………….

Sol.



(A) At I surface $\mu_1\sin60=\mu_2\sin r_1$

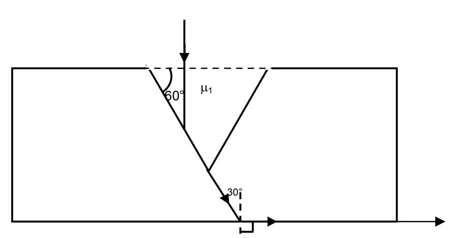
$$1\times\frac{\sqrt{3}}{2}=\sqrt{3}\ \sin r$$

$\Rightarrow$ r_1 = 30°

r_2 = 30°

e = 60°

θ = 120°



(B)

$$\frac{4}{\sqrt{3}}\times\frac{\sqrt{3}}{2}=2\sin r_1$$

$r_1=\pi/2$

e = 90°

θ = 180°

29. The angular ……………………….

Sol. Angular momentum = $\frac{nh}{2\pi} = \frac{h}{2\pi}$ ($\because$ n = 1)

30. An electromagnetic ……………………….

Sol. Usually for transparent mediums, $\mu_r = 1$

PART-B : CHEMISTRY

31. Which of the following compounds …………….

Sol. Consider vector summation.

32. The equilibrium: $NH_4HS(s)$ …………….

Sol. Increasing the volume will shift the equilibrium in the forward direction

33. Consider the following wave function …………….

Sol. The wave function is independant of angle which implies it belongs to spherically symmetric s orbital. Further, it has only one radial node at r = Z/2 which indicates 2s orbital.

34. Consider the following …………….

Sol. Theory based

35. Which of the following metallurgy …………….

Sol. Theory based

36. When an ideal gas at a pressure P …………….

Sol. Isothermally $\Rightarrow$ $PV = P_i V/n$

$\Rightarrow$ $P_i = nP$

Reversibly adiabatically $\Rightarrow$ $PV^\gamma = P_a \left(\frac{V}{n}\right)^\gamma$

$\Rightarrow$ $P_a = n^\gamma P$

Hence, $\frac{p_i}{p_a} = n^{1-\gamma}$

37. Four vessels 1,2,3 and 4 contains …………….

Sol.

Vessels	1	2	3	4
nitial nuclides	10 mol	1 mol	5mol	2 mol
$t_{1/2}$	10 hr	10 hr	2 hr	1 hr
Rate constant (λ)	$\frac{0.693}{10}$	$\frac{0.693}{5}$	$\frac{0.693}{2}$	$\frac{0.693}{1}$
Initial Activity	$\frac{0.693}{10}.10\ N_A$	$\frac{0.693}{5}.1\ N_A$	$\frac{0.693}{2}.5\ N_A$	$\frac{0.693}{1}.2\ N_A$

$\left(\frac{-dN}{dt} = \lambda N_0\right)$

38. van der Waals' gas equation …………….

Sol. C = b^2 , b = 25 cm^3/mol

b = 0.025 L/mol

$V_{real} = V_i - nb$

= 50 – 10 × 0.025

= 49.75 L

39. A detergent $C_{12}H_{25}SO_4^- Na^+$ solution …………….

Sol. Concentration of colloidal solution = 10^{-3} mol/L

or 10^{-6} mol/cm^3

or 10^{-9} mol/mm^3

or 6 × 10^{14} particles per mm^3

But colloidal particles or micelles are only 10^{13} per mm^3

Hence, 60 particles are per micelle

40. The dipole moment of H_2O_2 is more …………….

Sol. Theory based

41. Which of the following is not true …………….

Sol. Helium does not form clathrate compounds due to too small size.

42. How many stereoisomers are possible …………….

Sol. Five geometrical isomers (three optically active)

Trans pairs (1) (A,A) (B,B) (a,a) Optically inactive

(2) (A,B) (A,B) (a,a) Optically inactive

(3) (A,a) (A,a) (B,B) Optically active

(4) (B,a) (B,a) (A,A) Optically active

(5) (A,a) (B,a) (A,B) Optically active

43. The distance between an octahedral …………….

Sol. Octahedral void is at body centre of FCC cube and tetrahedral void at the centre of smaller cube of FCC.

