Vidyamandir Classes
Gurukul for IITJEE Preparation

Advanced Practice Test-12

TIME : 3 hrs	M.M. : 360

Read the following Instructions very carefully before you proceed.

1. **The Test consists of 90 questions.**
2. The question paper consists of 3 subjects: Subject I : Chemistry, Subject II : Physics, Subject III : Mathematics. Each Part contains **30 Straight Objective Type Questions**. Each question has 4 choices (A), (B), (C) and (D), out of which **Only One choice is correct**.
3. For each question you will be given **4 Marks** if you have darkened only the bubble corresponding to the correct answer and zero mark if no bubble is darkened. In all other cases, **minus one (–1) Mark (NEGATIVE MARKING)** will be given.
4. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc., except the Admit Card inside the examination hall/room.
5. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
6. For answering a question, an ANSWER SHEET (OMR SHEET) is provided separately. Please fill your **Test Code, Roll No.** and **Group** properly in the space given in the ANSWER SHEET.
7. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. However, the candidates are allowed to take away this Test Booklet with them.
8. Do not fold or make any stray marks on the Answer Sheet.

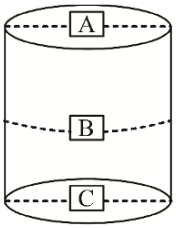
SUBJECT - I (CHEMISTRY)	120 MARKS

This section contains 30 Multiple Choice Questions. Each Question has 4 choices A, B, C & D, out of which ONLY ONE Choice is Correct :

1. The minimum and maximum wavelengths of Lyman series for hydrogen atoms are respectively (R ≡ Rydberg constant)

(A) $\frac{1}{R}, \frac{4}{3R}$ **(B)** $\frac{1}{2R}, \frac{4}{3R}$ **(C)** $\frac{1}{R}, \frac{4}{R}$ **(D)** None of these

2. At which location on the inside surface of the closed container will the number of molecular collisions per unit area be the greatest ? (Ignore effects of gravity)



(A) Square A (Top centre) **(B)** Square B (Centre side)

(C) Square C (Bottom centre)

(D) The number of collision is the same at A, B and C

3. Phenol can be distinguished from alcohol with :

(A) Tollen's reagent **(B)** Schiff's base

(C) Neutral $FeCl_3$ **(D)** $NaHCO_3$

4. Consider the following reaction, $C_6H_5NH_2 + CHCl_3 + KOH \xrightarrow{\Delta} (A) \xrightarrow{H^+/H_2O} (B) + (C)$. The compounds (B) and (C) are :

(A) C_6H_5COOH and NH_3 respectively **(B)** $C_6H_5NH_2$ and HCOOH respectively

(C) $C_6H_5NH_2$ and H_2O respectively **(D)** None of these

5. When 0.1 mole of $CrCl_3(NH_3)_5$ is treated with excess of $AgNO_3$, 0.2 mole of AgCl is obtained. The conductivity of the solution will correspond to : (cation: anion)

(A) 1 : 3 **(B)** 1 : 2 **(C)** 1 : 1 **(D)** 3 : 1

6. Find out the compound which gives red colour on reaction of its Lassaigne's extract with $FeCl_3$.

(A) *para*-$H_2N - C_6H_4 - COOH$ **(B)** NH_2SO_3H

(C) $(CH_3)_2SO_4$ **(D)** *para*-$H_2N - C_6H_4 - SO_3H$

7. Benzaldehyde and propionaldehyde can be distinguished by :

(A) Tollen's Reagent **(B)** Fehling's solution

(C) Iodoform test **(D)** By both (A) and (B)

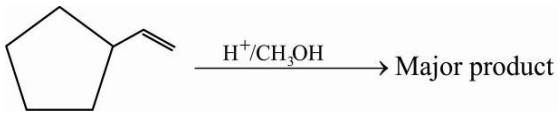
8. What would happen when a solution of potassium chromate is treated with an excess of dilute nitric acid?

(A) $Cr_2O_7^{2-}$ and H_2O are formed **(B)** CrO_4^{2-} is reduced to +3 states of Cr

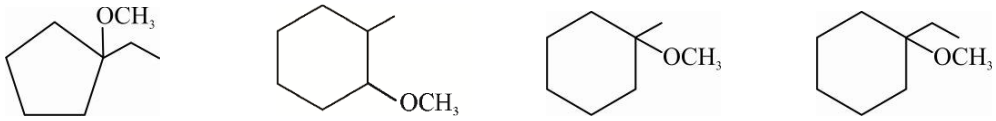
(C) CrO_4^{2-} is oxidized to +7 state of Cr **(D)** Cr^{3+} and $Cr_2O_7^{2-}$ are formed

9. Which of the following is formed by condensation polymerization?

(A) Teflon **(B)** Polystyrene **(C)** PVC **(D)** Dacron

10. 

The major product in above reaction is :



(A) **(B)** **(C)** **(D)**

11. The correct order in which oxygen-oxygen bond dissociation energy increases is :

(A) $H_2O_2 < O_2 < O_3$ **(B)** $H_2O_2 < O_3 < O_2$

(C) $O_2 < O_3 < H_2O_2$ **(D)** $O_2 < H_2O_2 < O_3$

For Questions 12 - 15

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

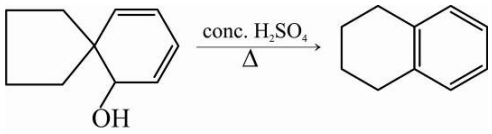
(B) Statement-1 is True, Statement-2 is True; Statement-2 is a not a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False **(D)** Statement-1 is False, Statement-2 is True

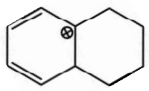
12. **Statement 1 :** Dry ice is formed by drying ice with dehydrating agent.

Statement 2 : Dehydrating agent absorbs moisture.

