Vidyamandir Classes
Gurukul for IITJEE Preparation

Advanced Practice Test-9

TIME : 3 hrs	M.M. : 360

Read the following Instructions very carefully before you proceed.

- **The Test consists of 90 questions.**
- The question paper consists of 3 parts: Part I : Chemistry, Part II : Physics, Part III : Mathematics. Each part contains 25 questions.

  Each question has 4 choices (A), (B), (C) and (D), out of which **Only One choice is Correct.**
- For each question you will be given **4 Marks** if you have darkened only the bubble corresponding to the correct answer and zero mark if no bubble is darkened. In all other cases, **minus one (–2) Mark (NEGATIVE MARKING)** will be given.
- For answering a question, an **ANSWER SHEET (OMR SHEET)** is provided separately. Please fill your **Name, Roll Number, Group, Batch** and the **PAPER CODE** properly in the space provided in the **ANSWER SHEET.**

  IT IS YOUR OWN RESPONSIBILITY TO FILL THE OMR SHEET CORRECTLY.
- A blank space has been provided on each page for rough work. You will not be provided with any supplement or rough sheet. However some blank pages for rough work are given at the end of this paper.
- The use of log tables, calculator, mobile or any other electronic device is strictly prohibited.
- Please do not disturb the invigilator or any other student for any confusion(s) in the paper. **Violating the examination room discipline will immediately result in the cancellation of your paper and no excuses will be entertained.**

PART - I (CHEMISTRY) | 120 MARKS

This section contains 30 Multiple Choice Questions. Each Question has 4 choices A, B, C & D, out of which ONLY ONE Choice is Correct:

1. How many isomeric bromides having molecular formula $C_6H_{13}Br$ on conversion into Grignard reagent followed by treatment with water would yield 2, 3-dimethylbutane?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

2. The root mean square velocity of nitrogen gas at 27°C and 70 cm pressure, (Given density of Hg = 13.6 g/cm^3) would be :

(A) 517 ms^{-1} **(B)** 476 m/s **(C)** 400 m/s **(D)** 422 m/s

3. X mL of 0.05 M solution of a double salt $Na_2CO_3 \cdot NaHCO_3 \cdot 2H_2O$ is titrated against 0.05 M HCl. When phenolphthalein is used as indicator 'A' mL of HCl is required and 'B' mL of the same acid is required (separately) when methyl orange is the indicator. The ratio B/A would be :

(A) 3 **(B)** $\frac{1}{3}$ **(C)** 5 **(D)** 2

4. A 9.3 g of salt (MF_n) of a metal (M) with fluorine is dissolved in 100 g of water. The boiling point of resulting solution is found to be higher than that of pure water by 1.24°C. The formula of the salt is : (K_b for water = 0.52 K kg mol^{-1} , atomic mass of M is 100 g mol^{-1} and F is 19 g mol^{-1})

(A) MF **(B)** MF_2 **(C)** MF_3 **(D)** MF_4

5. Which one of the following statement about H_3BO_3 is not correct?

(A) It is a weak tribasic acid

(B) It is prepared by acidifying an aqueous solution of borax

(C) It has a layer structure in which planar BO_3 units are joined by hydrogen bonds

(D) It does not act as proton donor as it acts as a Lewis acid by accepting hydroxyl ions

6. An aqueous solution of sucrose undergoes acid catalysed hydrolysis. 50 g sucrose in 125 mL water rotates the plane of plane polarized light by + 13.1° at t = 0. After complete hydrolysis, it shows a rotation of – 3.75°. The percentage hydrolysis of cane sugar at time 't' in the same solution having a rotation of 5° is :

(A) 55% **(B)** 48% **(C)** 42% **(D)** 58%

7. When conc. H_2SO_4 was treated with $[K_4Fe(CN)_6]$, CO gas was evolved. By mistake, somebody used dilute H_2SO_4 instead of conc. H_2SO_4, the gas evolved was :

(A) CO **(B)** HCN **(C)** N_2 **(D)** CO_2

8. What is the approximate ratio of solubility of silver chloride in 1M NH_3 solution to its solubility in pure water ? $[K_{sp}(AgCl) = 1\times10^{-10}$; $K_f[Ag(NH_3)_2^+] = 1\times10^8]$

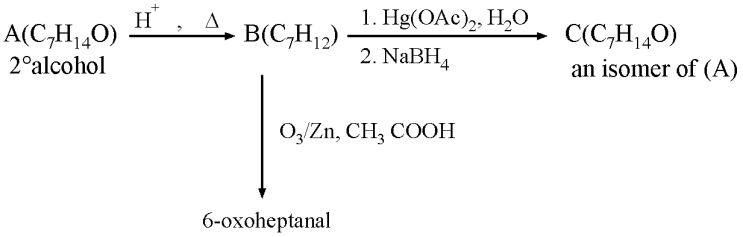
(A) 10 **(B)** 10^2 **(C)** 10^3 **(D)** 10^4

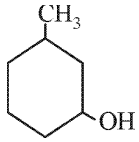
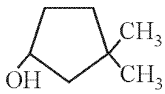
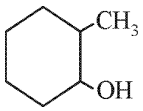
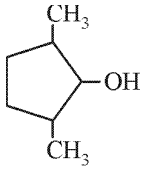
9. Select the group of species in which all show trigonal bipyramidal geometry :

(A) PF_5, IF_5, XeF_4 **(B)** ClO_4^-, IF_7, CO_3^{2-}

(C) I_3^-, XeF_2, SF_4 **(D)** XeF_6, PF_6^-, ICl_2^+

10. Identify (A) in the following scheme.



(A)  **(B)**  **(C)**  **(D)** 

11. The vapour pressure of water at 20°C is 17.54 mm. When 20 g of a non-ionic substance is dissolved in 100 g of water the vapour pressure is lowered by 0.30 mm. what is the molecular mass of the substance?