$\therefore$ distance = $\frac{\sqrt{3}a}{4}$.

44. Mark out the incorrect statement …………….

Sol. Conceptual

45. In the reaction …………….

Sol. $4Zn + NaNO_3 + 7NaOH \longrightarrow 4Na_2ZnO_2 + NH_3 + 2H_2O$

47. What is $[NH_4^+]$ in a solution that is …………….

Sol. $NH_4OH \rightleftharpoons NH_4^+ + OH^-$

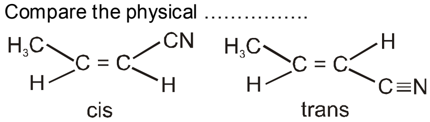
$$K_b = \frac{[NH_4^+][OH^-]}{[NH_4OH]}$$

$$1.8 \times 10^{-5} = \frac{[NH_4^+](0.01)}{(0.02)}$$

$$\therefore [NH_4^+] = 3.6 \times 10^{-5} \text{ M}$$

49. Compare the physical …………….

Sol.



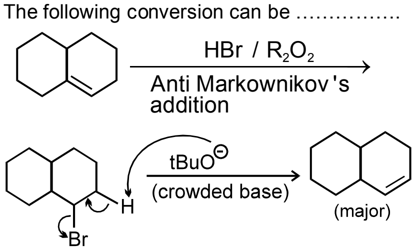
In trans isomer there is linear addition of bond dipoles hence, it is greater also, trans being more polar, its B.P. will be more. Packing in trans is better hence m.p. is more.

50. The compound 'X' can exist in …………….

Sol. 3 stereo centres ⇒ No. of stereoisomers = 2^3 = 8.

52. The following conversion can be …………….

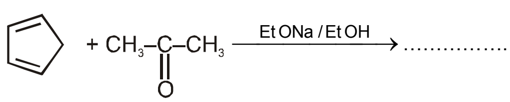
Sol.



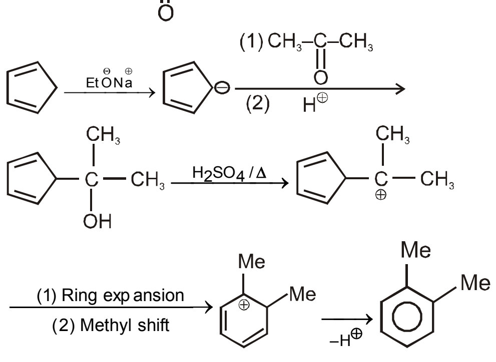
54. The rate of S_N1 and S_N2 reactions …………….

Sol. Rate of S_N2 is fastest at 2nd position because presence of carbonyl group near by and bromine at bridge head create a good amount of polarization which make transition state formation easier. S_N1 is fastest at position 6 due to formation of most stable carbocation

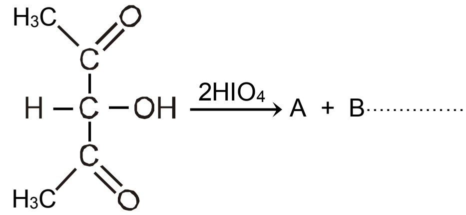
55.



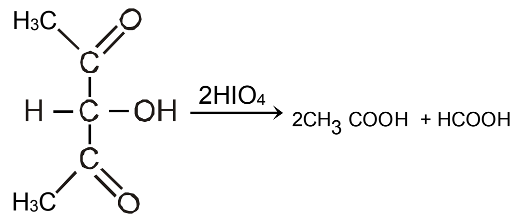
Sol.



56.

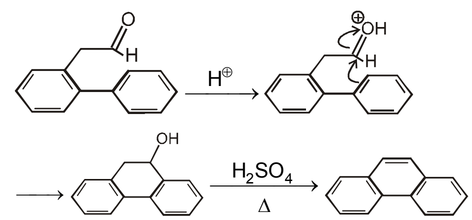


Sol. HIO_4 oxidised vicinal diol, α-hydroxy ketone and α-dicarbonyl compounds.



57. The final product in the following …………….

Sol.



58. The amino acid Lysine (Lys) …………….

Sol. The isoelectric point of lysine is $\frac{9.12 + 10.53}{2}$ = 9.82.

