IIT – JEE | **TW TEST (3 Yrs.)** | **MARKS:270**
TIME: 3 HRS. | **TOPIC: FULL CALCULUS** | **DATE:28/9/18**

(Multiple Choice Questions)

This section contains **90 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) for its answer, out which **ONLY ONE** is correct. **(+3, -1)**

1. $f(x) = 4-(6-x)^{2/3}$ in [5, 7]

2,3,4

(1) Lagranges theorem is applicable
(2) Rolle's theorem is applicable
(3) Lagranges theorem is applicable but not the Rolle's Theorem
(4) Both theorems are not applicable

2. If the function $f(x)$ and $g(x)$ are continuous in $[a, b]$ and differentiable in (a, b), then the Equation $\begin{vmatrix} f(a) & f(b) \\ g(a) & g(b) \end{vmatrix} = (b-a)\begin{vmatrix} f(a) & f'(x) \\ g(a) & g'(x) \end{vmatrix}$ has, in the interval $[a, b]$

(1) Atleast one root (2) Exactly one root (3) Atmost one root (4) No root

3. If $f(x)$ is a differentiable function $\forall x \in R$ so that, $f(2)=4, f^1(x) \ge 5\ \forall x \in [2,6]$, then, $f(6)$ is

(1) ≥ 24 (2) ≤ 24 (3) ≥ 9 (4) ≤ 9

4. The function $f(x) = \dfrac{\ln(\pi + x)}{\ln(e+x)}$

(1) Increasing on $(0, \infty)$
(2) Decreasing on $(0, \infty)$
(3) Increasing on $\left(0, \dfrac{\pi}{e}\right)$, decreasing on $\left(\dfrac{\pi}{e}, \infty\right)$
(4) Decreasing on $\left(0, \dfrac{\pi}{e}\right)$, increasing on $\left(\dfrac{\pi}{e}, \infty\right)$

5. Which of the following function is increasing in $\left(0, \dfrac{\pi}{2}\right)$

(1) $\cos x - \sin x$ (2) $\cos x + \sin x$ (3) $\dfrac{\sin x}{x}$ (4) $\dfrac{x}{\sin x}$

6. $f(x) = \dfrac{a\sin x + b\cos x}{c\sin x + d\cos x}$ is an increasing function if

(1) $ad - bc = 0$ (2) $ad - bc < 0$ (3) $ad - bc > 0$ (4) $ab + cd = 0$

7. If $f(x) = a\log_e|x| + bx^2 + x$ has extremum at $x = 1$ and $x = 3$ then

(1) $a = -\dfrac{3}{4}, b = -\dfrac{1}{8}$ (2) $a = \dfrac{3}{4}, b = -\dfrac{1}{8}$ (3) $a = -\dfrac{3}{4}, b = \dfrac{1}{8}$ (4) $a = \dfrac{3}{4}, b = \dfrac{1}{8}$

8. The number of points at which the function $f(x)=(x-|x|)^2(1-x+|x|)^2$ is not differentiable in the interval (-3, 4) is

(1) Zero (2) One (3) Two (4) Three

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9. Let $f(x)=1+2x^2+2^2x^4+2^3x^6+.......+2^{10}x^{20}$. Then $f(x)$ has

(1) More than one minimum (2) Exactly one minimum
(3) At least one maximum (4) No extreme

10. The point $(0,5)$ is closest to the curve $x^2=2y$ at

(1) $(2\sqrt{2},0)$ (2) $(2,2)$ (3) $(-2\sqrt{2},0)$ (4) $(-2\sqrt{2},4)$

11. The value of $\int_0^{\pi/4}\frac{e^{\frac{1}{\cos^2 x}}.\sin x}{\cos^3 x}dx$ equals

(1) $\frac{e^2-e}{2}$ (2) $\frac{e^4-1}{4}$ (3) $\frac{e^2+e}{4}$ (4) $\frac{e^2-e}{4}$

12. At $x=0, f(x)=\begin{cases}x^3(1-x) & x\le 0\\ x\ln x+3x & x>0\end{cases}$,

(1) Has point of maxima (2) Has point of minima
(3) Increases (4) Decreases

13. If $f(x)=4x^3-x^2-2x+1$ and $g(x)=\begin{cases}\min\{f(t):0\le t\le x: & 0\le x\le 1\}\\ 3-x & 1<x\le 2\end{cases}$ then $g\left(\frac{1}{4}\right)+g\left(\frac{3}{4}\right)+g\left(\frac{5}{4}\right)$ has the value equal to

