FIITJEE

ALL INDIA TEST SERIES

FULL TEST – VII

JEE (Main)-2019

TEST DATE: 24-03-2019

ANSWERS, HINTS & SOLUTIONS

Physics PART – I

SECTION – A

1. D

Sol. At t = 5 sec

$$x_{cm} = 50\cos 53° \times 5 = 150 \text{ m}$$

$$y_{cm} = 50 \sin 53° \times 5 - \frac{1}{2}\times 10\times 5^2 = 75 \text{ m}$$

$$x_{cm} = \frac{m\times 100 + 2m\times x_2}{3\text{ m}} \Rightarrow x_2 = 175 \text{ m}$$

$$y_{cm} = \frac{m\times 50 + 2m\times y_2}{3m} \Rightarrow y_2 = \frac{175}{2} \text{ m}$$

2. B

Sol. ma = – mg – kv^2

$$\Rightarrow v\frac{dv}{dy} = -\left(g + \frac{kv^2}{m}\right)$$

$$\int_0^{y_{max}} dy = -\int_{v_0}^{0} \frac{vdv}{g+\left(\frac{kv^2}{m}\right)}$$

$$y_{max} = \frac{m}{2k}\ell n\left(\frac{g+\frac{kv_0^2}{m}}{g}\right)$$

3. C

Sol. $mv_0\ell = \frac{4}{3}m\ell^2\omega \Rightarrow \omega = \frac{3v_0}{4\ell}$

$\frac{1}{2}\frac{4}{3}m\ell^2\left(\frac{3v_0}{4\ell}\right)^2 = 3mg\frac{\ell}{2} \Rightarrow v_0 = \sqrt{4g\ell}$

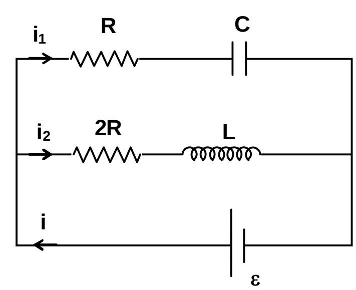
$3mg\frac{\ell}{2} = \frac{4}{3}m\ell^2\alpha \Rightarrow \alpha = \frac{9g}{8\ell}$

$a_{cm} = \alpha r = \frac{9g}{8\ell}\times\frac{3\ell}{4} = \frac{27g}{32}$

$2mg - F = 2m\times\frac{27g}{32} \Rightarrow F = \frac{5mg}{16}$

4. C

Sol. Since $RC = \frac{L}{2R}$



$\Rightarrow$ time constants for both branches are equal.

$i_1 = \frac{\varepsilon}{R}e^{-t/RC}$

$i_2 = \frac{\varepsilon}{2R}\left(1 - e^{-t/RC}\right)$

$i = i_1 + i_2 = \frac{\varepsilon}{2R} + \frac{\varepsilon}{2R}e^{-t/RC}$

$i_{max} = \frac{\varepsilon}{R}$

When, $i = \frac{5\varepsilon}{8R} = \frac{\varepsilon}{2R} + \frac{\varepsilon}{2R}e^{-t/RC}$

$\Rightarrow t = RC\ell n4 = 2RC\ell n2$

$i_1 = i_2$

$\Rightarrow \frac{\varepsilon}{R}e^{-t/RC} = \frac{\varepsilon}{2R}\left(1 - e^{-t/RC}\right)$

$\Rightarrow t = RC\ell n3$

5. B

Sol. $f_P = \frac{3}{4\ell_1}\sqrt{\frac{\gamma_1 RT_1}{M_1}}$, $f_Q = \frac{3}{2\ell_2}\sqrt{\frac{\gamma_2 RT_2}{M_2}}$

$\frac{f_P}{f_Q} = \frac{\ell_2}{2\ell_1}\sqrt{\frac{\gamma_1}{\gamma_2}\times\frac{T_1}{T_2}\times\frac{M_2}{M_1}} = \sqrt{\frac{7}{5}\times\frac{3}{5}\times\frac{300}{350}\times\frac{4}{2}} = \frac{6}{5}$

6. C

Sol. Let final pressure in both parts of the gas be P.

For left part $P_1V_0^{5/3} = PV_1^{5/3} \Rightarrow V_1 = \left(\frac{P_1}{P}V_0^{5/3}\right)^{3/5}$

For right part $P_2V_0^{5/3} = PV_2^{5/3} \Rightarrow V_2 = \left(\frac{P_2}{P}V_0^{5/3}\right)^{3/5}$

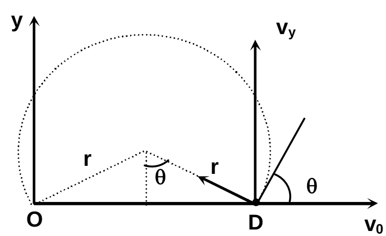
$V_1 + V_2 = 2V_0$

$$\Rightarrow \frac{\left(P_1^{3/5}+P_2^{3/5}\right)V_0}{P^{3/5}} = 2V_0$$

$$\Rightarrow P = \frac{\left(P_1^{3/5}+P_2^{3/5}\right)^{5/3}}{2^{5/3}}$$

7. B

Sol. When the particle strike at D



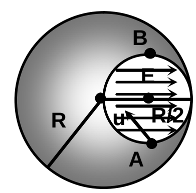
$$v_y = \sqrt{2\frac{qE}{m}\frac{d}{2}} = \sqrt{\frac{qEd}{m}}$$

$$d = 2r\sin\theta = 2\frac{mv}{qB}\sin\theta = \frac{2m}{qB}\sqrt{\frac{qEd}{m}}$$

$$\Rightarrow B = 2\sqrt{\frac{mE}{qd}}$$

8. A

Sol. Electric field inside the cavity is uniform and its magnitude



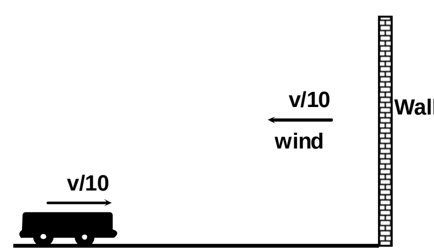
$$E = = \frac{\rho r}{2\varepsilon_0} = \frac{\rho\frac{R}{2}}{2\varepsilon_0} = \frac{\rho R}{4\varepsilon_0}.$$