The pka of carboxylic group is not considerable as neither of the equilibria involving the neutral form (N) involves ionization of the carboxylic group. Now the conclusion is

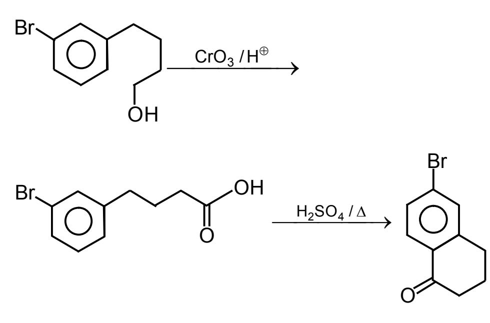
At 9.82 – it will be neutral (Zwitter ion form) (N), net charge = 0.

Above 9.82 – it will be in "B" form, – net charge = –1.

Below 9.82 – It will be in "A" form and net charge = +1.

60. The major product of the following …………….

Sol.



PART-C : MATHEMATICS

61. If $f(x)=\begin{cases}\dfrac{\sqrt{2+\cos x}-1}{(\pi-x)^2} & x>\pi\\ a & x=\pi\\ \dfrac{1+\sec^3 x}{2\tan^2 x}+b & x<\pi\end{cases}$ ....

Sol. $\underset{x\to\pi}{\text{lt}}\dfrac{\sqrt{2+\cos x}-1}{(\pi-x)^2}$ let $t=\pi-x$

$$\underset{t\to 0}{\text{lt}}\frac{\sqrt{2-\cos t}-1}{t^2}=\underset{t\to 0}{\text{lt}}\frac{2-\cos t-1}{t^2(\sqrt{2-\cos t}+1)}$$

$$=\underset{t\to 0}{\text{lt}}\frac{2\sin^2\frac{t}{2}}{t^2(\sqrt{2-\cos t}+1)}=\frac{1}{4}$$

$$\underset{x\to\pi^-}{\text{lt}}\frac{(1+\sec x)(1+\sec^2 x-\sec x)}{2(\sec x-1)(\sec x+1)}=-\frac{3}{4}$$

$\dfrac{1}{4}=a=-\dfrac{3}{4}+b \Rightarrow$ b = 1 and a + b = $\dfrac{5}{4}$

62. The distance...................

Sol. Let the given point (3, 8, 2) be P. Any point N on the given line is (1 + 2r, 3 + 4r, 2 + 3r). The direction ratio of PN are (2r – 2, 4r – 5, 3r). The PN is parallel to the plane

2x + 2y – 2z + 15 = 0, then 3(2r – 2) + 2(4r – 5) – 2(3r) = 0

r = 2

Coordinate of N are (5, 11, 8)

Required distance

PN = $\sqrt{(5-3)^2+(11-8)^2+(8-2)^2}$ = 7.

63. Consider the system...................

Sol. For nontrivial solution

$$\begin{vmatrix}|\cos\theta| & 2 & 1\\ \cos\theta & 12 & 7\\ \sin\theta & 5 & 3\end{vmatrix}=0$$

$|\cos\theta| - \cos\theta + 2\sin\theta = 0$

$|\cos\theta| = \cos\theta - 2\sin\theta$

Case-I $\cos\theta \ge 0$ then $\cos\theta = \cos\theta - 2\sin\theta \Rightarrow \sin\theta = 0$

so $\theta = 2n\pi$ $n \in I$

Case-II $\cos\theta < 0$ then $-\cos\theta = \cos\theta - 2\sin\theta \Rightarrow \tan\theta = 1$

$\Rightarrow 0 = 2n\pi + \dfrac{5\pi}{4}$ $n \in I$

So of solution in $\left[-4\pi, \dfrac{9\pi}{2}\right]$ are तब 9, -4π, -2π, 0, 2π,

4π, $-4\pi+\dfrac{5\pi}{4}$ $-2\pi+\dfrac{5\pi}{4}$, $\dfrac{5\pi}{4}$, $2\pi+\dfrac{5\pi}{4}$