(A) 200.8 **(B)** 206.88 **(C)** 210.2 **(D)** 215.2

12. In a cubic closed packed structure of mixed oxides, the lattice is made up of oxide ions, 20% of tetrahedral voids are occupied by divalent X^{2+} ions and 50% of the octahedral voids are occupied by trivalent Y^{3+} ions. The formula of the oxide is :

(A) X_2YO_4 **(B)** $X_4Y_5O_{10}$ **(C)** $X_5Y_4O_{10}$ **(D)** XY_2O_4

13. Which of the following is an incorrect statement? (Atomic numbers: P = 15, S = 16, I = 53)

(A) Axial P–Cl bonds in PCl_5 are longer than equatorial P–Cl bonds.

(B) SF_4 has distorted tetrahedral arrangement.

(C) I_3^- ion has nine lone pairs of electrons.

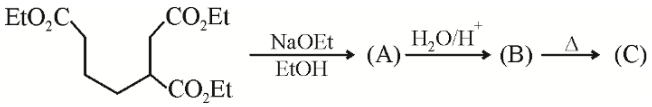
(D) According to molecular orbital theory B_2 and HF have same type of single bond.

14. Arrange the following species according to their bond angle order.

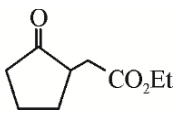
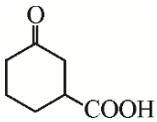
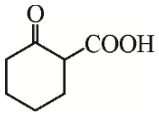
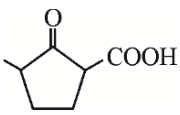
I. O_3 **II.** NO_2^- **III.** FNO

The correct choice is :

(A) I > II > III **(B)** II > I > III **(C)** III > II > I **(D)** II > III > I

15. 

The product (C) in the above sequence of reactions is :

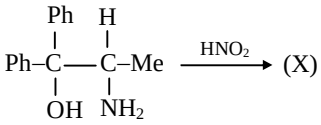
(A)  **(B)**  **(C)**  **(D)** 

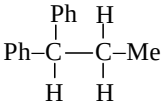
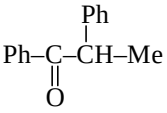
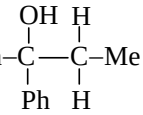
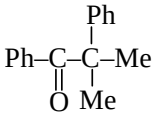
16. A mixture of 2g each of the sulphates of two metals whose equivalent weights are 12 and 32 respectively was dissolved in water. To this solution, an excess of $BaCl_2$ was added, the weight of $BaSO_4$ precipitated is : (atomic weight of O = 16, S = 32, Ba = 137)

(A) 4 g **(B)** 26.7 g **(C)** 3.4 g **(D)** 6.8 g

17. On heating 60 cc of a mixture of equal volume of Cl_2 and an oxide of chlorine and cooling to room temperature and pressure, the resulting gas measured 75 cc. Treatment of this gas with caustic soda solution resulted in a contraction to 15 cc. Assuming that all measurements were done at the same temperature and pressure, 300 cc of the oxide of chlorine diffuses in 40 sec and 400 cc of a gas, 'X' diffuses in 2/3rd of a minute. The gas 'X' could be :

(A) NO_2 **(B)** CH_2N_2 **(C)** C_4H_{10} **(D)** O_3

18. The major product in the reaction given below is :  (X)

(A)  **(B)**  **(C)**  **(D)** 

19. Pure water is saturated with pure solid AgCl, a silver electrode is placed in the solution and the potential is measured against normal calomel electrode at 25°C. The experiment is then repeated with a saturated solution of AgI. If the difference in potential of the two cases is 0.177 V , what is the ratio of solubilities of AgCl and AgI at the temperature of the experiment?

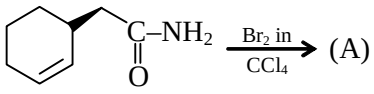
(A) 10^2 **(B)** 10^6 **(C)** 10^3 **(D)** 10^4

20. 0.2063 g of an organic compound (molar mass 168) was heated with sufficient amount of HI and the resulting solution was treated with alcoholic $AgNO_3$ solution. This led to precipitation of 0.8658 g of AgI. The number of methoxy groups in one molecule of the organic compound is(are) : (Given: atomic mass of Ag = 108, I = 127)

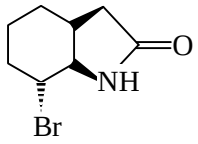
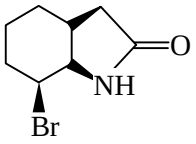
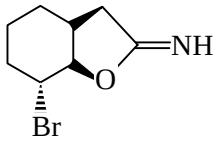
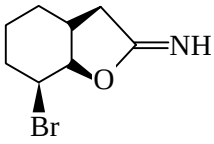
(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

21. Microbial growth in a culture follows first order kinetics. The rate constant for this growth is 0.0462 min^{-1}. How many microbes will be in culture after 3 hrs if initially there are 4×10^6 microbes? (Use: $\log 4 \times 10^3 = 3.6$)

(A) 1.6×10^{10} **(B)** 4.38×10^9 **(C)** 1.0×10^8 **(D)** 4×10^8

22. 