(1) $\frac{7}{4}$ (2) $\frac{2}{5}$ (3) $\frac{15}{2}$ (4) $\frac{5}{2}$

14. $f(x)=\begin{cases}2-|x^2+5x+6| & x\ne -2\\ a^2+1 & x=-2\end{cases}$; then the range of a, so that $f(x)$ has maxima at $x=-2$ is

(1) $(-\infty,-1]\cup[1,\infty)$ (2) $(-1,1)$ (3) $(1,\infty)$ (4) $(-2,2)$

15. The total number of local maxima and local minima of the function $f(x)=\begin{cases}(2+x)^3 & -3<x\le -1\\ x^{\frac{2}{3}} & -1<x<2\end{cases}$ is

(1) 0 (2) 1 (3) 2 (4) 3

16. $\int\left(\frac{x\ell nx+\ell nx-1}{(\ell nx)^2}\right)e^x dx=$

(1) $\frac{e^x}{\ell nx}+c$ (2) $\frac{x}{\ell nx}+c$ (3) $\frac{xe^x}{\ell nx}+c$ (4) $\frac{\ell nx}{xe^x}+c$

17. Given a function $f:[0,4]\to R$ is differentiable, then for some $a,b\in(0,4)\left[f(4)\right]^2-\left[f(0)\right]^2=$

(1) $8f'(b)f(a)$ (2) $4f'(b)f(a)$ (3) $2f'(b)f(a)$ (4) $f'(b)f(a)$

18. The number of value of $x \in [0, 2]$ at which the real function $f(x) = \left|x - \frac{1}{2}\right| + |x - 1| +$ tan x is not differentiable is

(A) 2 (2) 3 (3) 1 (4) 0

19. Number of points at which the function $f(x) = \begin{cases} \min(x, x^2) & -\infty < x < 0 \\ \min(2x - 1, x^2) & \text{other wise} \end{cases}$ is not derivable is

(1) 0 (2) 1 (3) 2 (4) 3

20. If $f(x) = \begin{cases} x^n \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$ is continuous but not differentiable at $x = 0$ then

(1) $n \in (-1, 0)$ (2) $n \in (0, 2]$ (3) $n \in (0, 1]$ (4) $n \in (1, 2)$

21. The number of points where the function $f(x) = (x - 3)\left|x^2 - 7x + 12\right| + \cos|x - 3|$ is not differentiable is

(1) One (2) Two (3) Three (4) Infinite

22. If the function $f(x) = \left[\frac{(x-2)^3}{a}\right]\sin(x - 2) + a\cos(x - 2)$, ([.] denotes the greatest integer function), is continuous and differentiable in (4, 6) then

(1) $a \in [8, 64]$ (2) $a \in (0, 8]$ (3) $a \in [64, \infty)$ (4) $a \in (-\infty, 0)$

23. Let $f(x) = \begin{cases} \frac{x}{1+|x|}, & |x| \geq 1 \\ \frac{x}{1-|x|}, & |x| < 1 \end{cases}$. Then which of the following statements is correct?

(1) f is continuous but not differentiable on $R - \{-1, 1\}$

(2) f is both continuous and differentiable on $R - \{-1, 1\}$

(3) f is not differentiable on $R - \{-1, 0, 1\}$

(4) f is not continuous on $R - \{-1, 0, 1\}$

24. If $f(x) = x + \tan$ and f is inverse of g, then $g'(x)$ equals:

(1) $\frac{1}{1 + [g(x) - x]^2}$ (2) $\frac{1}{2 - [g(x) - x]^2}$ (3) $\frac{1}{2 + [g(x) - x]^2}$ (4) None of these

25. If $y = \frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{c}{x-c} + 1$ Then $\frac{y'}{y} =$

25

(1) $\frac{1}{x}\left(\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x}\right)$ (2) $\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x}$

(3) $\frac{1}{x}\left(\frac{a}{x-a} + \frac{b}{x-b} + \frac{c}{x-c}\right)$ (4) $\frac{1}{x}\left(\frac{a}{x-a} - \frac{b}{x-b} - \frac{c}{x-c}\right)$