64. The line 2x + y – 3 = 0...................

Sol. $4x^2+(3-2x)^2=5\Rightarrow x=1,\dfrac{1}{2}$

$P(1, 1),\ Q\left(\dfrac{1}{2},2\right)$

Slope of tangent at Q $m_1=\dfrac{-4x_1}{y_1}=-1$

Slope of tangent at P $m_2=\dfrac{-4x_2}{y_2}=-4$

angle between normal equal to angle between tangent

$$\tan\theta=\frac{m_1-m_2}{1+m_1m_2}=\frac{3}{5}$$

65. The locus of the...................

Sol. Let mid point of chord be P(h, k) , $T = S_1$

$3hx - 2ky + 2(x + h) - 3(k + y) = 3h^2 - 2k^2 + 4h - 6k$

Slope = $\dfrac{3h+2}{2k+3}$ = 2 $\Rightarrow$ 3x – 4y = 4

66. If $x\in\left(\pi,\dfrac{3\pi}{2}\right)$ then...................

Sol. $\sqrt{1-\sin x}=\left|\sin\dfrac{x}{2}-\cos\dfrac{x}{2}\right|=\sin\dfrac{x}{2}-\cos\dfrac{x}{2}$ $\dfrac{x}{2}\in\left(\dfrac{\pi}{2},\dfrac{3\pi}{4}\right)$

$\sqrt{1+\sin x}=\left|\sin\dfrac{x}{2}+\cos\dfrac{x}{2}\right|=\sin\dfrac{x}{2}+\cos\dfrac{x}{2}$

$\tan^{-1}\left(-\tan\dfrac{x}{2}\right)=-\tan^{-1}\tan\dfrac{x}{2}\Rightarrow-\left(-\pi+\dfrac{x}{2}\right)=\pi-\dfrac{x}{2}$

67. Which of the following...................

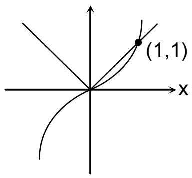
Sol.

p	q	~(p↔q)	~p↔q
T	T	F	F
T	F	T	T
F	T	T	T
F	F	F	F

68. If P, Q are sets defined...................

Sol. By graph it is clear that

$P\cap Q = \{1\}$ $\Rightarrow n(P\cap Q) = 1$



But $x \neq 0$

69. If a + b + 3c = 1....................

Sol. $$\frac{\frac{a}{2}+\frac{a}{2}+\frac{b}{2}+\frac{b}{2}+\frac{3c}{2}+\frac{3c}{2}}{6} \geq \left(\frac{a^2}{2^2}\cdot\frac{b^2}{2^2}\cdot\frac{3^2c^2}{2^2}\right)^{\frac{1}{6}}$$

$$\frac{2^6}{6^6 \cdot 9} \geq a^2b^2c^2 \Rightarrow \frac{1}{3^8} \geq a^2b^2c^2$$

$\therefore$ greatest value of $a^2b^2c^2$ is $\frac{1}{3^8}$

70. If non-symmetric form....................

Sol. Let D'ratio of the line are < a, b, c >

$\Rightarrow 2a - 3b + 4c = 0$

& $2a - 3b + 4c = 0 \Rightarrow \frac{a}{3} = \frac{b}{14} = \frac{c}{9}$

Let given line passes through $(\alpha, \beta, 0) \Rightarrow 2\alpha - 3\beta + 1 = 0$ and $4\alpha + 3\beta - 7 = 0$

$\Rightarrow \alpha = 1, \beta = 1$

71. If the letters of the....................

Sol. ACHINS

A ......5 !

C ......5 !

H ......5 !

I ......5 !

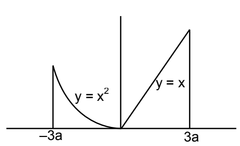
N ......5 !

SACHIN

5 . 5 ! + 1 = 601

72. Let $f(x) = \begin{cases} x^2, & x < 0 \\ x, & x \geq 0 \end{cases}$ ....

Sol.



$$\text{Area} = \int_{-3a}^{0} x^2\,dx + \int_{0}^{3a} x\,dx = \int_{0}^{3a} x^2\,dx + \int_{0}^{3a} x\,dx$$

$$= \left[\frac{x^3}{3}+\frac{x^2}{2}\right]_0^{3a} \Rightarrow \frac{27a^3}{3}+\frac{9a^2}{2} = \frac{9a}{2} \text{ (given)}$$

$$\Rightarrow a^2 + \frac{a}{2} - \frac{1}{2} = 0 \Rightarrow 2a^2 + a - 1 = 0 \Rightarrow a = \frac{1}{2}, a = -1$$

73. $\displaystyle\int_{\ell n\pi-\ell n2}^{\ell n\pi} \frac{e^x}{1-\cos\left(\frac{2}{3}e^x\right)}\,dx$....................