The direction of the field is as shown in the figure.
If the particle hits at B

$$R = \frac{u^2}{\frac{qE}{m}} = \frac{mu^2}{\frac{q\rho R}{4\varepsilon_0}} \quad \Rightarrow u = \sqrt{\frac{q\rho R^2}{4\varepsilon_0 m}}$$

9. B

Sol. Frequency of sound reaching to the wall



$$f_w = \frac{v_{eff}}{v_{eff}-v_s}f = \frac{\frac{9v}{10}}{\frac{9v}{10}-\frac{v}{10}}f = \frac{9f}{8}$$

Frequency of echo heard by the driver

$$f' = \frac{f_{eff}+v_0}{v_{eff}}f_w = \frac{\frac{11}{10}v+\frac{v}{10}}{\frac{11v}{10}} \times \frac{9f}{8} = \frac{27}{22}f$$

10. A

Sol. The mean time between successive collision of an ideal gas

$$\tau = \frac{1}{(\sqrt{2}n\pi d^2)v_{rms}} \propto \frac{V}{\sqrt{T}}$$

In isobaric process $\frac{V}{T} = \text{constant}$

$$\Rightarrow \tau' = \sqrt{2}\tau$$

11. B

Sol. $B = \frac{\mu_0 I}{2\pi r}$, the net magnetic field at any point on the line xx′ will be the vector sum of magnetic fields due to the three current carrying wires.

12. C

Sol. Least count = $\frac{0.5}{50} = 0.01\,mm$

Zero error = –(50 – 48) × 0.01 mm = – 0.02 mm
Measured thickness = 1 mm + 6 × 0.01 mm = 1.06 mm
Actual thickness = 1.06 mm – (–0.02 mm) = 1.08 mm

13. D

Sol. $\frac{1}{2}\varepsilon_0 E^2 \times \ell^3 =$ energy density in electric field × volume

$\frac{1}{2}LI^2 =$ energy stored in inductor

14. B

Sol. Comparing with damped SHM the amplitude of charge q_m on the capacitor as a function of time t will be

$q_m = q_0 e^{-Rt/2L}$

When $q_m = \frac{q_0}{2} \Rightarrow t = \frac{2L}{R}\ell n2$

15. B

Sol. $P = \frac{4\pi R^2 T}{t}$

16. D

Sol. $\frac{k_x}{k_y} = \frac{p_x^2 / 2m_x}{p_y^2 / 2m_y} = \frac{m_y}{m_x} = \frac{1}{2}$

$\frac{r_x}{r_y} = \left(\frac{A_x}{A_y}\right)^{1/3} = \left(\frac{m_x}{m_y}\right)^{1/3} = 2^{1/3}$

17. B

Sol. F = $I\ell B = \frac{B\ell v}{R}\left(e^{-t/RC}\right)\ell B = \frac{B^2\ell^2 v}{R}e^{-t/RC}$

18. B

Sol. $\frac{0.1\times P}{4\pi r^2} = \frac{B_0^2}{2\mu_0}\times c$, where c is speed of light.

$\Rightarrow P = \frac{4\pi r^2 B_0^2 C}{0.1\times 8\pi\times 10^{-7}} = \frac{r^2 B_0^2 C}{0.1\times 2\times 10^{-7}}$ = 216 W

19. A

Sol. Use Malus' Law

20. C

Sol. $T = 2\pi\sqrt{\frac{I}{MB_H}} = 2\pi\sqrt{\frac{I}{MB\cos\theta}}$

$\Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{B_2\cos\theta_2}{B_1\cos\theta_1}}$

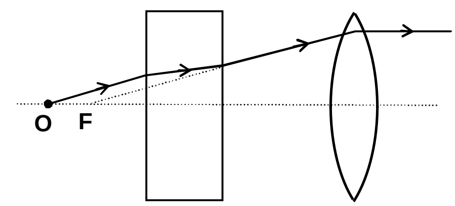
$\Rightarrow \frac{3}{4} = \sqrt{\frac{B_2\cos 60^\circ}{B_1\cos 30^\circ}}$

$\frac{B_1}{B_2} = \frac{16}{9\sqrt{3}}$

21. B

Sol. $25 - t\left(1 - \frac{1}{\mu}\right) = 24$

$\Rightarrow t = 3$ cm



22. C

Sol. For total internal reflection on surface AC

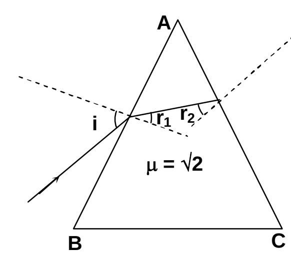
$r_2 > \theta_C$

$\Rightarrow 60^\circ - r_1 > \theta_C$

$\Rightarrow \sin r_1 < \sin\left(60^\circ - \sin^{-1}\frac{1}{\sqrt{2}}\right)$

$\Rightarrow \frac{\sin i}{\sqrt{2}} < \sin 15^\circ$

$\Rightarrow i < \sin^{-1}(0.366)$



23. A

Sol. From conservation of momentum

$\vec{p}_H + \vec{p}_{photon} = 0$

$|\vec{p}_H| = |\vec{p}_{photon}| = \frac{h}{\lambda} = hR\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3Rh}{4}$