13. **Statement 1 :** Alcohol (X) on heating in presence of conc. H_2SO_4 as catalyst produce product (Y)



(X) (Y)

Statement 2 : Most stable intermediate formed in dehydration of (X) is 

14. **Statement 1 :** Alcohols gets dehydrated in the presence of strong acid H_2SO_4 but not in the presence of equally stronger acid HI

Statement 2 : HSO_4^- is a weaker nucleophile while I^- is a stronger nucleophile

15. **Statement 1 :** Equal masses of nitrogen (N_2) and oxygen (O_2) are present in a mixture having 2L N_2 and $7/4$L O_2 at the same temperature and pressure

Statement 2 : Equal volume of different gases have equal masses under identical conditions of temperature and pressure

Paragraph for Questions 16 - 17

Effusion is the process in which a gas escapes through a small hole. Gases with a higher molecular weight effuse more quickly in terms of mass than gases with a lower molecular weight. This is why a balloon filled with low molecular weight hydrogen deflates faster than an equivalent balloon full of higher molecular weight oxygen.

16. Calculate relative rate of effusion O_2 to CH_4 through a container containing O_2 and CH_4 in 1:1 mass ratio.

(A) $\frac{3\sqrt{2}}{4}$ **(B)** $\frac{3}{4\sqrt{2}}$ **(C)** $\frac{3}{2\sqrt{2}}$ **(D)** $\frac{1}{2\sqrt{2}}$

17. Two gases A and B with molecular masses M and 2M respectively are mixed in ratio 2 : 1. The ratio of rate of effusion of the mixture with respect to the rate of effusion of gas (A) under identical conditions of pressure and temperature will be :

(A) 1.732 **(B)** 0.866 **(C)** 0.75 **(D)** 1

Paragraph for Questions 18 - 19

In quantum mechanics, the concept of matter waves or de-Broglie wave proposed the wave-particle duality of matter. The theory was proposed by Louis de-Broglie in 1924. The de-Broglie relations show that the wavelength associated with a moving particle is inversely proportional to the momentum of a particle and is also called de-Broglie wavelength.

18. Wavelength of a photon (wavelength λ_2) having energy equal to the KE of a proton (wavelength λ_1) will be :

(A) $\lambda_2 = \lambda_1$ **(B)** $\lambda_2 \propto \lambda_1^2$ **(C)** $\lambda_2 \propto \frac{1}{\lambda_1}$ **(D)** $\lambda_2 \propto \frac{1}{\lambda_1^2}$

19. If the radius of first orbit of H- atom is a_o, then de-Broglie wavelength of electron in 4th orbit is :

(A) $2\pi a_o$ **(B)** $16\pi a_0$ **(C)** $\frac{\pi a_0}{4}$ **(D)** $8\pi a_0$

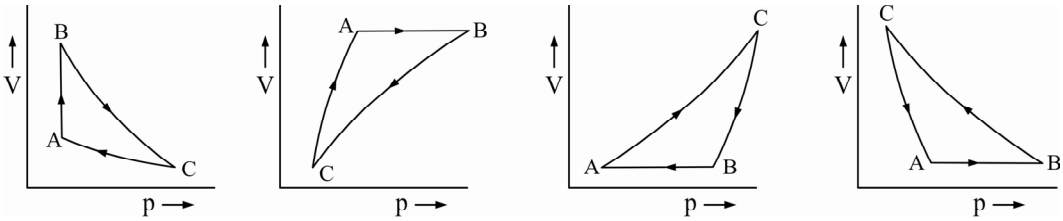
Paragraph for Questions 20 - 21

One mole of an ideal gas defined by the state A(300 K, 2 atm) is subjected to the following change of state :

$$A \xrightarrow[\text{heating}]{\text{Isochoric}} B(5.0\,\text{atm}) \xrightarrow[\text{expansion}]{\text{Adiabatic rev.}} C \xrightarrow[\text{Compression}]{\text{isothermal rev.}} A \qquad C_v = 1.5\,R$$

Answer the following questions based on the above information.

20. Which of the following p-V diagram describes the above-mentioned cyclic process most appropriately ?



(A) **(B)** **(C)** **(D)**

21. What is the net work (W) involved in the above cyclic process ?

(A) $-675\,R$ **(B)** $420\,R$ **(C)** $-255\,R$ **(D)** $-420\,R$

Paragraph for Questions 22 - 24

An aqueous solution of a mixture (A) containing two inorganic salts, when treated with dil HCl, gave a precipitate (X) and filtrate (Y). (X) separated and treated with NH_4OH, it gave black precipitate (B) which dissolves in aqua regia. The filtrate (Y) gave black precipitate with H_2S in alkaline medium. This precipitate is dissolved in dil HCl, and treated with dimethyl glyoxime and ammonium hydroxide, to form scarlet red precipitate (R). Aqueous solution of mixture also gave blood red colouration with neutral ferric chloride solution.

22. Aqueous solution (A) contains :

(A) $Ni^{2+}, Ag^{+}, NO_3^{-}$ **(B)** $Hg^{2+}, Ni^{2+}, CH_3COO^{-}$

(C) $Hg_2^{2+}, Ni^{2+}, CH_3COO^{-}$ **(D)** $Co^{2+}, Pb^{2+}, CrO_4^{2-}$

23. Compound (B) is :

(A) $NH_2 - Hg - O - Hg - Cl$ **(B)** $Hg(NH_2)Cl.Hg$

(C) $[Ag(NH_3)_2]Cl$ **(D)** $[Pb(NH_3)_4]Cl_2$

24. What is the hybridization of central metal in compound (R)

(A) sp^3d^2 **(B)** d^2sp^3 **(C)** sp^3 **(D)** dsp^2

Paragraph for Questions 25 - 27

Most of the reaction including nuclear reactions, decomposition of N_2O_5, NH_4NO_3, $C_6H_5N_2^+Cl^-$ etc. follows first order kinetics. Rate constant k, time period, half-life period, concentration, pressure etc. can be calculated by using following formula.

$$t = \frac{1}{k}\ln\frac{C_0}{C},\ t_{1/2} = \frac{1}{k}\ln 2$$

The rate constant (k) varies with temperature as given by Arrhenius equation.

$k = Ae^{-E_a/RT}$ where C_0 is initial concentration at time t = 0 and C is concentration at time t.

25. Which of the following is correct statement for first order reaction having half-life equal to 8 min?

(A) Reaction will be almost complete (99.9%) in approximately 1 Hr. and 20 min.