Sol. $$\int_{\ell n\pi-\ell n2}^{\ell n\pi} \frac{e^x}{1-\cos\left(\frac{2}{3}e^x\right)}\,dx$$

$$= \frac{3}{2}\int_{\pi/3}^{2\pi/3} \frac{dt}{1-\cos t} = \frac{3}{2}\int_{\pi/3}^{2\pi/3} \frac{dt}{1-\left(1-2\sin^2\frac{t}{2}\right)}$$

$$= \frac{3}{2.2}\int_{\pi/3}^{2\pi/3} \operatorname{cosec}^2\frac{t}{2}\,dt = \frac{3}{2}\left[-\cot\frac{t}{2}\right]_{\pi/3}^{2\pi/3}$$

$$= -\frac{3}{2}\left[\sqrt{3}-\frac{1}{\sqrt{3}}\right] = \frac{3}{4}\left(\frac{4}{\sqrt{3}}\right) = \sqrt{3}$$

74. $f(x) = e^x - x - \frac{x^3}{3}$ ....

Sol. $f(x) = e^x - x - \frac{x^3}{3} \Rightarrow f'(x) = e^x - 1 - x^2$

$f'(x) = 0 \Rightarrow e^x = 1 + x^2 \Rightarrow x = 0$

At x = 0, minima occurs and $f(x)_{min} = 1 = a$

So a + b = 1

75. $\displaystyle\int \frac{x^3-1}{\sqrt{x^6+2x^3}}\,dx$ is....................

Sol. $$\int \frac{x^3-1}{\sqrt{x^6+2x^3}} = \int \frac{\left(x-\frac{1}{x^2}\right)}{\sqrt{x^2+\frac{2}{x}}}\,dx$$

Let $x^2 + \frac{2}{x} = t \Rightarrow \left(2x - \frac{2}{x^2}\right)dx = dt$

$$= \frac{1}{2}\int \frac{dt}{\sqrt{t}} = \sqrt{\frac{x^3+2}{x}} + c$$

76. If Arg(–z) = $\frac{\pi}{3}$ and....................

Sol. Arg(–z) = $\frac{\pi}{3}$ $\therefore$ Arg(z) = $-\frac{2\pi}{3}$

Arg(ω) = $\frac{\pi}{3}$ hence Arg(ω^{100}) = $\frac{100\,\pi}{3}$

$\therefore$ Arg($z\omega^{100}$) = $\frac{98\ \ \pi}{3}$ = 32π + $\frac{2\ \ \pi}{3}$

$\therefore$ Ans. $\frac{2\ \ \pi}{3}$

77. If the line x – 1 = 0....................

Sol. $y^2 = k\left(x - \frac{8}{k}\right)$

Equation of directrix is

$x - \frac{8}{k} = -\frac{k}{4} \Rightarrow \frac{8}{k} - \frac{k}{4} = 1 \Rightarrow k = 4, -8$

78. Given the relation....................

Sol. R is reflexive if it contains (1, 1), (2, 2), (3, 3)

$\therefore$ (1,2) $\in$ R, (2,3) $\in$ R.

R is symmetric if (2,1), (3,2) $\in$ R.

Now, R = {(1,1), (2,2), (3,3), (2,1), (3,2),(2,3),(1,2)}

R will be transitive if (3,1), (1,3) $\in$ R. Thus R becomes an equivalence relation by adding (1,1), (2,2), (3,3) (2,1),(3,2), (1,3), (3,1).

Hence the total no. of ordered pairs is 7.