$v_H = \frac{3Rh}{4M}$

24. D

Sol. Energy of photon of 400 nm wavelength = $\frac{1240}{400} = 3.1\,\text{eV}$

Energy of photon of 310 nm wavelength = $\frac{1240}{310} = 4\,\text{eV}$

$h\nu = \phi + k_{max} \Rightarrow 3.1 = \phi + k_{max}$

$4 = \phi + 2k_{max}$

$\Rightarrow \phi = 2.2$ eV

25. B

Sol. $-\frac{dT}{dt} = \frac{e\sigma A}{ms}\left(T^4 - T_0^4\right)$, Also $\frac{4}{3}\pi r^3 = a^3$

$$\frac{-dT_1/dt}{-dT_2/dt} = \frac{A_1}{A_2} = \frac{4\pi r^2}{6a^2} = \left(\frac{\pi}{6}\right)^{1/3}$$

26. B

Sol. $\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3 \Rightarrow R = n^{1/3}r$ (R = radius of bigger drop)

$n \times 4\pi r^2 T - 4\pi R^2 T = \frac{1}{2}mv^2$

$\Rightarrow n \times 4\pi r^2 T - 4\pi n^{2/3} r^2 T = \frac{1}{2} n \times \frac{4}{3}\pi r^3 \rho v^2 \quad \Rightarrow v = \sqrt{\frac{6T}{r\rho}\left(1 - \frac{1}{n^{1/3}}\right)}$

27. A

Sol. $T^2 \propto r^3$

$\frac{T'^2}{T^2} = \frac{r'^3}{r^3} \Rightarrow \left(\frac{2T}{T}\right)^2 = \frac{r'^3}{r^3} \Rightarrow r = 4^{1/3} r$

28. D

Sol. Side band frequencies are $f_C + f_m$ and $f_C - f_m$
i.e., 2060 Hz and 1940 Hz

The modulation index = $\frac{A_m}{A_c} = \frac{8}{20} = 0.4$

29. A

Sol. When v = maximum
a = 0

$\Rightarrow F - kxmg = 0 \Rightarrow x = \frac{F}{kmg}$

$W_F - W_f = \frac{1}{2}mv^2$

$\Rightarrow Fx - kmg\frac{x^2}{2} = \frac{1}{2}mv^2$

$\Rightarrow F\frac{F}{kmg} - kmg\frac{F^2}{2k^2m^2g^2} = \frac{1}{2}mv_{max}^2 \quad \Rightarrow v_{max} = \sqrt{\frac{F^2}{km^2g}}$

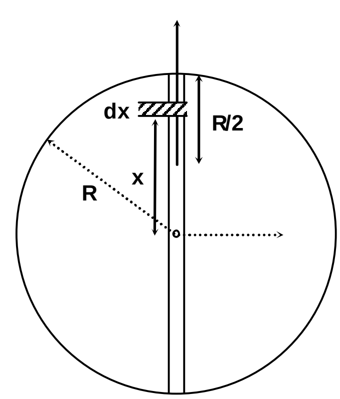
30. D

Sol. $U_i = \int (dm)v = \int_{R/2}^{R} -\frac{m}{R/2}dx\frac{GM}{2R}\left(3 - \frac{x^2}{R^2}\right)$

$= \frac{29GMm}{24R} = \frac{29gR^2m}{24} = \frac{29mgR}{24}$

$U_f = -\frac{GMm}{R} = -\frac{gR^2m}{R} = -mgR$

$W = \Delta U = U_f - U_i = \frac{5mgR}{24}$



Chemistry

PART – II

SECTION – A

31. B

Sol. Smaller be the size of the cation, lattice energy of the ionic carbonates decreases, and then thermal stability decreases.

32. A

Sol. $Ba(OH)_2(s) \rightleftharpoons Ba^{2+}(aq) + 2\overset{-}{O}H(aq)$

pH = 12

pOH = 2

$\therefore \left[\overset{-}{O}H\right] = 10^{-2}$

$\left[Ba^{2+}\right] = \frac{10^{-2}}{2} = 5 \times 10^{-3}$

$K_{sp} = \left[Ba^{2+}\right]\left[\overset{-}{O}H\right]^2 = 5 \times 10^{-3} \times \left(10^{-2}\right)^2$

$= 5 \times 10^{-7}$

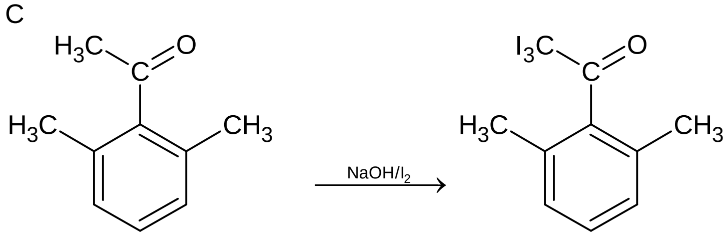
33. C

Sol. Borazine is non polar due to symmetrical distribution of charge.

$B_2O_3 + P_2O_5 \rightarrow 2BPO_4$

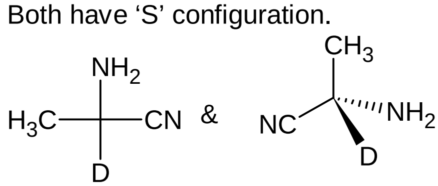
34. C

Sol.



35. A

Sol. Both have 'S' configuration.



36. C

Sol. Factual

37. B

38. B

Sol. I. $n + \ell = 4 + 1 = 5$

II. $n + \ell = 4 + 0 = 4$

III. $n + \ell = 3 + 2 = 5$

IV. $n + \ell = 3 + 1 = 4$

I is having greater n value as compared to III, II is having greater n value as compared to IV.