26. The value of $\lim_{n\to\infty}\prod_{r=3}^{n}\frac{r^3-8}{r^3+8}(\text{MDCL}-1)$

(1) $\frac{1}{7}$ (2) $\frac{2}{7}$ (3) $\frac{3}{7}$ (4) $\frac{4}{7}$

26,27,28, 31,34

27. $\lim_{x\to 0}\frac{1-\cos x\cos 2x\cos 3x}{\sin^2 2x}$ is equal to

(1) 7/2 (2) 7/3 (3) 7/4 (4) 7/5

28. The value of $\lim_{x\to 0}\left\{1^{1/\sin^2 x}+2^{1/\sin^2 x}+......+n^{1/\sin^2 x}\right\}^{\sin^2 x}$ is

(1) ∞ (2) 0 (3) $\frac{n(n+1)}{2}$ (4) n

29. $\lim_{x\to 0}\left\{\tan\left\{\frac{\pi}{4}-x\right\}\right\}^{1/x}$ is equal to

(1) 1 (2) e (3) e^2 (4) e^{-2}

30. $\lim_{x\to y}\frac{\sin^2 x-\sin^2 y}{x^2-y^2}$ is

(1) $\frac{2y}{\sin 2y}$ (2) $\frac{\sin y}{y}$ (3) $\frac{\sin 2y}{2y}$ (4) $\frac{y}{\sin y}$

31. $\int_{-\frac{\pi}{3}}^{0}\left[\cot^{-1}\left(\frac{2}{2\cos x-1}\right)\right]+\cot^{-1}\left(\cos x-\frac{1}{2}\right)dx$ is equal to

(1) $\frac{\pi^2}{6}$ (2) $\frac{\pi^2}{3}$ (3) $\frac{\pi^2}{8}$ (4) $\frac{3\pi^2}{8}$

32. If $z=x+3i$, then the value of $\int_{2}^{4}\left[\arg\left|\frac{z-i}{z+i}\right|\right]dx$, where [.] denotes the greatest integer function and $i=\sqrt{-1}$, is

(1) $3\sqrt{2}$ (2) $6\sqrt{3}$ (3) $\sqrt{6}$ (4) 0

33. Let $A=\int_0^1\frac{e^t dt}{1+t}$ then $\int_{a-1}^{a}\frac{e^{-t}dt}{t-a-1}=$

(1) Ae^{-a} (2) $-Ae^{-a}$ (3) $-ae^{-a}$ (4) Ae^{a}

34. $\int_0^{\frac{\pi}{4}}\left(\frac{x}{x\sin x+\cos x}\right)^2 dx=$

(1) $\frac{5-\pi}{5+\pi}$ (2) $\frac{2}{4+\pi}$ (3) $\frac{4-\pi}{4+\pi}$ (4) $\frac{4+\pi}{4-\pi}$

35. The value of constant $a > 0$ such that $\int_0^a \left[\tan^{-1}\sqrt{x}\right]dx = \int_0^a \left[\cot^{-1}\sqrt{x}\right]dx$ is [.] denotes G. I. F

(1) $\frac{2(3+\cos 4)}{1-\cos 4}$ (2) $\frac{(3-\cos 4)}{1+\cos 4}$ (3) $\frac{2(3+\cos 4)}{1+\cos 4}$ (4) $\frac{(3+\cos 4)}{1-\cos 4}$

36. Let $p(x)$ be a function defined on R such that $p'(x) = p'(1-x)$, for all $x \in [0,1], p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x)dx$ equals

(1) 21 (2) 41 (3) 42 (4) $\sqrt{41}$

35,36,38, 41

37. $\int_0^1 \cot^{-1}(1-x+x^2)dx =$

(1) $\pi - \ell n2$ (2) $\frac{\pi}{2} - \ell n2$ (3) $\pi + \ell n2$ (4) $\frac{\pi}{2} + \ell n2$

38. The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2}dx$ is

(1) $\frac{\pi}{2}\log 2$ (2) $\log 2$ (3) $\pi \log 2$ (4) $\frac{\pi}{8}\log 2$

39. $\int \frac{(x^2-1)}{(x^4+3x^2+1)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx =$

(1) $\log \sin^{-1}\left(x - \frac{1}{x}\right) + c$ (2) $\log \tan^{-1}\left(x - \frac{1}{x}\right) + c$