79. The coefficient of...................

Sol. $1 + x + x^2 + x^3 = (1+x) + x^2(1+x) = (1+x)(1+x^2)$

$(1 + x + x^2 + x^3)^{11} = (1+x)^{11}(1+x^2)^{11}$

$(1 + {}^{11}C_1.x + {}^{11}C_2.x^2 + {}^{11}C_3.x^3 + {}^{11}C_4.x^4 + ...)$
$\times (1 + {}^{11}C_1.x^2 + {}^{11}C_2.x^4 + ...)$

$\therefore$ The coefficient of x^4 is

${}^{11}C_2 + {}^{11}C_2\ \ {}^{11}C_1 + {}^{11}C_4 = 55 + 605 + 330 = 990$

80. n ordinary dice are....................

Sol. The total number of ways is 6 × 6 × ….. to n times = 6^n. The total number of ways to show only even numbers is 3 × 3…… to n times = 3^n. Therefore, the required number of ways is $6^n - 3^n$.

81. The general solution....................

Sol. $ydx + (x + 2y^2)\,dy = 0$

$ydx + xdy + 2y^2\,dy = 0$

$d(xy) + 2y^2\,dy = 0$

Integrating both sides we get

$\Rightarrow\ xy + \frac{2y^3}{3} = \text{constant} \Rightarrow 3xy + 2y^3 = C$

82. Let A is set of all....................

Sol. $x^2 - ax + 1 = 0$ has no real root

$\therefore$ D < 0 $\Rightarrow$ $a^2 - 4 < 0$

$\Rightarrow a \in (-2, 2)$

$\therefore$ A = {x : – 2 < x < 2}

$f(x) = bx^2 + bx + 0.5 > 0\ \forall\ x \in R$, if b > 0

$\therefore$ D < 0 $\Rightarrow$ $b^2 - 4.b.\frac{1}{2} < 0$

$\Rightarrow$ $b^2 - 2b < 0$ $\Rightarrow$ $b \in (0, 2)$

Also b = 0 then f(x) > 0 $\forall x \in R$ $\therefore$ $b \in [0, 2)$

$\therefore$ B = {x : 0 ≤ x < 2}

$\therefore$ A ∩ B = {x : 0 ≤ x < 2}

83. The equation of....................

Sol. Find image of centre of given circle.

Given circle is $x^2 + y^2 + 16x - 24y + 183 = 0$ ….(1)

Centre = (-8, 12); $r = \sqrt{64 + 144 - 183} = \sqrt{25} = 5$

Given line is 4x + 7y + 13 = 0 …..(2)

Let (α, β) be the image of (-8, 12) wrt (2).

$\frac{\alpha + 8}{4} = \frac{\beta - 12}{7} = -\frac{2(4(-8) + 7(12) + 13)}{4^2 + 7^2}$

$\Rightarrow \frac{\alpha + 8}{4} = \frac{\beta - 12}{7} = -\frac{2(65)}{65} = -2$

$\Rightarrow$ α = - 8 – 8, β = -14 + 12

$\Rightarrow$ α = -16, β = -2

$\therefore$ Equation of required circles is

$(x + 16)^2 + (y + 2)^2 = 5^2$

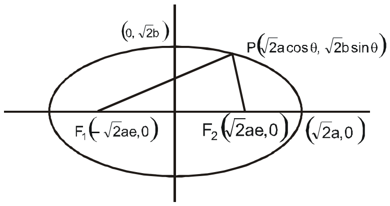
$\Rightarrow$ $x^2 + y^2 + 32x + 4y + 235 = 0$

84. If A and B are two....................

Sol. Number of onto functions = $3^4 - {}^3C_1 - {}^3C_2\,(2^4 - 2)$

85. P is a variable point....................

Sol.



area of ΔPF_1F_2 = A = $\frac{1}{2}(2\sqrt{2}ae)\ \sqrt{2}\,b\sin\theta$

$\Rightarrow$ A = 2abesinθ

Maximum area A = 2abe

= $\frac{2ab}{a}\ \sqrt{a^2-b^2}$ = 2b $\sqrt{a^2-b^2}$

86. The mean of 11....................

Sol. New mean = $\bar{X}+\frac{11(11+1)}{2.11}=\bar{X}+6$

87. If f(x) = x^{α} lnx and....................

Sol. f(0) = f(1) = 0 f(x) has to be continuous in [0, 1]