(B) Three fourth of reaction will be completed in 1 Hr and 20 min.

(C) Half-life decreases with decrease in concentration of reactants.

(D) Half-life increases with increase in concentration of reactants.

26. The half-life of the 1st order reaction given below is 24 minutes : $A_{(g)} \longrightarrow 2B_{(g)} + C_{(s)}$

The reaction is carried out taking certain mass of A enclosed in a vessel in which it exerts a pressure of 400 mm Hg. The pressure of the reaction mixture after expiry of 48 min will be : (Antilog 0.60 $\approx$ 4)

(A) 700 mm Hg **(B)** 600 mm Hg **(C)** 800 mm Hg **(D)** 1000 mm Hg

27. Which of the following statements are correct regarding Arrhenius equation?

(A) If $E_a = 0$, then 100%, reactant will convert into the product.

(B) If T is equal to infinite, then 100% reactant will convert into the product

(C) E_a can not be negative **(D)** All of these

Paragraph for Questions 28 - 30

The electrochemical series is the arrangement of various electrode systems in the increasing order of their standard reduction potentials. It has several important features. On moving from top to bottom in the series, tendency to gain electrons, i.e., to get reduced increases. The electrode systems having negative values of standard reduction potentials act as anode when connected to a standard hydrogen electrode, while those having positive values act as cathode.

28. HCl can not be stored in an aluminium vessel because :

(A) $E^0_{Al^{3+}/Al}$ is much smaller than $E^0_{H^+/H_2}$ **(B)** Al is a reactive metal

(C) HCl act as an oxidizing acid. **(D)** All of these

29. If $E^0_{Cu^{2+}/Cu} = 0.34$ V and $E^0_{Ag^+/Ag} = 0.8$ V, predict whether the reaction given below is feasible or not?

$Cu^{2+}(aq) + 2Ag(s) \rightarrow Cu(s) + 2Ag^+(aq)$

(A) Feasible **(B)** Not feasible

(C) Feasible at high temperature **(D)** Feasible when the conc. of Ag^+(aq) is high

30. If $E^0_{Fe^{2+}/Fe} = -0.44$V V and $E^0_{Mg^{+2}/Mg} = -2.37$ V, $E^0_{Cu^{2+}/Cu} = +0.34$V and $E^0_{Ag^+/Ag} = +0.80$V, the correct order in which the metals displace each other is :

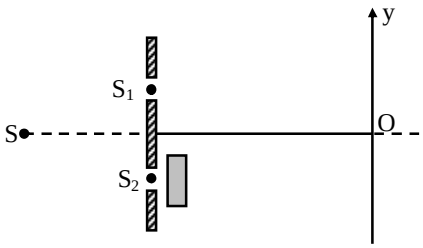
(A) Ag > Cu > Fe > Mg **(B)** Fe > Cu > Ag > Mg

(C) Fe > Ag > Cu > Mg **(D)** Mg > Fe > Cu > Ag

SUBJECT - II (PHYSICS)	120 MARKS

This section contains 30 Multiple Choice Questions. Each Question has 4 choices A, B, C & D, out of which ONLY ONE Choice is Correct :

1. A YDSE is performed in a medium of refractive index 4/3. A light of 600 *nm* wavelength is falling on the slits having 0.45 nm separation. The lower slit S_2 is covered by a thin glass plate of thickness $10.4\mu m$ and refractive index 1.5. The interference pattern is observed on a screen placed 1.5 *m* from the slits as shown in figure. (All the wavelengths in this problem are for the given medium of refractive index 4/3, ignore absorption). Find the light intensity at point *O* relative to maximum fringe intensity.



(A) $\frac{1}{4}I_{max}$ **(B)** $\frac{5}{4}I_{max}$ **(C)** $\frac{1}{2}I_{max}$ **(D)** $\frac{3}{4}I_{max}$

2. A physicist hanged a cylinder-shaped container of base area 100 cm^2 to a spring. He slowly poured water into the container and found that the surface of water remained at the same level. Find the spring constant k of the spring. Take density of water as 1000 kg/m^3.

(A) 50 N/m **(B)** 100 N/m **(C)** 1000 N/m **(D)** 500 N/m

3. A wooden block is floating in water tank. By applying an external force, the block is now pressed to its bottom. During the process, work done is equal to

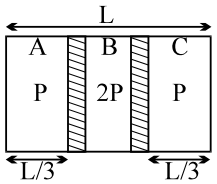
(A) work done against upthrust exerted by the water

(B) work done by upthrust plus loss of gravitational potential energy of the block

(C) work done by upthrust minus loss of gravitational potential energy of the block

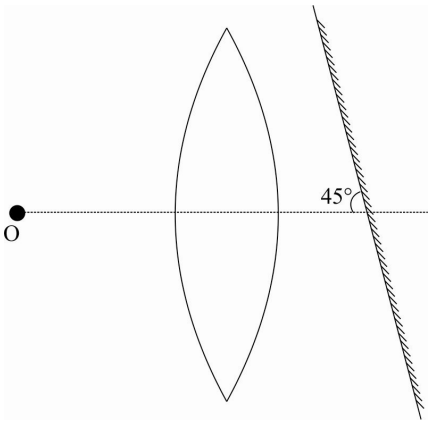
(D) None of these

4. Two conducting movable smooth pistons are kept inside a non conducting, adiabatic container with initial positions as shown. Gas is present in the three parts A, B and C having initial pressures as shown. Now the pistons are released and are allowed to attain equilibrium position slowly. Then the final equilibrium position length of part A will be :



(A) L/8 **(B)** L/4 **(C)** L/6 **(D)** L/5

5. A convex lens of focal length a/2 and a plane mirror are arranged as shown. The plane mirror is inclined at an angle 45° with the axis and the distance between lens and mirror is equal to focal length of the lens. A point source is placed at a distance 'a' from lens at its axis. The distance between the source and the image formed after reflection from the mirror will be



(A) *a*/2 **(B)** $\frac{\sqrt{5}a}{2}$ **(C)** $\frac{\sqrt{10}\,a}{2}$

(D) None of these

6. Magnetic field at the centre (at Nucleus) of the hydrogen like atoms (atomic number –z) due to motion of electron in n^{th} orbit is proportional to