The product **(A)** in the given reaction is :

 **(A)**  **(B)**  **(C)**  **(D)**

23. A solution of a monoprotic weak acid has acidity constant K_a. The minimum initial concentration 'c' in terms of K_a, such that the concentration of the undissociated acid can be equated to 'c' within a 10% limit of error, would be :

(A) 30 K_a **(B)** 50 K_a **(C)** 70 K_a **(D)** 90 K_a

24. Which of the following complex represents correct property indicated with it ? (Given: atomic number of Fe = 26, Ni = 28)

(A) $[Fe(CO)_5]$: square pyramidal **(B)** $[Ni(DMG)_2]$: Colourless

(C) $[Ni(CN)_4]^{2-}$: $\mu = \sqrt{8}$ BM **(D)** $[Fe(CN)_5NO]^{2-}$: d^2sp^3 hybridization

25. Which of the following metal carbide on hydrolysis gives methane?

(A) Mg_2C_3 **(B)** SiC **(C)** Mn_3C **(D)** Na_2C_2

26. Analysis show that nickel oxide consist of nickel ion with 96% ions having d^8 configuration and 4% ions having d^7 configuration. Which among the following best represents the formula of the nickel oxide? (Atomic number of Ni = 28)

(A) $Ni_{1.02}O$ **(B)** $Ni_{0.96}O$ **(C)** $Ni_{0.97}O$ **(D)** $Ni_{0.98}O$

27. For the reaction $N_2(g) + 3H_2(g) \longrightarrow 2NH_3(g)$; $\Delta_r H_{300} = -380$ kcal/mol, (Given : R = 2 cal/mol/K)

If the above reaction is carried out in open container the magnitude of heat evolved is x. If the above reaction is carried out in closed container the magnitude of heat evolved is y.

(A) x = y **(B)** y > x

(C) x > y **(D)** Can't be predicted from given data

28. Pick the correct regarding boiling point :

(A) $H_2 < H - D < D_2 < T_2$ **(B)** $T_2 < D_2 < H_2 < H - D$

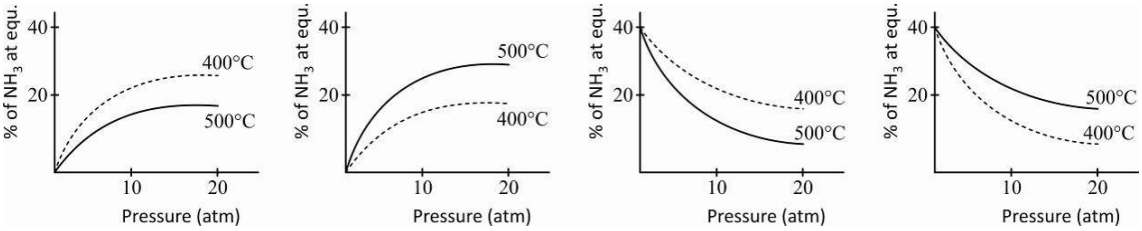
(C) $T_2 < D_2 < H - D < H_2$ **(D)** $H_2 < D_2 < H - D < T_2$

29. One of the allotropic forms of phosphorus, exists as a white waxy soild. It is poisonous, insoluble in water but soluble in carbon disulphide and glows in dark (chemiluminescence's). Now, if this form of phosphorus is exposed to air, it readily catches fire to give dense white fumes of X. Which of the following acid is not formed during stepwise hydrolysis of X ?

(A) Tetrameta-phosphoric acid **(B)** Tetrapolyphosphoric acid

(C) Pyrophosphoric acid **(D)** Hypophosphoric acid

30. The percentage of ammonia obtainable, if equilibrium were to be established during the Haber process, is plotted against the operating pressure for two temperatures, 400°C and 500°C. Which of the following graph correctly represent the two process ?



(A) **(B)** **(C)** **(D)**

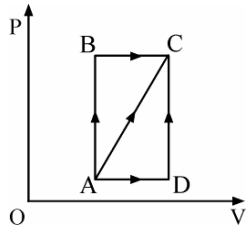
PART - II (PHYSICS) — 120 MARKS

This section contains 30 Multiple Choice Questions. Each Question has 4 choices A, B, C & D, out of which ONLY ONE Choice is Correct:

1. One end of a spring is tied to a wall and the other end moves at a speed *v*. If the mass of the spring is *m* then the kinetic energy of the spring is : (Assume that the speed of a point on the spring increases linearly with distance from O):

(A) $\frac{1}{2}mv^2$ **(B)** $\frac{1}{4}mv^2$ **(C)** $\frac{1}{6}mv^2$ **(D)** $\frac{1}{3}mv^2$

2. A thermodynamic process is shown in Figure. The pressures and volumes corresponding to some points in the figure are:



$P_A = 3 \times 10^4\ Pa$, $P_B = 8 \times 10^4 Pa$ and $V_A = 2 \times 10^{-3} m^3$, $V_D = 5 \times 10^{-3} m^3$

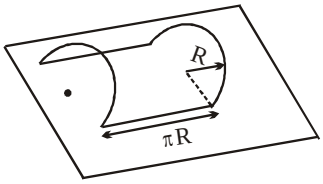
In process *AB*, 600 *J* of heat is added to the system and in process *BC*, 300 *J* of heat is added to the system. The change in internal energy of the system in process *AC* would be :

(A) 660 *J* **(B)** 800 *J* **(C)** 600 *J* **(D)** 640 *J*

3. A wire is bent into the structure as shown in the figure, and placed on a horizontal table. It consists of two half rings of radius R and two straight parts of length πR. The height of COM from the table is :

(A) $\frac{2R}{\pi}$ **(B)** $\frac{R}{\pi}$

(C) $\frac{R}{2}$ **(D)** Zero



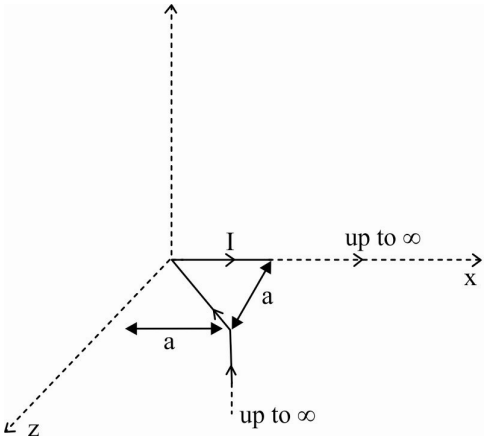
4. A capacitor of capacitance $10\mu F$ is charged to a potential 50 *V* with a battery. The battery is now disconnected and an additional charge 200 μC is given the positive plate of the capacitor. The potential difference across the capacitor will be