$$\lim_{x\to 0} f(x) = f(0) \Rightarrow \lim_{h\to 0} f(h) = f(0)$$

$$\Rightarrow \lim_{h\to 0} h^{\alpha}\ln h = 0 \ \Rightarrow \lim_{h\to 0} \frac{\ln h}{h^{-\alpha}} = 0$$

$$\Rightarrow \lim_{h\to 0} \frac{1/h}{-\alpha\ h^{-\alpha-1}} = 0 \quad \text{(By L-hospital)}$$

$$\Rightarrow \lim_{h\to 0} \frac{-h^{\alpha}}{\alpha} = 0 \Rightarrow \alpha > 0$$

88. The function f(x) = ...

Sol. f(x) = x $\sqrt{ax-x^2}$ a > 0

f '(x) = $\frac{x(a-2x)}{2\sqrt{ax-x^2}}$ + $\sqrt{ax-x^2}$

= $\frac{3ax-4x^2}{2\sqrt{ax-x^2}}$, f'(x) > 0

x $\in$ $\left(0, \frac{3a}{4}\right)$

f '(x) < 0

x $\in$ $\left(\frac{3a}{4}, a\right)$

89. If the slope of the....................

Sol. From curve 1 – a – b = 0 $\Rightarrow$ b = 1 – a

and $\frac{dy}{dx}$ = – $\frac{(a+y)}{b+x}$ at (–1, – 1) – $\frac{(a+1)}{b-1}$ = 2

a – 1 = 2a $\Rightarrow$ a = –1, b = 2

90. Find the equation of....................

Sol. equation of angle bisectors

$\frac{x+y-2}{\sqrt{2}}$ = ± $\frac{(7x-y+4)}{5\sqrt{2}}$

2x – 6y + 14 = 0 and 2x + 4y – 6 = 0

x – 3y + 7 = 0 and x + 2y – 3 = 0

required lines x – 3y = – 5, 6x + 2y = 10

3x + y = 5

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ALL INDIA OPEN TEST-2 (AIOT-2)
(JEE MAIN PATTERN)
TARGET : JEE (MAIN+ADVANCED) 2019

DATE : 17-02-2019 **COURSE CODE : JP,JF,JR,EP,EF,ER,JCC**

ANSWER KEY

CODE-1

PHYSICS

1.	(3)	**2.**	(2)	**3.**	(4)	**4.**	(3)	**5.**	(3)	**6.**	(2)	**7.**	(1)
8.	(2)	**9.**	(3)	**10.**	(3)	**11.**	(3)	**12.**	(1)	**13.**	(4)	**14.**	(1)
15.	(4)	**16.**	(1)	**17.**	(2)	**18.**	(4)	**19.**	(3)	**20.**	(3)	**21.**	(2)
22.	(4)	**23.**	(3)	**24.**	(3)	**25.**	(2)	**26.**	(1)	**27.**	(4)	**28.**	(3)
29.	(3)	**30.**	(3)										

CHEMISTRY

31.	(2)	**32.**	(2)	**33.**	(3)	**34.**	(3)	**35.**	(3)	**36.**	(2)	**37.**	(2)
38.	(1)	**39.**	(3)	**40.**	(4)	**41.**	(4)	**42.**	(3)	**43.**	(4)	**44.**	(2)
45.	(2)	**46.**	(4)	**47.**	(1)	**48.**	(4)	**49.**	(4)	**50.**	(4)	**51.**	(2)
52.	(3)	**53.**	(3)	**54.**	(4)	**55.**	(1)	**56.**	(3)	**57.**	(4)	**58.**	(2)
59.	(4)	**60.**	(1)										

MATHEMATICS

61.	(1)	**62.**	(2)	**63.**	(4)	**64.**	(3)	**65.**	(1)	**66.**	(1)	**67.**	(2)
68.	(1)	**69.**	(1)	**70.**	(3)	**71.**	(4)	**72.**	(1)	**73.**	(1)	**74.**	(4)
75.	(2)	**76.**	(1)	**77.**	(2)	**78.**	(3)	**79.**	(4)	**80.**	(3)	**81.**	(4)
82.	(2)	**83.**	(4)	**84.**	(1)	**85.**	(2)	**86.**	(2)	**87.**	(4)	**88.**	(2)
89.	(3)	**90.**	(2)										