(A) $\frac{n^3}{z^5}$ **(B)** $\frac{n^4}{z}$ **(C)** $\frac{z^2}{n^3}$ **(D)** $\frac{z^3}{n^5}$

7. Two thin cylindrical pipes *A* and *B* have same length. Pipe *A* is open at both ends and is filled with a monoatomic gas of molar mass M_A. Pipe *B* is open at one end and closed at the other end and is filled with a diatomic gas of molar mass M_B. Both gases are at same temperature. If the frequency of the second harmonic in pipe *A* is equal to frequency of third harmonic in pipe *B*, then the ratio M_A/M_B is :

(A) $\frac{200}{189}$ **(B)** $\frac{200}{289}$ **(C)** $\frac{400}{189}$ **(D)** $\frac{400}{289}$

8. A neutron moving with an energy of 20.4 *ev* undergoes a perfectly inelastic collision with a stationary hydrogen atom $\left(M_p = M_n\right)$, which is in ground state. Then due to impact the hydrogen atom :

(A) gets ionized **(B)** gets excited to first excited state
(C) gets excited to second excited state **(D)** nor gets excited or gets ionized

9. A point object is placed at a distance of 25 *cm* from a convex lens of focal length 20 *cm*. If a glass slab of thickness *t* and refractive index 1.5 is inserted between the lens and the object, the image is formed at infinity. The thickness *t* is :

(A) 10 *cm* **(B)** 5*cm* **(C)** 20 *cm* **(D)** 15 *cm*

10. A uniform magnetic field exists in region given by $\vec{B} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ *T*. A rod of length 5*m* placed along *y*-axis is moved along *x*-axis with constant speed 1 m/s. Then induced e.m.f. in the rod will be:

(A) zero **(B)** 25 volt **(C)** 20 volt **(D)** 15 volt

11. In an α-decay the Kinetic energy of α particle is 48 MeV and Q-value of the reaction is 50 MeV. The mass number of the mother nucleus is: (Assume that daughter nucleus is in ground state)

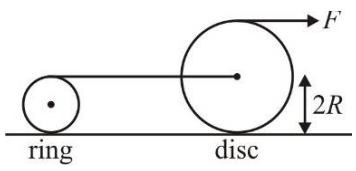
(A) 96 **(B)** 100 **(C)** 104 **(D)** None of these

12. Light from a discharge tube containing hydrogen atoms falls on the surface of a piece of sodium. The K.E. of the fastest photoelectrons emitted from sodium is 0.73 eV. The work function of sodium is 1.82 eV then,

(A) the energy of the photons causing the photoelectric emission is 2.55 eV
(B) the quantum number of the two levels involved in the emission of these photons are 1 and 4
(C) the change in the angular momentum of the electron in the hydrogen atom is $h/2\pi$ (in the above transition)
(D) assuming it to be at rest before transition, the recoil speed of the emitting hydrogen atom of mass 1.6×10^{-27} kg is 8.5 m/s

13. A disc and ring of mass *M* and radius 2*R* and *R* respectively are connected by a light inextensible thread as shown. A force *F* is applied at the topmost point of disc. It was observed that the surface was rough enough for both of them to roll without slipping.

Which of the following statements is correct?



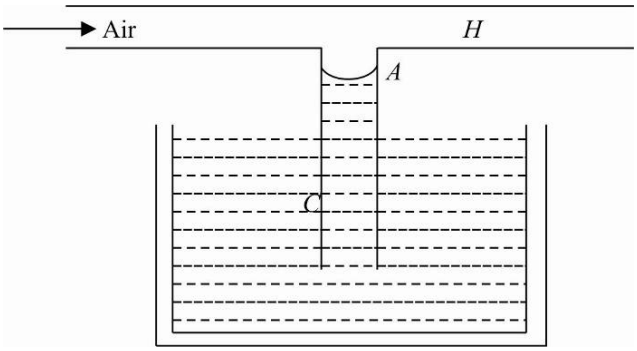
(A) Friction force between disc and ground is backward
(B) Friction force between ring and ground is forward
(C) Minimum value of coefficient of friction is $\frac{F}{2Mg}$
(D) Minimum value of coefficient of friction is $\frac{3F}{4Mg}$

14. Two boys, each of mass 50 kg, stand diametrically opposite to each other on the rim of a uniform circular platform of mass 200 kg which is free to rotate about a vertical axis through its centre. Each boy walks simultaneously at equal constant speed in clockwise direction once around the rim. The angle through which the platform is turned in space will be :

(A) 180° in anticlockwise direction **(B)** 120° in clockwise direction

(C) 120° in anticlockwise direction **(D)** 180° in clockwise direction

15. Figure shows a capillary tube C dipped in a liquid that wets it. The liquid rises to a point A. If we blow air through the horizontal tube H, what will happen to the liquid column in the capillary tube?



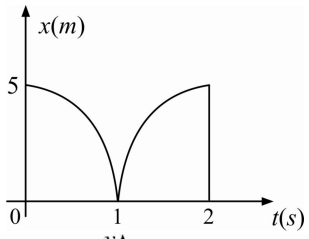
(A) Level will rise above *A*

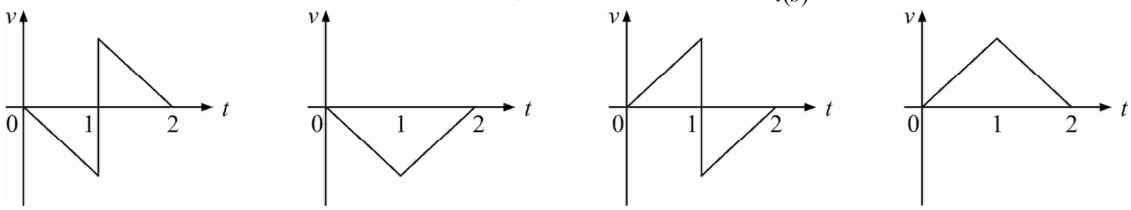
(B) Level will fall below *A*

(C) Level will remain at *A*

(D) It is difficult to predict

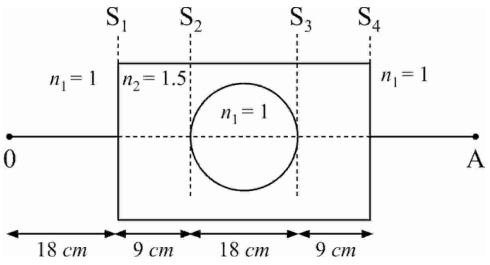
16. The displacement-time graph of a moving particle with constant acceleration is shown in the figure. The velocity-time graph is given by :





(A) **(B)** **(C)** **(D)**

17. A cubical block of glass, refractive index 1.5, has a spherical cavity of radius r = 9 cm inside it as shown in figure. A luminous point object O is at a distance of 18 cm from the cube (see figure). What is the apparent positive of O as see from A?