(A) 50 *V* **(B)** 80 *V* **(C)** 100 *V* **(D)** 60 *V*

5. Distance between the centres of two spherical stars is 10*a*. The masses of these stars are *M* and 16 *M* and their radii a and 2*a* respectively. A body of mass *m* is fired straight from the surface of the larger star towards the smaller star along the line joining their centres. The minimum initial speed for the body to reach the surface of smaller star is

(A) $\frac{2}{3}\sqrt{\frac{GM}{a}}$ **(B)** $\frac{3}{2}\sqrt{\frac{5GM}{a}}$ **(C)** $\frac{2}{3}\sqrt{\frac{5GM}{a}}$ **(D)** $\frac{3}{2}\sqrt{\frac{GM}{a}}$

6. The magnetic field at the origin due to the current flowing in the wire is



(A) $-\frac{\mu_0 I}{8\pi a}\left(\hat{i}+\hat{k}\right)$ **(B)** $\frac{\mu_0 I}{2\pi a}\left(\hat{i}+\hat{k}\right)$ **(C)** $\frac{\mu_0 I}{8\pi a}\left(-\hat{i}+\hat{k}\right)$ **(D)** $\frac{\mu_0 I}{4\pi a\sqrt{2}}\left(\hat{i}-\hat{k}\right)$

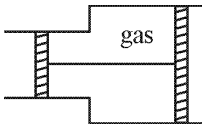
(B)

7. $10gm$ of ice at $-20°C$ is dropped in to a calorimeter containing $10gm$ of water at $10°C$, the specific heat of water is twice that of ice. When equilibrium is reached the calorimeter will contain.

(A) 20 gm of water at $0°C$ **(B)** 15 gm of water at $0°C$

(C) 10 gm ice and 10 gm of water at $0°C$ **(D)** 5gm ice and 15 gm of water at $0°C$

8. A gas is filled in the cylinder shown in the figure. The two pistons are joined by an inextensible string. If the gas is heated, the pistons will :



(A) move towards left **(B)** move towards right

(C) remain stationary **(D)** move away from each other

9. A body is orbiting in a circular orbit with speed V_0 round the earth at a height above the surface equal to the radius of the earth. If the body is stopped suddenly in its orbit and allowed to move freely, then it hits the surface of earth with the speed :

(A) $2V_0$ **(B)** $0.707\ V_0$ **(C)** $1.414\ V_0$ **(D)** $0.5\ V_0$

10. A particle is thrown upwards from ground. The air applies a resistive force which by it self can produce retardation of 2 m/s^2. The ratio of time of ascent to the time of descent is: [g = 10 m/s^2]

(A) 1 : 1 **(B)** $\sqrt{\frac{2}{3}}$ **(C)** $\frac{2}{3}$ **(D)** $\sqrt{\frac{3}{2}}$

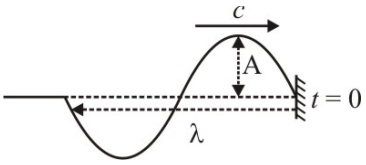
11. Three long thin wires, each carrying current I in the same direction are in the x-y plane of a gravity free-space, mass per unit length of the central wire is λ. The central wire is along y-axis while the other two are along $x = \pm d$. If the central wire is displaced along the z-direction by a small amount and released keeping the other two wires fixed, the frequency of oscillation is :

(A) $\frac{I}{\pi d}\sqrt{\frac{\mu_0}{\lambda}}$ **(B)** $\mu_0 I\sqrt{\pi d\lambda}$ **(C)** $2\pi d\sqrt{\mu_0\lambda}$ **(D)** $\frac{I}{2\pi d}\sqrt{\frac{\mu_0}{\pi\lambda}}$

12. A lift ascends with constant acceleration a, then with constant velocity and finally stops under constant retardation a. If the total distance ascended is h and total time taken is t, the time during which the lift is ascended with constant velocity is :

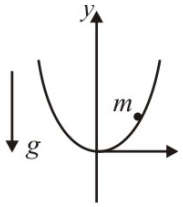
(A) $\sqrt{t^2-\frac{4h}{a}}$ **(B)** $\sqrt{t^2-\frac{8h}{a}}$ **(C)** $\sqrt{t^2-\frac{4h}{3a}}$ **(D)** $\sqrt{t^2-\frac{8h}{3a}}$

13. A sinusoidal wave pulse of amplitude A and wavelength λ, is propagating in a string towards right with speed c. At t = 0, it starts reflecting from the fixed end as shown. Find the velocity of the point which is at a distance of λ/6 left from fixed end in its mean position at instant $t = \frac{\lambda}{6c}$.