(3) $\log \sin^{-1}\left(x + \frac{1}{x}\right) + c$ (4) $\log \tan^{-1}\left(x + \frac{1}{x}\right) + c$

40. $\int e^{\sin^2 x}(\cos x + \cos^3 x)\sin x dx =$

(1) $2\left[e^{\sin^2 x}(3-\sin^2 x)\right] + C$ (2) $\frac{1}{2}\left[e^{\sin^2 x}(3-\sin^2 x)\right] + C$

(3) $2\left[e^{\sin^2 x}(3+\sin^2 x)\right] + C$ (4) $\frac{1}{2}\left[e^{\sin^2 x}(3+\sin^2 x)\right] + C$

41. If $\int e^x \left[\frac{x^3+3x^2+4}{(x+1)^3}\right]dx = e^x\left[\frac{x^2+Ax+B}{(x+1)^2}\right] + C$ then $A + B =$

(1) 4 (2) 0 (3) 2 (4) – 4

42. If the integral $\int \frac{5\tan x}{\tan x - 2}dx = x + a\log|\sin x - 2\cos x| + k$ the a is equal to

(1) 1 (2) 2 (3) – 1 (4) – 2

43. $\int\left\{\frac{\log x-1}{1+(\log x)}\right\}^{2} dx =$

43,48,50

(1) $\frac{\log x}{(\log x)^{2}+1}+c$ (2) $\frac{x}{x^{2}+1}+c$ (3) $\frac{xe^{x}}{1+x^{2}}+c$ (4) $\frac{x}{(\log x)^{2}+1}+c$

44. $\int\frac{x-\sin x}{1+\cos x}dx = x\tan\left(\frac{x}{2}\right)+p\log\left|\sec\left(\frac{x}{2}\right)\right|+c \Rightarrow p =$

(1) – 4 (2) 4 (3) 2 (4) -2

45. $\int\frac{dx}{x(\log x-2)(\log x-3)} = I+c \Rightarrow I =$

(1) $\frac{1}{x}\log\left|\frac{\log x-3}{\log x-2}\right|$ (2) $\log\left|\frac{\log x-3}{\log x-2}\right|$ (3) $\log\left|\frac{\log x-2}{\log x-3}\right|$ (4) $\log\left|(\log x-3)(\log x-2)\right|$

46. $\int e^{x}\left[\frac{(x+2)x^{3}+8}{(x+2)^{2}}\right]dx$

(1) $e^{x}\left[x^{2}-2x+4\right]+\frac{8}{x+2}$ (2) $e^{x}\left[x^{2}-4x+8\right]+\frac{8}{x+2}$

(3) $e^{x}\left[x^{2}-2x+4\right]-\frac{8}{x+2}$ (4) $e^{x}\left[x^{2}-4x+8\right]-\frac{8}{x+2}$

47. The slope of the tangent to a curve $y=f(x)$ at $(x, f(x))$ is $2x+1$. If the curve passes through the point (1, 2) then the area of the region bounded by the curve, X – axis and the line x = 1 is

(1) $\frac{5}{6}$ (2) $\frac{6}{5}$ (3) 6 (4) $\frac{1}{6}$

48. The area bounded by the curve $x=\left|y^{2}-1\right|$ and the line $y=x-5$ is

(1) $\frac{73}{6}$ (2) $\frac{85}{6}$ (3) $\frac{109}{6}$ (4) $\frac{125}{6}$

49. Let $f(x)$ be a non – negative continuous function such that the area bounded by the curve $y=f(x)$, x – axis and the ordinates $x=\frac{\pi}{4}$ and $x=\beta>\frac{\pi}{4}$ is $\beta\sin\beta+\frac{\pi}{4}\cos\beta+\sqrt{2}\beta$. Then $f^{1}\left(\frac{\pi}{2}\right)$ is

(1) $\left(\frac{\pi}{2}-\sqrt{2}-1\right)$ (2) $\left(\frac{\pi}{4}+\sqrt{2}-1\right)$ (3) $-\frac{\pi}{2}$ (4) $\left(1-\frac{\pi}{4}+\sqrt{2}\right)$

50. The value of the parameter a such that the area bounded by $y=a^{2}x^{2}+ax+1$, coordinate axes, and the line $x=1$ attains its least value is equal to