39. D
Sol. Since there is stronger intermolecular force of attraction present in Cl_2, it is easily liquifiable.

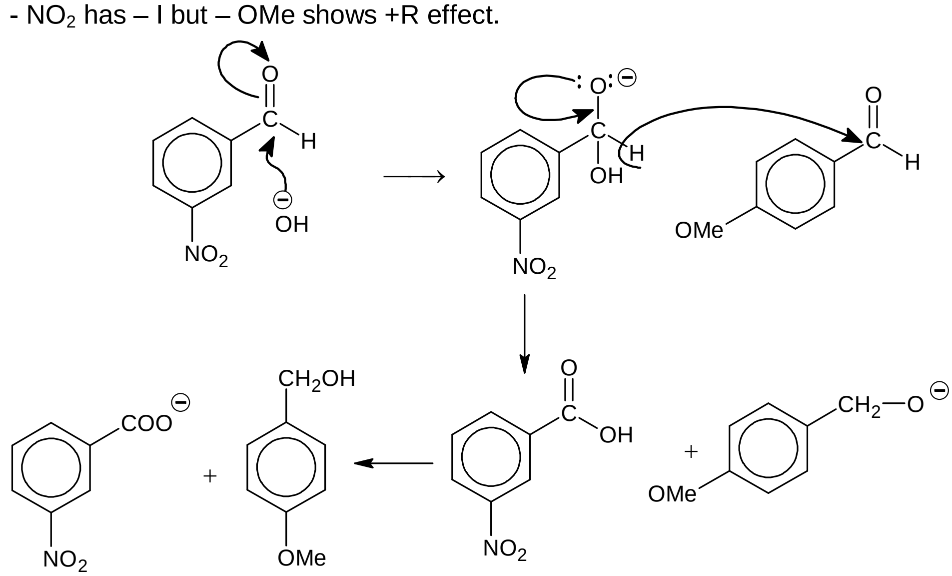
40. D
Sol. The compound must be cyclic and contains a – OH and – CHO group. It must contain seven carbon atoms.

41. C
Sol. $$Zn \xrightarrow[HNO_3]{diluted} Zn(NO_3)_2 \xrightarrow[NaOH]{aq.} \underset{\text{White precipitate}}{Zn(OH)_2} \xrightarrow[NaOH]{excess} Na_2[Zn(OH)_4] \xrightarrow{H_2S} \underset{\text{(white ppt.)}}{ZnS}\downarrow$$

42. B
Sol. $$E_{cell} = -\frac{0.059}{2}\log\frac{[M^{2+}]}{[0.001]}$$
$$[M^{2+}] = 10^{-5}$$
$$MX_2(s) \rightleftharpoons M^{2+} + 2X^-$$
$$[X^-] = 2\times10^{-5}$$
$$K_{sp} == [M^{2+}][X^-]^2$$
$$= 10^{-5}\times(2\times10^{-5})^2$$
$$= 4\times10^{-15}$$

43. A
Sol. $M_{eqv.}$ of CaO = $M_{eqv.}$ of CaC_2O_4 = $M_{eqv.}$ Of $KMnO_4$
Let in line mass of CaO = w
$$\frac{w}{56/2}\times1000 = 40\times0.25$$
w = 0.28
$$\% \text{ of CaO} = \frac{0.28\times100}{0.518} = 54\%$$

44. A
Sol. - NO_2 has – I but – OMe shows +R effect.



45. C

Sol. In $\left[V(CO)_6\right]^-$, tendency of back donation of electron to π^* orbital of CO is greater, thus, V – C bond length decreases.

46. A

Sol. +R effect of – OCH_3 group.

47. B

Sol. $Ba(NO_3)_2 + Na_2SO_4 \longrightarrow 2NaNO_3 + BaSO_4 \downarrow$

48. B

Sol. Formula of the complex Ba(x – 2) $[Co(CN)_x]_2$.

$$Ba_{(x-2)}\left[Co(CN)_x\right]_2 \rightleftharpoons (x-2)Ba^{2+} + 2\left[Co(CN)_x\right]^{(x-2)-}$$

$$1-0.75 \qquad 0.75(x-2) \qquad 0.75\times 2$$

i = 4

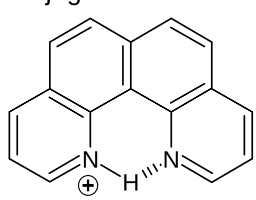
0.25 + 0.75 (x – 2) + 1.5 = 4

i.e. x = 5.

49. B

Sol. (i) Lone pair of N on P is in conjugation.

Conjugate acid of R can be stablised through intramoleuclar H-bonding.



50. B

Sol. ICl_2^- & CO_2 both are linear.

51. C

Sol. Two – NO_2 groups are at ortho and para position with respect to – F. Due to – I effect of F, the intermediate is more stabilized.

52. C

Sol. At Boyle temperature, Z = 1

Thus, $0.34\,P - \dfrac{160P}{T_B} = 0$

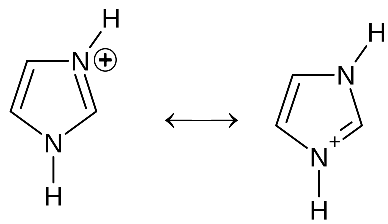
$$T_B = \frac{160}{0.34} = 470.58\,K.$$

53. C

Sol. Gabriel's phthalimide synthesis takes place through S_N2 pathway which is not possible in bridgehead position.

54. B

55. C

Sol. 

Conjugate acid is stabilized by equivalent resonating structure.

56. B

Sol. $dV = \left(\frac{\partial U}{\partial T}\right)_v dT + \left(\frac{\partial U}{\partial V}\right)_T dV$

For isothermal process, dT = 0

$dV = \left(\frac{\partial U}{\partial T}\right)_T dV$

For real gas, $\left(\frac{\partial U}{\partial T}\right)_T \neq 0.$

57. C

Sol. For a 'n' the order reaction,

$t_{1/2} \propto \frac{1}{a^{n-1}}$ where a = initial conc.

$\left(\frac{5\times10^{-3}}{25\times10^{-4}}\right)^{n-1} = \frac{8}{1}$

$\Rightarrow n = 4$

$\frac{t_{1/2}}{1} = \left(\frac{5\times10^{-3}}{1.25\times10^{-3}}\right)^{n-1}$

$t_{1/2} = 64 \text{ hr}$

58. C

Sol. Factual

59. C

Sol. The reaction takes place through the formation of free radical.