(A) $\frac{3}{2}\frac{2\pi cA}{\lambda}$ down **(B)** $\frac{3}{2}\frac{2\pi cA}{\lambda}$ up **(C)** $\frac{1}{2}\frac{2\pi cA}{\lambda}$ down **(D)** $\frac{1}{2}\frac{2\pi cA}{\lambda}$ up

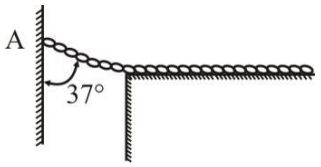
14. A particle of mass m is allowed to oscillate near the minimum of a frictionless vertical parabolic path having the equation $x^2 = 4ay$. The angular frequency of small oscillation is given by :



(A) $\sqrt{\frac{g}{a}}$ **(B)** $\sqrt{2gh}$

(C) $\sqrt{\frac{g}{2a}}$ **(D)** $2\sqrt{\frac{gh}{a}}$

15. A uniform chain of length L has one of its end attached to the wall at point A, while $\frac{3L}{4}$ of the length of the chain is lying on table as shown in figure. The minimum coefficient of friction between table and chain so that chain remains in equilibrium is :

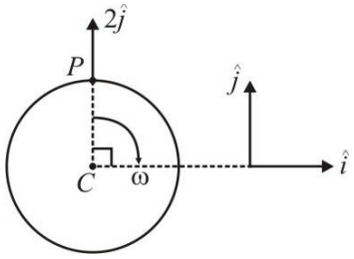


(A) 1/3 **(B)** 1/4 **(C)** 3/4 **(D)** 1/5

16. A ball is released from a point, it goes vertically downwards and collides with a fixed smooth inclined plane of angle of inclination of 30° from the ground, then ball goes horizontally just after collision. Find the coefficient of restitution between the ball and the inclined plane.

(A) $\frac{1}{2}$ **(B)** $\frac{1}{3}$ **(C)** 1 **(D)** $\frac{1}{5}$

17. A disc, having plane parallel to the horizontal is moving such that velocity of point P with respect to ground on its periphery is $2m/s\ \hat{j}$ as shown in the figure. If radius of disc is $R = 1m$ and angular speed of disc about vertical axis passing through disc is $\omega = 2$ rad/s, the velocity of centre of disc in m/s is :



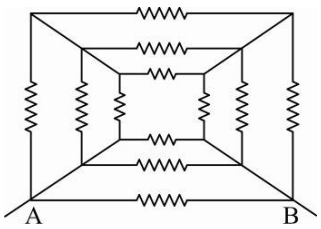
(A) $2\hat{j}$ **(B)** $2\hat{i}+2\hat{j}$

(C) $-2\hat{i}+2\hat{j}$ **(D)** $2\hat{i}-2\hat{j}$

18. A particle is projected from ground towards a vertical wall 80m away at an angle of 37° with horizontal with initial velocity of 50 m/s. After its collision with wall & then once with ground find at what distance from wall will it strike the ground again. (If coefficient of restitution for both collisions is equal to 1/2)

(A) 70 m **(B)** 120 m **(C)** 140 m **(D)** None

19. For the circuit shown below each resistance is of 12Ω. Find the net resistance across A and B.



(A) 4Ω

(B) 3Ω

(C) 16Ω

(D) 2Ω

20. A sphere A moving with speed u and rotating with an angular velocity ω makes a head-on elastic collision with an identical stationary sphere B. There is no friction between the surfaces of A and B. Choose the correct alternative. Disregard gravity.

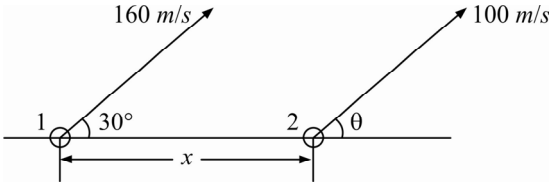
(A) A will stop moving but continue to rotate with an angular velocity ω

(B) A will come to rest and stop rotating ω

(C) B will move with speed u and rotate with $\omega/2$

(D) B will move with speed u and rotate with an angular velocity ω

21. Suppose two particles 1 and 2 are projected in vertical plane simultaneously



Their angles of projection are 30° and θ respectively, with the horizontal. Let they collide after a time t in air. Then which of the following option is INCORRECT ?

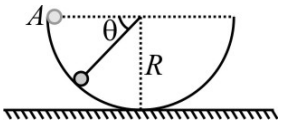
(A) $\theta = \sin^{-1}(4/5)$ and they will have same speed just before the collision

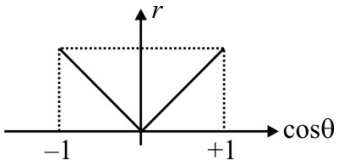
(B) $\theta = \sin^{-1}(4/5)$ and they will have different speed just before the collision

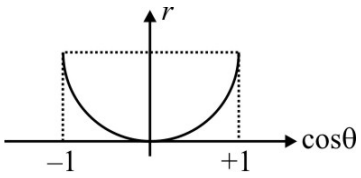
(C) $x < 1280\sqrt{3} - 960$ m

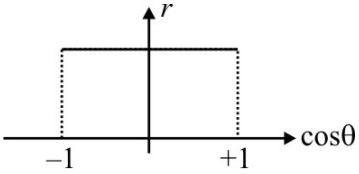
(D) It is possible that the particles collide when both of them are at their highest point

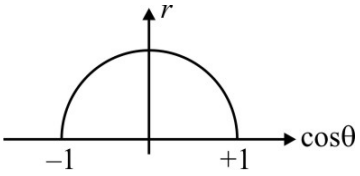
22. A small particle of mass *m* is released from rest from point *A* inside a frictionless fixed hemispherical bowl as shown. The graph between the ratio (*r*) of magnitude of centripetal force and normal reaction on the particle at any point on the bowl as a function of $\cos\theta$ is :



(A)


(B)


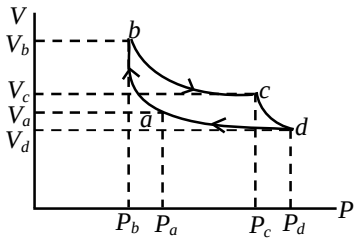
(C)


(D)


23. Volt meter reads the potential difference across the terminals of battery 1.42 volt while a potentiometer reads its voltage to be 1.50 volt. The voltmeter resistance is $280\,\Omega$. Then the internal resistance of the battery is approximately.