CODE-2

MATHEMATICS

1.	(2)	**2.**	(3)	**3.**	(3)	**4.**	(4)	**5.**	(2)	**6.**	(2)	**7.**	(3)
8.	(2)	**9.**	(2)	**10.**	(4)	**11.**	(3)	**12.**	(2)	**13.**	(2)	**14.**	(3)
15.	(3)	**16.**	(2)	**17.**	(3)	**18.**	(4)	**19.**	(3)	**20.**	(4)	**21.**	(3)
22.	(3)	**23.**	(3)	**24.**	(2)	**25.**	(3)	**26.**	(3)	**27.**	(3)	**28.**	(3)
29.	(4)	**30.**	(3)										

PHYSICS

31.	(2)	**32.**	(4)	**33.**	(2)	**34.**	(1)	**35.**	(4)	**36.**	(1)	**37.**	(4)
38.	(3)	**39.**	(1)	**40.**	(4)	**41.**	(2)	**42.**	(3)	**43.**	(1)	**44.**	(3)
45.	(2)	**46.**	(4)	**47.**	(3)	**48.**	(1)	**49.**	(2)	**50.**	(1)	**51.**	(4)
52.	(2)	**53.**	(4)	**54.**	(1)	**55.**	(4)	**56.**	(3)	**57.**	(2)	**58.**	(1)
59.	(4)	**60.**	(1)										

CHEMISTRY

61.	(4)	**62.**	(1)	**63.**	(3)	**64.**	(3)	**65.**	(2)	**66.**	(2)	**67.**	(1)
68.	(2)	**69.**	(3)	**70.**	(4)	**71.**	(4)	**72.**	(2)	**73.**	(3)	**74.**	(2)
75.	(1)	**76.**	(4)	**77.**	(1)	**78.**	(4)	**79.**	(4)	**80.**	(3)	**81.**	(2)
82.	(2)	**83.**	(4)	**84.**	(3)	**85.**	(1)	**86.**	(4)	**87.**	(3)	**88.**	(3)
89.	(4)	**90.**	(2)										

CODE-3

CHEMISTRY

1.	(3)	**2.**	(1)	**3.**	(2)	**4.**	(4)	**5.**	(2)	**6.**	(2)	**7.**	(1)
8.	(2)	**9.**	(2)	**10.**	(4)	**11.**	(4)	**12.**	(3)	**13.**	(4)	**14.**	(2)
15.	(2)	**16.**	(4)	**17.**	(1)	**18.**	(4)	**19.**	(4)	**20.**	(3)	**21.**	(1)
22.	(3)	**23.**	(2)	**24.**	(3)	**25.**	(1)	**26.**	(2)	**27.**	(3)	**28.**	(2)
29.	(4)	**30.**	(2)										

MATHEMATICS

31.	(4)	**32.**	(4)	**33.**	(1)	**34.**	(1)	**35.**	(2)	**36.**	(4)	**37.**	(4)
38.	(4)	**39.**	(4)	**40.**	(2)	**41.**	(1)	**42.**	(4)	**43.**	(4)	**44.**	(4)
45.	(4)	**46.**	(4)	**47.**	(4)	**48.**	(2)	**49.**	(1)	**50.**	(2)	**51.**	(2)
52.	(4)	**53.**	(4)	**54.**	(4)	**55.**	(4)	**56.**	(4)	**57.**	(1)	**58.**	(2)
59.	(4)	**60.**	(4)										

PHYSICS

61.	(1)	**62.**	(3)	**63.**	(1)	**64.**	(4)	**65.**	(1)	**66.**	(1)	**67.**	(2)
68.	(1)	**69.**	(1)	**70.**	(4)	**71.**	(1)	**72.**	(4)	**73.**	(2)	**74.**	(3)
75.	(2)	**76.**	(2)	**77.**	(4)	**78.**	(2)	**79.**	(1)	**80.**	(4)	**81.**	(1)
82.	(2)	**83.**	(1)	**84.**	(4)	**85.**	(3)	**86.**	(3)	**87.**	(1)	**88.**	(1)
89.	(4)	**90.**	(2)										