(1) $-\frac{1}{4}$ sq. units (2) $-\frac{1}{2}$ sq. units (3) $-\frac{3}{4}$ sq. units (4) – 1 sq. units

51. The area enclosed by the curve $y=\sqrt{4-x^2}, y\geq\sqrt{2}\sin\left(\frac{x\pi}{2\sqrt{2}}\right)$, and the x – axis is divided by the y – axis in the ratio

(1) $\frac{\pi^2-8}{\pi^2+8}$ (2) $\frac{\pi^2-4}{\pi^2+4}$ (3) $\frac{\pi-4}{\pi-4}$ (4) $\frac{2\pi^2}{2\pi+\pi^2-8}$

51,52, 53,55, 60

52. The area of the loop of the curve $ay^2=x^2(a-x)$ is

(1) $4a^2$ sq. units (2) $\frac{8a^2}{15}$ sq. units (3) $\frac{16a^2}{9}$ sq. units (4) None of these

53. The area bounded by the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ and the coordinate axes is

(1) $\frac{a^2}{6}$ (2) $\frac{a^2}{8}$ (3) $\frac{a^2}{4}$ (4) $\frac{a}{6}$

54. $\underset{n\to\infty}{\text{Lt}}\left[\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right).....\left(1+\frac{n}{n}\right)\right]^{\frac{1}{n}}=$

(1) 2e (2) $\frac{2}{e}$ (3) $\frac{4}{e}$ (4) 4e

55. The order, degree of the differential equation satisfying the relation $\sqrt{1+x^2}+\sqrt{1+y^2}=\lambda\left(x\sqrt{1+y^2}-y\sqrt{1+x^2}\right)$ is

(1) 1, 1 (2) 2, 1 (3) 3, 2 (4) 0, 1

56. Solution of $\log\left(\frac{dy}{dx}\right)=3x+4y, y(0)=0$ is

(1) $e^{3x}+3e^{-4y}$ (2) $4e^{3x}-4e^{-4y}=3$ (3) $3e^{3x}+4e^{4y}=7$ (4) $4e^{3x}+3e^{-4y}=7$

57. The solution of $\frac{dy}{dx}=\frac{px+q}{rx+s}$ represents a parabola when

(1) $p=0, q=0$ (2) $r=0, s=0$ (3) $p=0, q\neq 0$ (4) $r=0, s\neq 0$

58. The solution of $(x^2-y^2)dx+2xy\,dy=0$ is

(1) $x^2+y^2=cx$ (2) $x^2-y^2=cx$ (3) $x^2+y^2=cy$ (4) $x^2+y^2=cx^2$

59. Let I be the purchase value of an equipment and $V(t)$ be the value after it has been used for t years. The value V(t) depreciates at a rate given by differential equation $\frac{dV(t)}{dt}=-k(T-t)$, where k > 0 is a constant and T is the total life in years of the equipment. Then the scrap value V(T) of the equipment is

(1) $I-\frac{k(T-t)^2}{2}$ (2) e^{-kT} (3) $T^2-\frac{I}{k}$ (4) $I-\frac{kT^2}{2}$

60. The solution of the differential equation $(x\cot y+\log\cos x)dy+(\log\sin y-y\tan x)dx=0$ is

(1) $(\sin x)^y(\cos y)^x=c$ (2) $(\sin y)^x(\cos x)^y=c$

(3) $(\sin x)^x(\cos y)^y=c$ (4) $(\sin x)^x+(\cos y)^y=c$

62,63

61. The solution to the differential equation $\frac{dy}{dx}=\frac{(x+y+1)^2}{xy-y+2x-2}$ is

(1) $e^{2y/x}=cx^4+cx^3y$ (2) $e^{y/x}=cx^3+cx^4y$

(3) $e^{\frac{2y+2}{x-1}}=cx^4+cx^2y$ (4) None of these

62. Let a continuous function $f(x)$ on $R\to R$ be defined such that it satisfies the relation $f(x)+f(x+2y)+3xy=2f(2y-x)+2y^2\forall x,y\in R$. Then which of the following is true

(1) f(x) is an odd function (2) f(x) is one-one

(3) f(x) is into (4) f(x) is invertible

63. $\lim\limits_{n\to\infty}\left[\frac{1.n+2(n-1)+.....+n.1}{1^3+2^3+.......+n^3}\right]^n$ is equal to