(A) 1.233 **(B)** 1.577 **(C)** 12.33 **(D)** 15.77

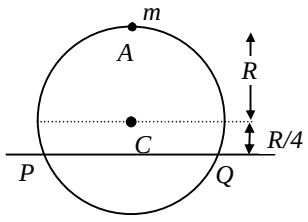
24. A cyclic process *ABCD* shown in *V-P* diagram with two adiabatic and two isothermal process for *n* mole of monoatomic gas is shown in the figure. The ratio of $\frac{P_b}{P_c}$ is :



(A) $\frac{P_a}{P_d}$ **(B)** $\left(\frac{P_a}{P_d}\right)^{\gamma-1}$

(C) $\frac{P_d}{P_a}$ **(D)** $\left(\frac{P_d}{P_a}\right)^{\gamma}$

25. A uniform circular disc has radius *R* and mass *m*. A particle also of mass *m* is fixed at a point *A* on the edge of the disc as shown in figure. The disc can rotate freely about a fixed horizontal chord *PQ* that is at a distance *R*/4 from the centre *C* of the disc. The line *AC* is perpendicular to *PQ*. Initially the disc is held vertical with the point *A* at its highest position. It is then allowed to fall so that it starts rotating about *PQ*. The linear speed of the particle at its lowest position is :



(A) $\sqrt{5gR}$ **(B)** $\sqrt{3gR}$ **(C)** $\sqrt{2.5gR}$ **(D)** $\sqrt{1.5gR}$

26. A galvanometer has a resistance of $20\,\Omega$ and reads full scale when 0.1 *V* is applied across it. To convert it into voltmeter of 10 *V* range, the galvanometer should have a resistance :

(A) 1980 Ω in series **(B)** 2080 Ω in series

(C) 980 Ω in series **(D)** 1980 Ω in parallel

27. A pendulum consists of a wooden bob of mass m_2 and length l. A bullet of m_1 is fired towards the pendulum with a speed V_1. The bullet emerges out of bob with a speed $\frac{V_1}{3}$ and the bob just completes motion along a vertical circle, then V_1 is :

(A) $\left(\frac{m_1}{m_2}\right)\sqrt{5g\ell}$ **(B)** $\frac{3}{2}\left(\frac{m_2}{m_1}\right)\sqrt{5g\ell}$ **(C)** $\frac{2}{3}\left(\frac{m_1}{m_2}\right)\sqrt{5g\ell}$ **(D)** $\left(\frac{m_2}{m_1}\right)\sqrt{5g\ell}$

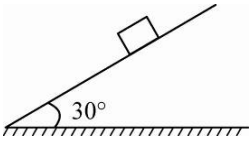
28. 20 *gm* ice at $-10^\circ C$ is mixed with *m gm* steam at $100^\circ C$. The minimum value of *m* so that finally all ice and steam converts into water is : (Use $S_{ice} = 0.5\ cal/gm^\circ C$, $S_{water} = 1\ cal/gm^\circ C$, L (melting) = 80 *cal*/*gm* and L (vaporization) = 540 *cal*/*gm*)

(A) $\frac{185}{27}\ gm$ **(B)** $\frac{135}{17}\ gm$ **(C)** $\frac{85}{32}\ gm$ **(D)** $\frac{113}{17}\ gm$

29. 'A' and 'B' are moving along same straight line in opposite direction, each with speed 4 *m*/*s* with respect to ground. 'A' sees that the rain drops are falling vertically while 'B' sees the rain drops falling at an angle $45°$ with the vertical. The speed of rain w.r.t. ground, is :

(A) $4\sqrt{2}m/s$ **(B)** $4\sqrt{5}ms$ **(C)** $4m/s$ **(D)** $8\sqrt{2}m/s$

30. Figure shows a block of mass 2*kg* kept on a rough inclined plane. The maximum external force down the inclined plane for which the block remains at rest is 2*N*. The coefficient of static friction μ is:



(A) $\frac{2\sqrt{3}}{5}$ **(B)** $\frac{\sqrt{3}}{5}$ **(C)** $\frac{2}{5}$ **(D)** $\frac{4\sqrt{3}}{15}$

PART - III (MATHEMATICS) — 120 MARKS

This section contains 30 Multiple Choice Questions. Each Question has 4 choices A, B, C & D, out of which ONLY ONE Choice is Correct:

1. A real valued function $f(x)$ is given as $f(x) = \begin{cases} \int_0^x 2\{x\}\,dx & x+\{x\} \in I \\ x^2 - x + \frac{1}{2} & \frac{1}{2} < x < \frac{3}{2} \text{ and } x \neq 1 \\ x^2 - \frac{x}{3} + \frac{1}{6} & \text{otherwise} \end{cases}$

Where [.] denotes greatest integer less than or equal to x and {.} denotes fractional part function of x. Which of the following is true.