60. D

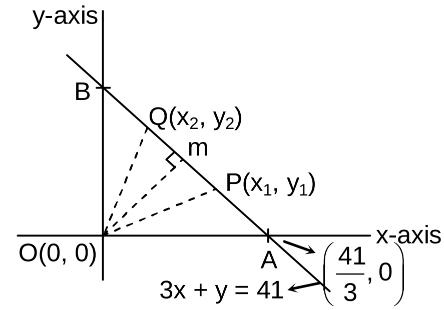
Sol. It is highly insoluble ($K_{sp} \approx 10^{-52}$)

Mathematics

PART – III

SECTION – A

61. B



Sol. $OM = \left|\frac{3.0 + 0 - 41}{\sqrt{9+1}}\right| = \frac{41}{\sqrt{10}}$

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\because$ P, Q lies on 3x + y = 41

So, $3x_2 + y_2 = 41$

$3x_1 + y_1 = 41$

$3(x_2 - x_1) + (y_2 - y_1) = 0$

$\Rightarrow (y_2 - y_1) = -3(x_2 - x_1)$

So, $PQ = \sqrt{(x_2 - x_1)^2 + 9(x_2 - x_1)^2} = \sqrt{10}|x_2 - x_1|$

Area of $\Delta OPQ = \frac{1}{2} \times PQ \times OM = \frac{1}{2} \times \sqrt{10}|x_2 - x_1| \times \frac{41}{\sqrt{10}} = \frac{41}{2}|x_2 - x_1|$

Since x_1, x_2 are integer

So, $|x_2 - x_1|$ must be even integer

Possible number of x_1 and x_2 are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13

So, there are 7 even and 7 odd numbers

So, $|x_2 - x_1|$ will be even if both are even or odd

So, number of ways of solution = ${}^7C_2 + {}^7C_2 = 2\,{}^7C_2 = 2 \times 21 = 42$

62. C

Sol. Length of direct common tangent = $\sqrt{d^2 - (r_1 - r_2)^2}$

Length of transverse common tangent = $\sqrt{d^2 - (r_1 + r_2)^2}$

Where d is distance between their centers

Then $\left(\sqrt{d^2 - (r_1 - r_2)^2}\right) = 2\left(\sqrt{d^2 - (r_1 + r_2)^2}\right)$

$d^2 = \left(r_1^2 + r_2^2 + \frac{10}{3}r_1r_2\right)$, where $d = 3r_1 - r_2$

$(3r_1 - r_2)^2 = r_1^2 + r_2^2 + \frac{10}{3}r_1r_2$

$r_1 : r_2 = 7 : 6$

63. A

Sol. Given equation of parabola $x^2 = 3ax + 2ay + 3$ where a is variable on solving

$\left(x - \frac{3a}{2}\right)^2 = 2a\left(y + \frac{12 + 9a^2}{8a}\right)$, vertex $v\left(\frac{3a}{2}, -\frac{(12 + 9a^2)}{8a}\right)$

Let v(h, k)

$h = \frac{3a}{2}$, $k = -\frac{(12 + 9a^2)}{8a}$

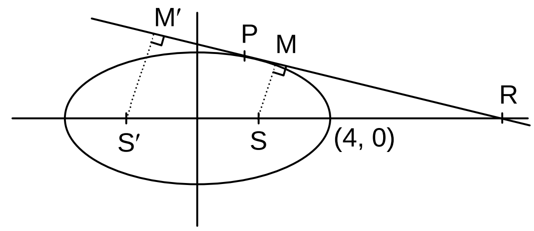
$3h^2 + 4hk + 9 = 0$

So, locus is $3x^2 + 4xy + 9 = 0$

64. D

Sol. We know that SM·S′M′ = b^2 ..... (1)

and ΔRMS ~ ΔRM′S′



So, $\frac{RS}{RS'} = \frac{MS}{M'S'} = \frac{2}{3}$

MS = $\frac{2}{3}M'S'$

$\frac{2}{3}(M'S')^2 = 8$

$(M'S')^2 = \frac{8\times 3}{2} = 4\times 3$

$M'S' = 2\sqrt{3}$

$MS = \frac{2}{3}\times 2\sqrt{3} = \frac{4}{\sqrt{3}}$

65. B

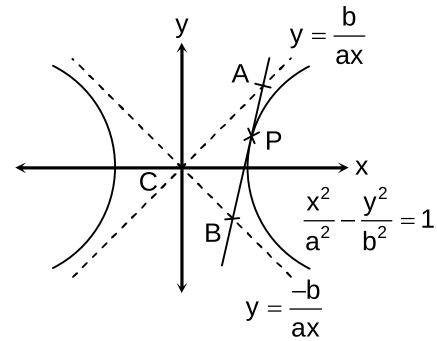
Sol. Area of ΔABC = ab

Where a = 3

b = 2

= 3 × 2

= 6 sq. units



66. A

Sol. $= \sum_{r=1}^{n} \frac{\left(1^3 + 2^3 + 3^3 + 4^3 + 5^3 + ... + (2r)^3\right) - \left(2^3 + 4^3 + 6^3 + 8^3 + ... + (2r)^3\right)}{1 + 3 + 5 + ... + (2r-1)}$

$= \sum_{r=1}^{n} \frac{\frac{4r^2(2r+1)^2}{4} - 8\left(\frac{r^2(r+1)^2}{4}\right)}{r^2}$

$= \sum_{r=1}^{n} (2r+1)^2 - 2(r+1)^2$

$= \sum_{r=1}^{n} \left(4r^2 + 1 + 4r - 2r^2 - 2 - 4r\right)$

$= \sum_{r=1}^{n} \left(2r^2 - 1\right) = 2\sum_{r=1}^{n} r^2 - \sum_{r=1}^{n} 1$