(1) $\frac{2}{3}$ (2) e^2 (3) $e^{\frac{1}{2}}$ (4) $e^{\frac{2}{3}}$

64. Which of the following statements is NOT true (where [.] denote the greatest integer function and {.} denote fractional part function)

(1) $\{x^2\}$ is continuous as well as differentiable at x = 0

(2) $[x^2]$ is continuous as well as differentiable at x = 0

(3) $\sqrt{\{x\}^2}$ is continuous as well as differentiable at x = 0

(4) none of the above

65. If $\frac{d^3x}{dy^3}\left(\frac{dy}{dx}\right)^5=P\left(\frac{d^2y}{dx^2}\right)^2-\frac{dy}{dx}\frac{d^3y}{dx^3}$ then value of 'P' is equal to

(1) 1 (2) 2 (3) 3 (4) None of these

66. $\int x^9(1+x^5)^{2/5}dx$ is equal to

(1) $\frac{5}{12}(1+x^5)^{12/5}+\frac{1}{14}(1+x^5)^{7/5}+C$ (2) $\frac{1}{12}(1+x^5)^{12/5}-\frac{1}{7}(1+x^5)^{7/5}+C$

(3) $\frac{5}{12}(1+x^5)^{12/5}-\frac{5}{7}(1+x^5)^{7/5}+C$ (4) None of these

67. If $f(x)=x(x-1)(x-2)(x-3)(x-4)(x-5)$ then value of $\int_0^5\left[\frac{3f(x)-|f(x)|}{3f(x)+|f(x)|}\right]dx$ (where [.] denotes the greatest integer function) is equal to

(1) 10 (2) 15 (3) 6 (4) 9

68. If $f(x)=\cos(\pi\sin^2 x)$ and $g(x)=\cos(\pi\cos^2 x)$ then which of the following statements is true

(1) $f(x)$ is aperiodic

(2) g(x) is aperiodic

(3) $f(x)+g(x)$ is periodic with fundamental period π

(4) None of these

69. Let $S_n = \sum_{k=1}^{n} \frac{\tan^{-1}\left(\frac{k}{n}\right)}{n}$ and $T_n = \sum_{k=0}^{n-1} \frac{\tan^{-1}\left(\frac{k}{n}\right)}{n}$ for $n \in N$, then which of the following statements is false

(1) $S_n > \frac{\pi - \ln 4}{4}$ (2) $T_n < \frac{\pi - \ln 4}{4}$ (3) $\lim_{n\to\infty} S_n > \lim_{n\to\infty} T_n$ (4) None of these

70. Let if $f(x) = \sum_{i=0}^{9} a_i x^i$ be a real valued function, $a_i \notin R^-$, where $f(x) = 0$ has two distinct negative roots. Then minimum number of distinct real roots of $f(x) = 0$ are

(1) 3 (2) 2 (3) 9 (4) Not enough information

71. Area bounded by curve $y = 1$ and $y = \frac{\sin x + \cos x + |\sin x - \cos x|}{2}$ in $x \in [0, \pi]$ is

(1) $\frac{3\pi}{4} - \frac{1}{\sqrt{2}} - 1$ (2) $\frac{3\pi}{4} - \frac{1}{\sqrt{2}}$ (3) $\pi + \sqrt{2} - 3$ (4) $\pi - \sqrt{2} - 1$

72. Number of points where function $f(x) = (1 - \sin x - \cos x)\,\mathrm{sgn}\left(x^2 - \frac{\pi x}{2}\right)$ is discontinuous is

(1) 0 (2) 1 (3) 2 (4) 3

73. If $f(x) = \frac{1}{\sqrt{|x| - x}}$ and $g(x) = \cos\left(\ln\left(\frac{\sqrt{1-x^2}}{e^x}\right)\right)$, then domain of $(fog)(x)$ is

(1) $(-\infty, 0)$ (2) $(-1, 0)$ (3) $(-1, 1)$ (4) $(0, 1)$

74. The curve for which square of sub tangent varies as subnormal is

(1) $(x+2)^2 = y^3$ (2) $(x-2)^3 = y^2$ (3) $x^2 = (y-2)^3$ (4) $x^3 = (y+2)^2$

75. Let f(x) be a ‘n’ degree polynomial function having ‘n’ real and distinct roots.
If g(x) = f’(x) + 100f(x), then minimum number of roots that g(x) must possesses is