(A) $f(x)$ is continuous and differentiable in $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

(B) $f(x)$ is continuous and not differentiable in $x \in \left(-\frac{1}{2}, \frac{1}{2}\right)$

(C) $f(x)$ is continuous and differentiable in $x \in \left[\frac{1}{2}, \frac{3}{2}\right]$

(D) $f(x)$ is continuous but not differentiable in $x \in (0, 1)$

2. If $a_1, a_2, a_3, \ldots, a_{2n+1}$ are in A.P., then $\frac{a_{2n+1}-a_1}{a_{2n+1}+a_1}+\frac{a_{2n}-a_2}{a_{2n}+a_2}+\ldots+\frac{a_{n+2}-a_n}{a_{n+2}+a_n}$ is equal to :

(A) $\frac{n(n+1)}{2}\times\frac{a_2-a_1}{a_{n+1}}$ **(B)** $\frac{n(n+1)}{2}$

(C) $(n+1)(a_2-a_1)$ **(D)** None of these

3. Let $x^2+3y^2=3$ be the equation of an ellipse in the *x-y* plane. A and B are two points whose position vectors are $-\sqrt{3}\hat{i}$ and $-\sqrt{3}\hat{i}+2\hat{k}$. Then the position vector of a point P on the ellipse such that $\angle APB=\frac{\pi}{4}$ is :

(A) $\pm\hat{j}$ **(B)** $\pm(\hat{i}+\hat{j})$ **(C)** $\pm\hat{i}$ **(D)** None of these

Paragraph for Questions 4 - 5

Read the following write up carefully and answer the following questions :

Consider two vector $\vec{A}(t)=f(t)\hat{i}+f'(t)\hat{j}$, $\vec{B}(t)=g(t)\hat{i}+g'(t)\hat{j}$, $f(t)$ and $g(t)$ are two continuous function such that $f(0)=-1$, $g(1)=2$, $g'(t)=\left|\lim_{t\to 0}\frac{\sin 2t-a\sin t}{t^3}\right|$, $f(t)=\int_0^t 2x\,dx+c$ then :

4. The value of *t* such that $\vec{A}(t)$ is parallel to $\vec{B}(t)$

(A) –1 **(B)** 1 **(C)** 0 **(D)** 2

5. The value of *gof* (2) is :

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

Paragraph for Questions 6 - 7

If a_{ij} is the element of i^{th} row & j^{th} column of a 3×3 matrix *A* and

(i) If $(i<j)$ then a_{ij} is number of ways of distributing $(j+1)$ identical objects to $(i+1)$ person so that each one may receive none, one or more.

(ii) If $(i>j)$ then a_{ij} is the number of ways of distributing $(i+1)$ distinct objects to $(j+1)$ persons so that each one may receive any number of objects.

(iii) If $i=j$ than $a_{ij}=\max\left[i^{j+1},(i+1)^j\right]$ then,

6. The value of $\left|\frac{det\,(adj\ A)}{12\ det(A)}\right|$ is :

(A) 2 **(B)** 4 **(C)** 6 **(D)** 7

7. Three planes are :

P_1 : $a_{11}x+a_{12}y+a_{13}z=11$ P_2 : $a_{21}x+a_{22}y+a_{23}z=28$

The unit vector in the direction of the line of intersection of P_1 and P_2 is :

(A) $\frac{15\hat{i}+10\hat{j}+14\hat{k}}{\sqrt{521}}$ **(B)** $\frac{15\hat{i}+10\hat{j}-14\hat{k}}{\sqrt{521}}$ **(C)** $\frac{-15\hat{i}+10\hat{j}-14\hat{k}}{\sqrt{521}}$ **(D)** None of these

8. If $A(x_1) = \begin{bmatrix} \cos x_1 & -\sin x_1 & 0 \\ \sin x_1 & \cos x_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $A(x_2) = \begin{bmatrix} \cos x_2 & 0 & \sin x_2 \\ 0 & 1 & 0 \\ -\sin x_2 & 0 & \cos x_2 \end{bmatrix}$, then $(A(x_1) \cdot A(x_2))^{-1}$ is equal to :

(A) $A(x_1)\, A(x_2)$ **(B)** $A(-x_1)\, A(-x_2)$ **(C)** $-A(x_2)\, A(x_1)$ **(D)** $A(-x_2)\, A(-x_1)$

9. Let $f(x)$ be a polynomial one-one function such that $f(x) f(y) + 2 = f(x) + f(y) + f(xy)$, $\forall\, x,\, y \in R - \{0\}$, $f(1) \neq 1$, $f'(1) = 3$. Let $g(x) = \frac{x}{4}\{f(x) + 3\} - \int_0^x f(x)dx$, then :

(A) $g(x) = 0$ has exactly one root for $x \in (0, 1)$

(B) $g(x) = 0$ has exactly two roots for $x \in (0, 1)$

(C) $g(x) \neq 0,\ \forall\, x \in R - \{0\}$ **(D)** $g(x) = 0,\ \forall\, x \in R - \{0\}$

10. If the algebraic sum of perpendicular distance from the points $A(0, -2)$, $B(2, 0)$ and $C(1, 1)$ on a variable straight line is zero, then the line always passes through a fixed point P, which w.r.t. the ΔABC is its

(A) centroid **(B)** incentre **(C)** orthocentre **(D)** circumcentre

11. Let $g(x) = f(x)\sin x$, where $f(x)$ is a twice differentiable function on $(-\infty, \infty)$ such that $f'(-\pi) = 1$. The value of $g''(-\pi)$ equals to :

(A) 1 **(B)** 2 **(C)** –2 **(D)** 0

12. Consider the parabola $x^2 = 8y$, then number of normals that can be drawn from the point $(1, -5)$ to the parabola is :

(A) 1 **(B)** 2 **(C)** 3 **(D)** None of these

13. The area of the triangle whose vertices are the roots of $z^3 + iz^2 + 2i = 0$ is :

(A) $\frac{\sqrt{3}}{4}$ **(B)** $\frac{3}{\sqrt{7}}$ **(C)** $\frac{3\sqrt{7}}{4}$ **(D)** 2

14. There are 10 stations on a circular path. A train has to stop at 4 stations such that no two stations are adjacent. The number of such selections must be :

(A) 25 **(B)** 35 **(C)** 210 **(D)** 50

15. In a triangle ABC, if $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$ then C is equal to

(A) 30° **(B)** 60° **(C)** 75° **(D)** 90°

16. If the eccentric angle of a point lying in the first quadrant on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a > b$) be θ and the line joining centre to point makes angle ϕ with x–axis then $\theta - \phi$ will be maximum when θ is equal to :

(A) $\tan^{-1}\sqrt{\frac{a}{b}}$ **(B)** $\tan^{-1}\sqrt{\frac{b}{a}}$ **(C)** $\frac{\pi}{4}$ **(D)** None of these.