$= \frac{2n(n+1)(2n+1)}{6} - n$ where n = 60

$= \frac{50\times 51\times 101}{3} - 50 = 50\times 17\times 101 - 50$

= 50(17 17 – 1) = 50 × 17 16

= 85800

67. B

Sol. $x^2 + ax + 10 = 0 \Rightarrow 2x^2 + 2ax + 20 = 0$ ..... (1)

$x^2 + bx + 8 = 0 \Rightarrow 3x^2 + 3bx + 24 = 0$ ..... (2)

Adding equation (1) and (2), we get

$5x^2 + (2a + 3b)x + 44 = 0$ ..... (3)

$x^2 + (2a + 3b)x + 60 = 0$ ..... (4)

Subtracting equation (4) from (3), we get

$4x^2 = 16$, $x = \pm 2$, a = 7, b = 6 or a = –7, b = –6

So, |a – b| = 1

68. C

Sol. Let z = x + iy

$x + iy + \sqrt{x^2 + y^2} = 1 + 7i$

$x + \sqrt{x^2 + y^2} = 1$ and y = 7

$\Rightarrow \sqrt{x^2 + y^2} = 1 - x \Rightarrow x^2 + y^2 = 1 + x^2 - 2x$

$y^2 = 1 - 2x$

49 – 1 = –2x

x = –24

$|z|^2 = x^2 + y^2 = 576 + 49 = 625$

|z| = 25

$\sqrt{|z + \bar{z}|^2 + |z - \bar{z}|^2} = \sqrt{(1^2 + 1^2)(|z|^2 + |\bar{z}|^2)} = \sqrt{2 \times 2 \times 625} = \sqrt{4 \times 625} = 2 \times 25 = 50$

$\sqrt{|z + \bar{z}|^2 + |z - \bar{z}|^2} = 50$

69. D

Sol. Using formula ${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$

${}^{18}C_{r-2} + {}^{18}C_{r-1} + {}^{18}C_{r-1} + {}^{18}C_r \ge {}^{20}C_{13}$

${}^{19}C_{r-1} + {}^{19}C_r \ge {}^{20}C_{13}$

${}^{20}C_r \ge {}^{20}C_{13}$ or ${}^{20}C_{20-r} \ge {}^{20}C_{13}$

r = 7, 8, 9, 10, 11, 12, 13

Sum of all values of r = 70

70. D

Sol. A – 3 numbers   A – 3 numbers, B – 3 numbers

B – 3 numbers   A

– –

Number of palindromes are $\frac{6!}{3!3!} = \frac{6 \times 5 \times 4 \times 3!}{6 \times 3!} = 20$

Palindrome has same sequence from starting and from ending

71. A

Sol. There will be n heads and n tails in 2n throws

Probability = $\frac{2n!}{n!n!}\left(\frac{1}{2}\right)^n\left(\frac{1}{2}\right)^n = \frac{{}^{2n}C_n}{2^{2n}}$

Option (A) = $\prod_{r=1}^{n}\left(\frac{2r-1}{2r}\right) = \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}\cdot\frac{7}{8}.....\frac{2n-1}{2n} = \frac{(2n)!}{2^{2n}(n!)^2} = \frac{{}^{2n}C_n}{2^{2n}}$

72. C

Sol. $\sim(\sim p \wedge q) \wedge (p \vee q)$

$\equiv [\sim(\sim p) \vee (\sim p)] \wedge (p \vee q)$

$\equiv [p \vee \sim q] \wedge (p \vee q)$

$\equiv p \vee (\sim q \wedge q)$

$\equiv p \vee f$

$\equiv p$

73. B

Sol. Variance = $\dfrac{1^2+3^2+5^2+.....+(199)^2}{100} - \left(\dfrac{1+3+5+.....+199}{100}\right)^2$

Sum = $\sum_{n=1}^{n}(2n-1)^2$ = $4\Sigma n^2 - 4\Sigma n + n$ = $\dfrac{4n(n+1)(2n+1)}{6} - \dfrac{4n(n+1)}{2} + n$

Put n = 100

= $\dfrac{4\times100\times101\times201}{6} - 2\times100\times101+100$

= $100\left(\dfrac{2\times101\times201}{3} - 2\times101+1\right)$ = 100(2 × 6767 – 202 + 1)

= 100(13534 – 201) = 100 × 13333 = 1333300

Variance = $\dfrac{1333300}{100} - (100)^2 = 13333 - 10000 = 3333$

74. C

Sol. $\dfrac{\cos n\theta\cdot(\sec\theta)^n - \cos(n-1)\theta\cdot\sec^{n-1}\theta}{\sin(n-1)\theta\cdot\sec^{n-1}\theta} + \dfrac{1}{n}\tan(n\theta)$

= $\dfrac{\dfrac{\cos n\theta}{\cos\theta} - \cos(n-1)\theta}{\sin(n-1)\theta} + \dfrac{1}{n}\tan(n\theta)$

= $\dfrac{\cos n\theta - \cos\theta(\cos(n-1)\theta)}{\cos\theta\cdot\sin(n-1)\theta} + \dfrac{1}{n}\tan(n\theta)$

= $\dfrac{\cos(n-1+1)\theta - \cos\theta\cdot\cos(n-1)\theta}{\cos\theta\cdot\sin(n-1)\theta} + \dfrac{1}{n}\tan(n\theta)$

= $\dfrac{\cos(n-1)\theta\cos\theta - \sin(n-1)\theta\sin\theta - \cos\theta\cos(n-1)\theta}{\cos\theta\cdot\sin((n-1)\theta)} + \dfrac{1}{n}\tan(n\theta)$

= $-\tan\theta + \dfrac{1}{n}\tan(n\theta)$

75. C

Sol. $\dfrac{\sqrt{5}-1}{4}\times\dfrac{1}{\sin x} + \dfrac{\sqrt{10+2\sqrt{5}}}{4}\times\dfrac{1}{\cos x} = 2$