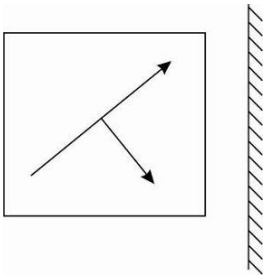
(A) 17 cm, left of S_4

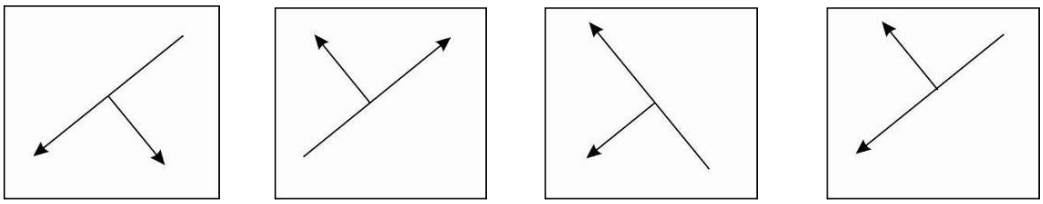
(B) 25 cm, right of S_4

(C) 13 cm, left of S_4

(D) 10 cm, right of S_4

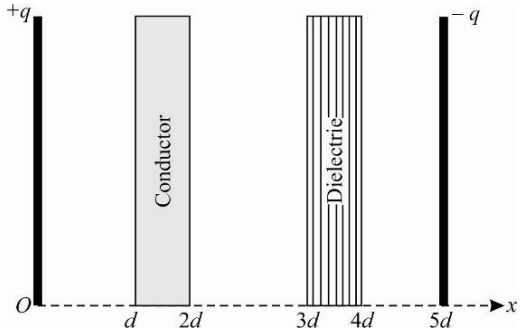
18. Choose the correct mirror image of figure given below :





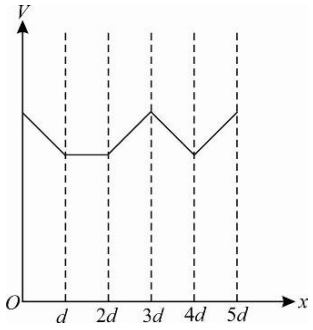
(A) **(B)** **(C)** **(D)**

19. The distance between plates of a parallel-plate capacitor is 5*d*. The positively charged plate is at $x = 0$ and negatively charged plate is at $x = 5d$

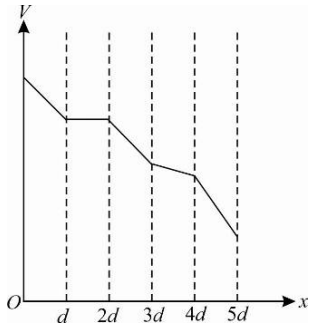


Two slabs, one of conductor and the other of a dielectric of same thickness *d*, are inserted between the plates as shown in figure. Potential *V* versus distance *x* graph will be

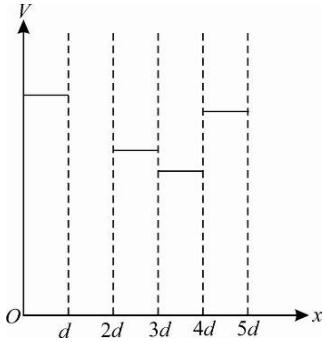
(A)



(B)



(C)



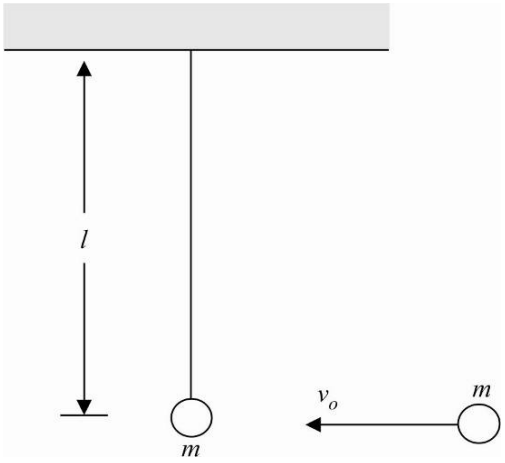
(D) None of these

20. A particle of mass *m* moves with a variable velocity v, which changes with distance covered x along a straight line as $v = k\sqrt{x}$, where k is a positive constant. At $t = 0$, particle is at $x = 0$. The work done by all the forces acting on the particle, during the first *t* seconds is :

(A) $\frac{mk^4}{t^2}$ **(B)** $\frac{mk^4t^2}{4}$ **(C)** $\frac{mk^4t^2}{8}$ **(D)** $\frac{mk^4t^2}{16}$

(D) Each object continuous to emit and absorb radiation even after attaining the temperature *T*

21. A simple pendulum consists of a bob of mass m and a light string of length l as shown in the figure. Another identical ball moving with the small velocity v_0 collides with the pendulum's bob and sticks to it. For this new pendulum of mass 2m, mark out the correct statement(s).