17. If the function $f : [0, 8] \to R$ is differentiable then for $0 < \alpha < 1 < \beta < 2$, $\int_0^8 f(t)dt$ is equal to :

(A) $3[\alpha^3 f(\alpha^2) + \beta^3 f(\beta^2)]$ **(B)** $3[\alpha^3 f(\alpha^2) + \beta^3 f(\beta)]$

(C) $3[\alpha^2 f(\alpha^3) + \beta^2 f(\beta^3)]$ **(D)** $3[\alpha^2 f(\alpha^2) + \beta^2 f(\beta^2)]$

18. If foci of hyperbola lie on $y = x$ and one of the asymptotes is $y = 2x$, then equation of the hyperbola, given that it passes through (3, 4), is :

(A) $x^2 - y^2 - \frac{5}{2}xy + 5 = 0$ **(B)** $2x^2 - 2y^2 + 5xy + 5 = 0$

(C) $2x^2 + 2y^2 - 5xy + 10 = 0$ **(D)** None of these

19. If $g(x)$ is a differentiable real valued function satisfying $g''(x) - 3g'(x) > 3 \ \forall \ x \geq 0$ and $g'(0) = -1$ then $g(x) + x$ for $x > 0$ is :

(A) increasing function of x **(B)** decreasing function of x

(C) data insufficient **(D)** None of these

20. If $\hat{\alpha}$ and $\hat{\beta}$ be two perpendicular unit vectors such that $\vec{x} = \hat{\beta} - (\hat{\alpha} \times \vec{x})$, then $|\vec{x}|$ is equal to

(A) 1 **(B)** $\sqrt{2}$ **(C)** $\frac{1}{\sqrt{2}}$ **(D)** None of these

21. If $f''(x) > 0, \ \forall \ x \in R, f'(3) = 0$ and $g(x) = f(\tan^2 x - 2\tan x + 4), 0 < x < \frac{\pi}{2}$, then $g(x)$ is increasing in

(A) $\left(0, \frac{\pi}{4}\right)$ **(B)** $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ **(C)** $\left(0, \frac{\pi}{3}\right)$ **(D)** None of these

For Questions 22 - 24

(A) Statement-1 is True, Statement-2 is True and Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True and Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

22. **Statement 1 :** If $f(x)$ is a quadratic expression such that $f(1) + f(2) = 0$. If -1 is a root of $f(x) = 0$ then the other root is $\frac{8}{5}$.

Statement 2 : If $f(x) = ax^2 + bx + c$ then $\alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$.

23. **Statement 1 :** The equation $(x-p)(x-r) + \lambda(x-q)(x-s) = 0$ where $p < q < r < s$ has non real roots if $\lambda > 0$.

Statement 2 : The equation $ax^2 + bx + c = 0$ $(a, b, c \in$ real, number) has non real roots if $b^2 - 4ac < 0$.

24. **Statement 1 :** If $x < 0, \tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2}$

Statement 2 : $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, \ \forall \ x \in R$

25. The set of values of m for which it is possible to draw the chord $y = \sqrt{m}x + 1$ to the curve $x^2 + 2xy + \left(2 + \sin^2\alpha\right)y^2 = 1$, which subtends a right angle at the origin for some value of α is :

(A) [2, 3] **(B)** [0, 1] **(C)** [1, 3] **(D)** None of these

26. If $f(x) = \left|x^2 - 3x + 2\right| + \frac{1}{4} \forall \ x \in R$, then the least value of $\frac{f^5(x)+1}{f^2(x)}$ is equal to :

(A) 1 **(B)** $\frac{5}{2}\left(\frac{3}{2}\right)^3$ **(C)** $\frac{5}{3}\left(\frac{3}{2}\right)^{2/5}$ **(D)** $\frac{15}{(108)^{1/5}}$

27. A normal to the hyperbola $\frac{x^2}{4}-\frac{y^2}{1}=1$ has equal intercepts on positive *x*- and *y*-axis. If this normal touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then a^2+b^2 is equal to :

(A) 5 **(B)** 25 **(C)** 16 **(D)** None of these

28. Let *A* be an *n*th order square matrix and *B* be its adjoint, then $\left|AB+KI_n\right|$ is (where *K* is a scalar quantity)

(A) $\left(|A|+K\right)^{n-2}$ **(B)** $\left(|A|+K\right)^{n}$ **(C)** $\left(|A|+K\right)^{n-1}$ **(D)** None of these

29. If *a*, *b*, *c* and *d* are the solutions of the equation $x^4-bx-3=0$, then an equation whose solutions are $\frac{a+b+c}{d^2}, \frac{a+b+d}{c^2}, \frac{a+c+d}{b^2}$, and $\frac{b+c+d}{a^2}$, is :

(A) $3x^4+bx+1=0$ **(B)** $3x^4-bx+1=0$

(C) $3x^4+bx^3-1=0$ **(D)** $3x^4-bx^3-1=0$

30. The solution of differential equation $2x^3y\,dy+\left(1-y^2\right)\left(x^2y^2+y^2-1\right)dx=0$ is :

(A) $x^2y^2=(cx+1)\left(1-y^2\right)$ **(B)** $x^2y^2=(cx+1)\left(1+y^2\right)$

(C) $x^2y^2=(cx-1)\left(1-y^2\right)$ **(D)** $x^2y^2=(cx-1)\left(1+y^2\right)$