$\dfrac{\sin\frac{\pi}{10}}{\sin x} + \dfrac{\cos\frac{\pi}{10}}{\cos x} = 2$

$\sin x\cdot\cos\dfrac{\pi}{10} + \cos x\cdot\sin\dfrac{\pi}{10} = 2\sin x\cos x$

$\sin\left(x+\frac{\pi}{10}\right)=\sin 2x$

$x+\frac{\pi}{10}=2x \Rightarrow x=\frac{\pi}{10}$

or $x+\frac{\pi}{10}=\pi-2x$

$3x=\frac{9\pi}{10}$

$x=\frac{3\pi}{10}$

76. C

Sol. Let $\sin^{-1}(\log_2 x)=p$, $\cos^{-1}(\log_2 y)=q$

So, $3p+q=\frac{\pi}{2}$ ..... (1)

$p+2q=\frac{11\pi}{6}$ ..... (2)

$p=\frac{11\pi}{6}-2q$

$3\left(\frac{11\pi}{6}-2q\right)+q=\frac{\pi}{2}$

$\frac{11\pi}{2}-6q+q=\frac{\pi}{2}$

$\frac{11\pi}{2}-\frac{\pi}{2}=5q$

$5q=\frac{10\pi}{2}$

$q=\pi$

$p=\frac{11\pi}{6}-2\pi=-\frac{\pi}{6}$

$\sin^{-1}(\log_2 x)=-\frac{\pi}{6}$

$\log_2 x=-\sin\frac{\pi}{6}=-\frac{1}{2}$

$x=2^{-\frac{1}{2}} \Rightarrow \frac{1}{x}=2^{\frac{1}{2}} \Rightarrow \frac{1}{x^2}=2$

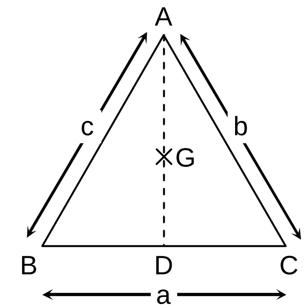
$\cos^{-1}(\log_2 y)=\pi$

$\log_2 y=\cos\pi=-1$, $y=\frac{1}{2}$, $\frac{1}{y}=2$

$\frac{1}{x^2}+\frac{1}{y^2}=2+4 \Rightarrow \frac{1}{x^2}+\frac{1}{y^2}=6$

77. A

Sol. $AB^2 + AC^2 = 2(BD^2 + AD^2)$



$$c^2 + b^2 = 2\left(\frac{a^2}{4} + AD^2\right)$$

$$b^2 + c^2 - \frac{a^2}{2} = 2AD^2$$

$$b^2 + c^2 - \frac{a^2}{2} = 2AD^2$$

$$2AD^2 = \frac{2b^2 + 2c^2 - a^2}{2}$$

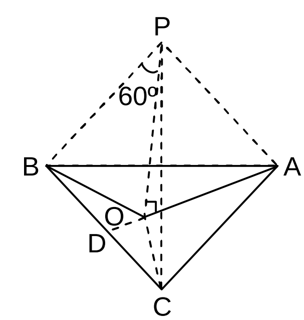
$$AD^2 = \frac{2b^2 + 2c^2 - a^2}{4}$$

$$\ell = AD = \frac{\sqrt{2b^2 + 2c^2 - a^2}}{2}$$

$$4\ell^2 = 2b^2 + 2c^2 - a^2$$

78. B

Sol. OP = h



ΔPOA

$\angle POA = 90^o$

$AP^2 = OP^2 + OA^2$

PA = PB = PC = a

$$a^2 = h^2 + \frac{a^2}{3}$$

$$h^2 = \frac{2a^2}{3},\ 3h^2 = 2a^2$$

$$AD^2 = BA^2 - BD^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$$

$$AD = \frac{a\sqrt{3}}{2};\ OA = \frac{2}{3}AD = \frac{2}{3} \times \frac{a\sqrt{3}}{2}$$

$$OA = \frac{a}{\sqrt{3}}$$

79. D

Sol. $x - ay + az = 0$ ..... (1)

$bz - bx - y = 0$ ..... (2)

$cx - cy - z = 0$ ..... (3)

Since equation are consistent

$$\text{So, } \begin{vmatrix} 1 & -a & a \\ -b & -1 & b \\ c & -c & -1 \end{vmatrix} = 0$$

$1 + bc + a(b - bc) + a(bc + c) = 0$

$1 + bc + ab - abc + abc + ac = 0$

$ab + bc + ca = -1$

80. C

Sol. Trace A = $x^2 + y^2 + z^2$

Trace B = $2x + 2y + 2z - 3$

$x^2 + y^2 + z^2 = 2x + 2y + 2z - 3$
$x^2 + y^2 + z^2 - 2x - 2y - 2z + 3 = 0$
$(x - 1)^2 + (y - 1)^2 + (z - 1)^2 = 0$
It is true if x = 1, y = 1, z = 1
So, x + y + z = 3

81. B

Sol. Since plane ax – by + cz = 0 contains the line $\frac{x-a}{a} = \frac{y-2d}{b} = \frac{z-c}{c}$
So, point (a, 2d, c) will satisfy the plane and normal of plane will 90º with line
So, $a \cdot a - b \cdot 2d + c^2 = 0$
$c^2 + a^2 = 2bd$ ..... (1)
and $a^2 - b^2 + c^2 = 0$ since angle between normal of plane and line is 90º
$b^2 = a^2 + c^2$ ..... (2)
From equation (1) and (2), we get
$b^2 = 2bd$
b = 2d
$\frac{b}{d} = 2$

82. D

Sol. $|\vec{a}| = |\vec{b}| = 1$

$|\vec{a} + \vec{b}| = \sqrt{3} \Rightarrow |\vec{a}|^2 + |\vec{b}| + 2\vec{a} \cdot \vec{b} = 3$