(1) n (2) n + 1 (3) n – 1 (4) information is insufficient

76. Which of the following statement is true?

(1) $e^{\pi} < \pi^{e}$ (2) $\pi^3 > 3^{\pi}$ (3) $2^e > e^2$ (4) $\pi^{10} > 10^{\pi}$

77. $\int_{-2}^{2} x^4 d(\ln x)$ is equal to

(1) $\frac{e^8 - e^{-8}}{4}$ (2) $\frac{64}{5}$ (3) 0 (4) None of these

78. Area bounded by the curve $f(x) = \frac{1}{x^2 + [x]^2 + 2\{x\} + 1 - 2x[x]}$ and x – axis between $x = -\frac{3}{2}$ and $x = \frac{5}{2}$ is equal to (where [.] denote the greatest integer function and {.} denote fractional part function)

(1) $\frac{1}{2}$ (2) 1 (3) 2 (4) 4

79. Area bounded by the curves $C_1 : x^2 + y^2 = 36; C_2 : y^2 = 5x; C_3 : y^2 = -5x$ is equal to

(1) $4\int_0^4 \left(\sqrt{36-x^2} - 5x\right)dx$ (2) $2\int_0^4 \left(\sqrt{36-x^2} - 5x\right)dx$

(3) $4\left(\int_0^{\sqrt{20}} \frac{y^2}{5}dy\right)$ (4) $4\left[9\pi - \left(\int_0^{\sqrt{20}}\left(\sqrt{36-y^2} - \frac{y^2}{5}\right)dy\right)\right]$

80. $\int \frac{6x^{23} - 9x^8}{\left(x^{15} + x^9 + 1\right)^2}dx$ is equal to

(1) $-\frac{x^9}{x^{15} + x^9 + 1} + c$ (2) $\frac{3x^9}{\left(x^{15} + x^9 + 1\right)^3} - \frac{2x^3}{\left(x^{15} + x^9 + 1\right)^2} + c$

(3) $-\frac{x^{15} + x^6}{x^{15} + x^9 + 1} + c$ (4) None of these

81. Which of the following limits vanish? (i) $\lim_{x\to\infty}\frac{\sin x}{x}$ (ii) $\lim_{x\to\infty}\frac{\int_0^x e^t dt}{e^{x^2}}$ (iii) $\lim_{x\to 0}(\sin x)^{1/x^2}$

(1) (ii), (iii) (2) (i), (ii) (3) (i), (iii) (4) (i), (ii), (iii)

82. If $f(x) = \begin{cases} x^2, & x < 2 \\ x^3 + 3x, & x > 2 \\ a, & x = 2 \end{cases}$, then range of values of 'a' for which $f(x)$ is strictly monotonically increasing at x = 2.

(1) [0, 14] (2) [4, 14] (3) R (4) (4, 14)

83. $\int \frac{3x^2 + 1}{x^6 + 2x^4 + 2x^3 + x^2 + 2x + 2}$ is equal to

(1) $\frac{x^5 - 2x^2}{x^3 + x + 1} + c$ (2) $-\cot^{-1}\left(x^3 + x + 1\right) + c$

(3) $\frac{x^3 + x}{\left(x^6 + 2x^4 + 2x^3 + x^2 + 2x + 2\right)^2}$ (4) None of these

84. If $\sqrt{2y + x} + \sqrt{2y - x} = \text{constant}$ then $\frac{dy}{dx}$ is equal to

(1) $\frac{x}{4y - \sqrt{4y^2 - x^2}}$ (2) $\frac{x}{4y + \sqrt{4y^2 - x^2}}$ (3) $\frac{x}{4y + 2\sqrt{4y^2 - x^2}}$ (4) $\frac{x}{4y - 2\sqrt{4y^2 - x^2}}$

85. Consider the functions $f(x) = |x|^5$; $g(x) = \{\cos x\}$; $h(x) = [|\sin x|]$ (where [.] denote the greatest integer function and {.} denote fractional part function) then which of these functions is differentiable at x = 0?