(A) Time period of the pendulum is $2\pi\sqrt{\frac{4l}{g}}$

(B) The equation of motion for this pendulum is

$$\theta = \frac{v_o}{2\sqrt{gl}}\sin\left[\sqrt{\frac{g}{l}}t\right]$$

(C) The equation of motion for this pendulum is

$$\theta = \frac{v_o}{2\sqrt{gl}}\cos\left[\sqrt{\frac{g}{l}}t\right]$$

(D) Time period of the pendulum is $2\pi\sqrt{\frac{2l}{g}}$

22. X-ray from a tube with a target A of atomic number Z shows strong K lines for target A and weak K lines for impurities. The wavelength of K_a line is λ_z for target A and λ_1 and λ_2 for two impurities

$$\frac{\lambda_z}{\lambda_1} = 4 \text{ and } \frac{\lambda_z}{\lambda_2} = \frac{1}{4}$$

Assuming the screening constant of K_a line sot be unity, select the correct statement(s).

(A) The atomic number of first impurity is 2z – 1

(B) The atomic number of first impurity is 2z + 1

(C) The atomic number of second impurity is $\frac{z}{2}+2$

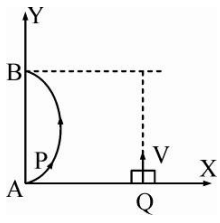
(D) The atomic number of second impurity is $\frac{z}{2}+1$

23. A light wave of wavelength λ_0 propagates from point A to point B. We introduce in its path a glass plate of refractive index n and thickness *l*. The introduction of the plate alters the phase of the plate at B by an angle ϕ. If λ is the wavelength of light on emerging from the plate, then :

(A) $\Delta\phi = 0$ **(B)** $\Delta\phi = \frac{2\pi l}{\lambda_o}$

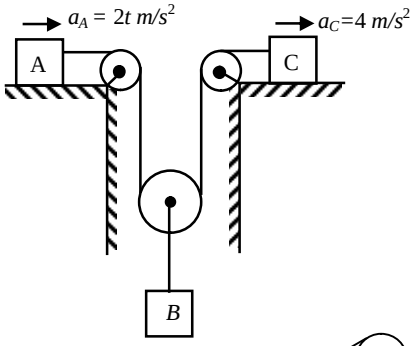
(C) $\Delta\phi = 2\pi l\left(\frac{1}{\lambda}-\frac{1}{\lambda_o}\right)$ **(D)** $\Delta\phi = \frac{2\pi l}{\lambda_o}(n+1)$

24. A particle *P* starts from origin as shown and moves along a circular path. Another particle *Q* crosses *x*-axis at the instant particle *P* leaves origin. *Q* moves with constant speed *v* parallel to *y*-axis and is all the time having *y*-coordinate same as that of *P*. When *P* reaches diametrically opposite to point *B*, its average speed is :



(A) πV **(B)** $\frac{\pi V}{2}$ **(C)** $\frac{V}{2}$ **(D)** None of these

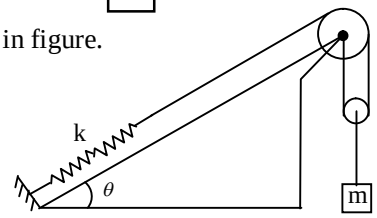
25. Figure shows an arrangement in which three blocks are joined together with an inextensible string. All the surfaces are smooth and pulleys are massless. If a_A, a_B and a_C are the respective accelerations of the blocks *A*, *B*, and *C*, then velocity of *B* at the moment it come to equilibrium (take $m_B = 2kg$, $a_A = 2t\ m/s^2$, $a_C = 4\ m/s^2$)



(A) 2 *m/s* **(B)** 4 *m/s*

(C) 8 *m/s* **(D)** 10 *m/s*

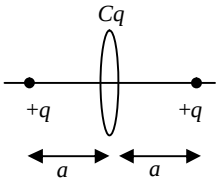
26. Calculate the time period of *SHM* of block in the situation as shown in figure.



(A) $2\pi\sqrt{\frac{m}{K}}$ **(B)** $2\pi\sqrt{\frac{m}{2K}}$

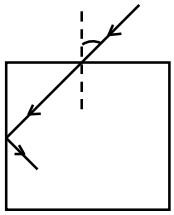
(C) $2\pi\sqrt{\frac{m}{4K}}$ **(D)** $2\pi\sqrt{\frac{3m}{2K}}$

27. Two point charges of charge $+q$ each are placed $2a$ distance apart and a ring of radius a having charge Cq distributed uniformly over its circumference is placed midways between them as shown in figure. If none of them accelerated when released, the value of *C* will be :



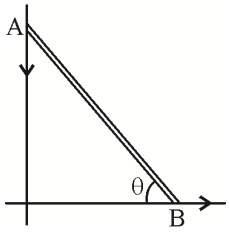
(A) $-\frac{1}{\sqrt{2}}$ **(B)** $-\sqrt{2}$ **(C)** $-\frac{1}{2\sqrt{2}}$ **(D)** $-2\sqrt{2}$

28. A light ray falls on a square slab at an angle 45°, what must be the minimum index of refraction of glass, if total internal reflection takes place at the vertical face?



(A) $\frac{\sqrt{3}}{2}$ **(B)** $\sqrt{\frac{3}{2}}$ **(C)** $\frac{3}{2}$ **(D)** $\frac{3}{\sqrt{2}}$

29. A straight rod of length ℓ has its ends in straight grooves in the same plane at right angles to one another. At an instant, acceleration of point A is a_A and that of B is a_B. Then,



(A) $a_B = a_A \tan\theta$ **(B)** $a_A = a_B \tan\theta$

(C) $a_B > a_A \tan\theta$ **(D)** $a_B < a_A \tan\theta$

30. A plane wave of monochromatic light falls normally on a uniformly thin film of oil which covers a glass plate. The wavelength of source can be varied continuously. Complete constructive interference is observed for $\lambda_1 = 5000$ Å and $\lambda_2 = 10000$ Å and for no other wavelength in between. If μ of oil is 1.25 and that of glass is 1.5, the thickness of film will be _____μm.