$1 + 1 + 2\vec{a} \cdot \vec{b} = 3$

$2\vec{a} \cdot \vec{b} = 1$

$\vec{a} \cdot \vec{b} = \frac{1}{2}$

Given $3(\vec{a} \times \vec{b}) = \vec{c} - \vec{a} - 2\vec{b}$

$3(\vec{a} \times \vec{b}) \cdot \vec{b} = \vec{c} \cdot \vec{a} - \vec{a} \cdot \vec{b} - 2\vec{b} \cdot \vec{b}$

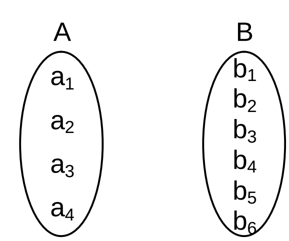
$O = \vec{c} \cdot \vec{b} - \frac{1}{2} - 2 \cdot 1$

$\vec{c} \cdot \vec{b} = 2 + \frac{1}{2} = \frac{5}{2}$

$\vec{c} \cdot \vec{b} = \frac{5}{2}$

83. B

Sol. Number of one-one functions
$= 6 \times 5 \times 4 \times 3$
$= 30 \times 12$
$= 360$



84. D

Sol. $$g(x)=\begin{cases}\dfrac{\dfrac{ax^2+bx}{(\cot x)^n}+c}{1+\dfrac{4}{(\cot x)^n}}=c & \text{if } x<\dfrac{\pi}{4}\\[2ex] \dfrac{\dfrac{\sin x+\cos x}{(\tan x)^n}+1}{\dfrac{1}{(\tan x)^n}+c}=\dfrac{1}{c} & \text{if } x>\dfrac{\pi}{4}\end{cases}$$

Limit to exist $c=\dfrac{1}{c}\Rightarrow c^2=1,\ c=\pm 1$

85. D

Sol. $y=e^{3x}$

$\dfrac{dy}{dx}=3e^{3x}$

$\dfrac{d^2y}{dx^2}=9e^{3x}$ ..... (1)

$e^{3x}=y$

$3x=\log_e y$

$3\dfrac{dx}{dy}=\dfrac{1}{y}$

$3\dfrac{d^2x}{dy^2}=-\dfrac{1}{y^2}$, $\dfrac{d^2x}{dy^2}=-\dfrac{1}{3y^2}$ ..... (2)

$\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{d^2x}{dy^2}\right)=9e^{3x}\times-\dfrac{1}{3\cdot e^{6x}}=-3e^{-3x}$

86. B

Sol. P(x, y), Q(1, 0)

$PQ=\sqrt{(x-1)^2+y^2}$

$PQ^2=x^2+1-2x+y^2$

$PQ^2=x^2-2x+1+2x^3-3x^2+9$

$PQ^2=2x^3-2x^2-2x+10$

$\dfrac{d\left(PQ^2\right)}{dx}=6x^2-4x-2=0$

$6x^2-4x-2=0$

$3x^2-2x-1=0$

$3x^2-3x+x-1=0$

$3x(x-1)+(x-1)=0$

$(x-1)(3x+1)=0$

$x=1,\ x=-\dfrac{1}{3}$

$\dfrac{d^2\left(PQ^2\right)}{dx^2}=12x-4$

So, PQ^2 is minimum at x = 1

$y^2=2+9-3=8$

$(PQ)_{\text{minimum}}=\sqrt{0+8}=2\sqrt{2}$

87. C

Sol. $\int x^2 \sin 3x \cdot d\left(x^2 \sin 3x\right)$

$= \dfrac{\left(x^2 \sin 3x\right)^2}{2} + c$

$= \dfrac{x^4 \sin^2 3x}{2} + c$

a = 4, b = 2, d = 2

a + b + d = 8

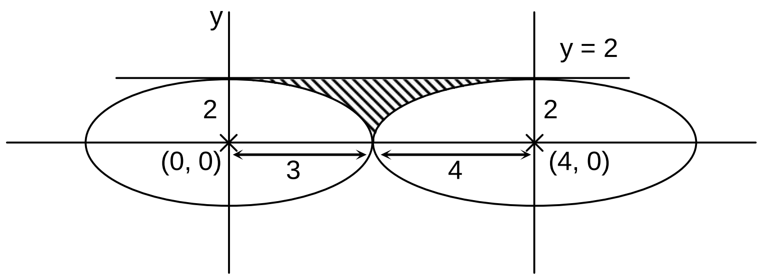
88. C

Sol. $1\cdot f(x) = 1 + 0 - 1\cdot x^2 f(x)$

$f(x)(1 + x^2) = 1,\ f(x) = \dfrac{1}{1+x^2} = \int_{-1}^{1} \dfrac{dx}{1+x^2} = 2\int_{0}^{1} \dfrac{dx}{1+x^2} = 2\left|\tan^{-1} x\right|_0^1 = 2 \times \left(\dfrac{\pi}{4} - 0\right) = \dfrac{\pi}{2}$

89. D

Sol. Area of an ellipse = πab



So, area of shaded region = $7 \times 2 - \dfrac{\pi}{4} \cdot 2 \times 3 - \dfrac{\pi}{4} \cdot 4 \cdot 2 = 14 - \dfrac{14\pi}{4} = 14 - \dfrac{7\pi}{2} = \dfrac{28 - 7\pi}{2}$

90. D

Sol. Let y(t) = z

$\dfrac{dz}{dt} + 2z = 2e^{-2t}$

$I\cdot F = e^{\int 2dt} = e^{2t}$

$z\cdot(IF) = \int 2(IF)\,dt + c$

$z e^{2t} = 2t + c$

$y(t) e^{2t} = 2t + c$

Put t = 0

$y(0) e^{0} = c$

2 = c

$y(t) e^{2t} = 2t + 2$

$y(t) = \dfrac{2t + 2}{e^{2t}}$

$y(1) = \dfrac{2+2}{e^2} = \dfrac{4}{e^2}$