(1) f(x) and g(x) (2) g(x) and h(x)
(3) f(x), g(x) and h(x) (4) f(x) and h(x)

86. If the line segment $y = 2x, -1 \le x \le 1$ is rotated about y-axis, then which of the following statements hold true for the solid so formed

(1) Volume $= \frac{2\pi}{3}$, surface area $= \sqrt{5}\pi$ (2) Volume $= \frac{4\pi}{3}$, surface area $= \sqrt{5}\pi + 2\pi$

(3) Volume $= \frac{2\pi}{3}$, surface area $= \sqrt{5}\pi + \pi$ (4) Volume $= \frac{4\pi}{3}$, surface area $= 2\sqrt{5}\pi$

87. Let a relation be defined on a set of functions defined on $R \to R$ such that R = {(f, g)|f – g is an even function} then, relation R is

(1) Reflexive, symmetric (2) Reflexive, transitive

(3) Symmetric, transitive (4) Equivalence relation

88. If area bound by the curves $y = e^{ax^2}, y = e^{\frac{1}{8}}$ between $x = 0$ and $x = 1$ minimum, then the value of 'a' is

(1) 1 (2) 2 (3) $\frac{1}{2}$ (4) None of these

89. Order and degree of the differential equation $\sin\left(\frac{dy}{dx}\right) = 2xy - 3\cos\frac{dy}{dx}$ respectively are

(1) Order 1, degree 1 (2) Order 1, degree not defined

(3) Order 2, degree not defined (4) None of these

90. Let the number of elements in a set A be 'n'. A set 'C' is defined such the $C = \{(x, y) | x, y \in P(A) \text{ and } x \cap y = \phi\}$, where $P(A)$ is power set of A, then cardinal number of C is equal to

(1) 2^{2n} (2) 3^n (3) 2.3^n (4) None of these

IIT – JEE **TW TEST (3 Yrs.)**

TOPIC: FULL CALCULUS **DATE:28/9/18**

ANSWER KEY

1.	**(4)**	**2.**	**(1)**	**3.**	**(1)**	**4.**	**(2)**	**5.**	**(4)**	**6.**	**(3)**	**7.**	**(1)**
8.	**(1)**	**9.**	**(2)**	**10.**	**(D)**	**11.**	**(1)**	**12.**	**(1)**	**13.**	**(4)**	**14.**	**(1)**
15.	**(3)**	**16.**	**(3)**	**17.**	**(1)**	**18.**	**(2)**	**19.**	**(2)**	**20.**	**(3)**	**21.**	**(1)**
22.	**(3)**	**23.**	**(2)**	**24.**	**(3)**	**25.**	**(1)**	**26.**	**(2)**	**27.**	**(3)**	**28.**	**(4)**
29.	**(4)**	**30.**	**(3)**	**31.**	**(1)**	**32.**	**(4)**	**33.**	**(2)**	**34.**	**(3)**	**35.**	**(1)**
36.	**(1)**	**37.**	**(2)**	**38.**	**(3)**	**39.**	**(4)**	**40.**	**(2)**	**41.**	**(2)**	**42.**	**(2)**
43.	**(4)**	**44.**	**(1)**	**45.**	**(2)**	**46.**	**(4)**	**47.**	**(1)**	**48.**	**(3)**	**49.**	**(3)**
50.	**(3)**	**51.**	**(4)**	**52.**	**(2)**	**53.**	**(1)**	**54.**	**(3)**	**55.**	**(1)**	**56.**	**(4)**
57.	**(4)**	**58.**	**(1)**	**59.**	**(4)**	**60.**	**(2)**	**61.**	**(4)**	**62.**	**(3)**	**63.**	**(4)**
64.	**(3)**	**65.**	**(3)**	**66.**	**(2)**	**67.**	**(3)**	**68.**	**(4)**	**69.**	**(3)**	**70.**	**(1)**
71.	**(4)**	**72.**	**(1)**	**73.**	**(2)**	**74.**	**(2)**	**75.**	**(3)**	**76.**	**(4)**	**77.**	**(1)**
78.	**(3)**	**79.**	**(4)**	**80.**	**(1)**	**81.**	**(4)**	**82.**	**(2)**	**83.**	**(2)**	**84.**	**(3)**
85.	**(4)**	**86.**	**(4)**	**87.**	**(4)**	**88.**	**(3)**	**89.**	**(1)**	**90.**	**(2)**		