(A) 0.2 **(B)** 0.1 **(C)** 0.8 **(D)** 0.4

SUBJECT - III (MATHEMATICS)	120 MARKS

This section contains 30 Multiple Choice Questions. Each Question has 4 choices A, B, C & D, out of which ONLY ONE Choice is Correct :

Paragraph for Questions 1 - 2

Number of ways of arranging 12 boys and 12 girls are like

a_1 = a line such that boys and girls sit alternatively.

a_2 = around a circular table alternatively.

a_3 = around an equilateral triangular table alternatively and eight on each side.

a_4 = around a square table alternatively and six on each side.

(For a_3 and a_4 on a corner if on one side it's a boy then on the other side it should be a girl to maintain alternation). Now answer the following questions :

1. Which of the following is true ?

(A) $a_1 > a_2 > a_3 > a_4$ **(B)** $a_4 > a_3 > a_2 > a_1$

(C) $a_1 > a_3 > a_4 > a_2$ **(D)** $a_1 > a_4 > a_3 > a_2$

2. Which of the following is true if $P(a_r)$ is the probability of arranging boys and girls in their ways explained in the paragraph.

(A) $P(a_1) > P(a_2) > P(a_3) = P(a_4)$ **(B)** $P(a_1) = P(a_2) = P(a_3) = P(a_4)$

(C) $P(a_1) > P(a_3) > P(a_4) > P(a_2)$ **(D)** $P(a_1) > P(a_2) = P(a_3) = P(a_4)$

3. A doctor is called to see a sick child. The doctor knows (prior to the visit) that 90% of the sick children in that neighborhood are sick with the flu, denoted by F, while 10% are sick with the measles, denoted by M. A well-known symptom of measles is a rash, denoted by R. The probability of having a rash for a child sick with the measles is 0.95. However, occasionally children with the flue also develop a rash, with conditional probability 0.08. Upon examination the child, the doctor finds a rash. Then what is the probability that the child has the measles ?

(A) 91/165 **(B)** 90/163 **(C)** 82/161 **(D)** 95/167

4. If $S = \begin{bmatrix} \frac{\sqrt{3}-1}{2\sqrt{2}} & \frac{\sqrt{3}+1}{2\sqrt{2}} \\ -\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right) & \frac{\sqrt{3}-1}{2\sqrt{2}} \end{bmatrix}$ and $A = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$ and $P = S(adj\ A)S^T$ then if $\sum_{r=1}^{2015} \left(S^T P^r S\right) = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$

then total number of factors of (A + B + C + D) is :

(A) 12 **(B)** 48 **(C)** 120 **(D)** None of these

5. If $g(x) = \left(4\cos^4 x - 2\cos 2x - \frac{1}{2}\cos 4x - x^7\right)^{\frac{1}{7}}$, then the value of $g(g(100))$ is equal to :

(A) –1 **(B)** 0 **(C)** 1 **(D)** 100

6. The domain of the function $f(x) = \cos^{-1}\left\{\sec\left(\cos^{-1} x\right)\right\} + \sin^{-1}\left\{\cos ec\left(\sin^{-1} x\right)\right\}$, is :

(A) $x \in R$ **(B)** $x \in \{1, -1\}$ **(C)** $-1 \le x \le 1$ **(D)** $x \in \phi$

7. A box contains 12 pairs of shoes. 4 shoes are taken at random. What is the probability that there is at least one pair?

(A) $\frac{15}{322}$ **(B)** $\frac{33}{78}$ **(C)** $\frac{12}{23}$ **(D)** $\frac{41}{161}$

8. Let $f(x)=\lim\limits_{h\to 0}\frac{\left(\sin(x+h)\right)^{\ln(x+h)}-(\sin x)^{\ln x}}{h}$, then $f\left(\frac{\pi}{2}\right)$ is :

(A) equal to 1 **(B)** equal to 0 **(C)** $\ln\frac{\pi}{2}$ **(D)** Non-existent

9. The number of ways in which 2*n* white and 2*n* black balls can be arranged such that no consecutive '*n*' white balls are together, (n > 1) are :

(A) ${}^{2n+1}C_2+{}^{4n}C_{2n}$ **(B)** ${}^{2n+1}C_2-{}^{2n+1}C_1\,{}^{3n}C_n+{}^{4n}C_{2n}$

(C) ${}^{2n+1}C_2+{}^{2n+1}C_1\,{}^{3n}C_n(-1)^n+{}^{4n}C_{2n}$ **(D)** ${}^{2n+1}C_2+{}^{2n+1}C_1\,{}^{3n}C_n(-1)^n$

10. Let *X* be a set containing *n* elements. If two subsets *A* and *B* of *X* are picked at random, the probability that *A* and *B* have the same number of elements, is :

(A) $\frac{{}^{2n}C_n}{2^n}$ **(B)** $\frac{1}{{}^{2^n}C_n}$ **(C)** $\frac{1\cdot 3\cdot 5\cdot\cdots\cdot(2n-1)}{2^n\cdot n!}$ **(D)** $\frac{3^n}{4^n}$

Paragraph for Questions 11 - 12

A function $f(x)$ is monotonic increasing or decreasing according as $f'(x)>0$ or $f'(x)<0$. Let us consider a function $f(x)=\int_0^x\{2\sqrt{2}\sin^2 t+(2-\sqrt{2})\sin t-1\}dt, 0<x<2\pi$. Then :

11. Interval of *x* in which $f(x)$ is increasing is given by :

(A) $\left(\frac{\pi}{6},\frac{5\pi}{6}\right)$ **(B)** $\left(0,\frac{\pi}{6}\right)$ **(C)** $\left(\frac{5\pi}{6},\frac{5\pi}{4}\right)$ **(D)** $\left(\frac{7\pi}{4},2\pi\right)$

12. $f(x)$ is decreasing in the region :

(A) $\left(\frac{\pi}{6},\frac{5\pi}{6}\right)$ **(B)** $\left(\frac{5\pi}{6},\frac{5\pi}{4}\right)$ **(C)** $\left(0,\frac{7\pi}{4}\right)$ **(D)** $\left(\frac{5\pi}{4},\frac{7\pi}{4}\right)$

13. The larger of $\sin x+\tan x$ and $2x$ in the interval $0<x<\frac{\pi}{2}$ is :

(A) $\sin x+\cos x$ **(B)** $\sin x+\tan x$ **(C)** $2x$ **(D)** None of these

14. The plane $x - y - z = 4$ is rotated through 90° about its line of intersection with the plane $x + y + 2z = 4$. Then its equation in the new position

(A) $5x+y+4z=20$ **(B)** $3x+y+4z=20$ **(C)** $4x-y-z=10$ **(D)** None of these

15. If the probability of choosing an integer k out of 2m integers 1,2,3, …….2m is inversely proportional to $k^4(1\le k\le 2m)$, then the probability that chosen number is odd, is

(A) equal to 1/2 **(B)** less than 1/2 **(C)** greater than 1/2 **(D)** less than 1/3

16. In a triangle ABC, if $\frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}$ then C is equal to :

(A) 30° **(B)** 60° **(C)** 75° **(D)** 90°

17. If $[x]$ denotes the greatest integer not exceeding x and if the function f defined by :

$$f(x)=\begin{cases}\dfrac{a+2\cos x}{x^2} & (x<0)\\ b\tan\dfrac{\pi}{[x+4]} & (x\geq 0)\end{cases}$$ is continuous at $x = 0$, then the ordered pair (a, b) =

(A) $(-2, 1)$ **(B)** $(-2, -1)$ **(C)** $\left(-1, \sqrt{3}\right)$ **(D)** $\left(-2, -\sqrt{3}\right)$

18. If foci of hyperbola lie on $y = x$ and one of the asymptotes is $y = 2x$, then equation of the hyperbola, given that it passes through (3, 4), is

(A) $x^2 - y^2 - \frac{5}{2}xy + 5 = 0$ **(B)** $2x^2 - 2y^2 + 5xy + 5 = 0$

(C) $2x^2 + 2y^2 - 5xy + 10 = 0$ **(D)** None of these

19. If $\hat{\alpha}$ and $\hat{\beta}$ be two perpendicular unit vectors such that $\vec{x} = \hat{\beta} - (\hat{\alpha} \times \vec{x})$, then $|\vec{x}|$ is equal to

(A) 1 **(B)** $\sqrt{2}$ **(C)** $\frac{1}{\sqrt{2}}$ **(D)** None of these

20. The set of values of m for which it is possible to draw the chord $y = \sqrt{m}x + 1$ to the curve $x^2 + 2xy + \left(2 + \sin^2\alpha\right)y^2 = 1$, which subtends a right angle at the origin for some value of α is :

(A) [2, 3] **(B)** [0, 1] **(C)** [1, 3] **(D)** None of these

21. If L_1 is the line of intersection of the planes $2x + y + z = 1$ and $3x + y + 2z = 2$ and L_2 is the line $x = y = z$, then the shortest distance between the lines L_1 and L_2 is equal to :

(A) $\frac{1}{\sqrt{2}}$ **(B)** $\sqrt{2}$ **(C)** $\frac{3}{\sqrt{2}}$ **(D)** $\frac{\sqrt{3}}{2}$

22. If $f(x) = \left|x^2 - 3x + 2\right| + \frac{1}{4} \forall\ x \in R$, then the least value of $\dfrac{f^5(x)+1}{f^2(x)}$ is equal to :

(A) 1 **(B)** $\frac{5}{2}\left(\frac{3}{2}\right)^3$ **(C)** $\frac{5}{3}\left(\frac{3}{2}\right)^{2/5}$ **(D)** $\frac{15}{(108)^{1/5}}$

23. A circle with radius $|a|$ and center on the y-axis slides along it and a variable line through $(a, 0)$ cuts the circle at points P and Q. The region in which the point of intersection of the tangents to the circle at points P and Q lies, is represented by :

(A) $y^2 \geq 4(ax - a^2)$ **(B)** $y^2 \leq 4(ax - a^2)$ **(C)** $y \geq 4(ax - a^2)$ **(D)** $y \leq 4(ax - a^2)$

24. Let $f_n(x) = \dfrac{n+1}{(n-1)!}x^n\ (n \geq 1)$. The value of $\int_0^1\left(\sum_{n=1}^{\infty} f_n(x)\right)dx$ is :

(A) e **(B)** 0 **(C)** $2e$ **(D)** $e/2$

25. $\lim\limits_{x\to\infty}\left(\left(\dfrac{1+\tan\frac{\pi}{2x}}{1+\sin\frac{\pi}{x}}\right)^x + \left(\frac{2}{\pi}\cos^{-1}\frac{1}{x}\right)^{\frac{x\pi^2}{4}}\right)$ equals :

(A) $e^{-\frac{\pi}{2}}$ **(B)** $e^{-\pi}$ **(C)** $2e^{-\pi}$ **(D)** $2e^{-\frac{\pi}{2}}$

Paragraph for Questions 26 - 27

Let a function *f* even and integrable everywhere and periodic with period 2.

Let $g(x)=\int_0^x f(t)\,dt$ and $g(1) = k$

26. The value of $g(x+2)-g(x)$ is equal to :

(A) $g(1)$ **(B)** 0 **(C)** $g(2)$ **(D)** $g(3)$

27. The value of $g(2)$ in terms of k is equal to :

(A) k **(B)** $2k$ **(C)** $3k$ **(D)** $5k$

Paragraph for Questions 28 - 29

If $x \in R$, and the equation $f(x)=x^4+4x^3-8x^2+K=0$ then :

28. The roots of $f(x) = 0$ when $K \in (0,3)$ are :

(A) All real **(B)** Two real and two complex
(C) No real **(D)** Repeated roots

29. If $K \in (3,4]$, then the roots of $f(x) = 0$ are :

(A) All real **(B)** Two real and two complex
(C) No real **(D)** Coincident

30. Let x_1, x_2, x_3 be the roots of the equation $x^3 - x^2 + \beta x + \gamma = 0$, which are in *A.P*, then $\beta x + \gamma y = 1$ passes through the fixed point:

(A) $\left(\frac{9}{2},\frac{7}{2}\right)$ **(B)** $\left(\frac{9}{2},\frac{27}{2}\right)$ **(C)** $\left(\frac{5}{2},\frac{27}{2}\right)$ **(D)** $\left(\frac{9}{2},\frac{5}{2}\right)$