JEE PREPARATORY TEST (JPT-1)
(JEE MAIN PATTERN)
TARGET : JEE (MAIN+ADVANCED) 2019

DATE : 10-03-2019 SET-1 COURSE : (JP, JF, JR, JCC, EP, EF, ER)

HINTS & SOLUTIONS

PART-A : PHYSICS

1. The graph shows ………………………

Sol. At t = 3

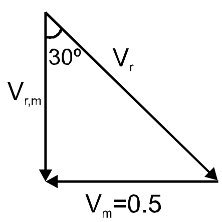
$$\text{Slope} = \frac{d\left(\frac{1}{V}\right)}{dt} = -1$$

$$\Rightarrow -\frac{1}{V^2}\frac{dV}{dt} = -1$$

$$\Rightarrow \left(\frac{1}{\sqrt{3}}\right)^2 \frac{dV}{dt} = 1 \Rightarrow a = \frac{dV}{dt} = 3 \text{ m/s}^2$$

2. To a stationary ………………………

Sol. $V_{r,m}$ = (0.5) (cot 30º)



$= 0.5\sqrt{3}$ m/s

3 A heavy spherical ………………………

Sol. $mg\sin\theta - ma\cos\theta = 0$

$a = g\tan\theta$

4. In the figure ………………………

Sol. velocity of both same so that relative velocity is zero.

5. A particle of mass ………………………

Sol. $$\int_a^x -\frac{k}{x^2}.dx\cos 180 = -\frac{1}{2}mv^2$$

$$\therefore v = \sqrt{\frac{2k}{m}}\sqrt{\left|\frac{a-x}{ax}\right|}$$

6. The velocity ………………………

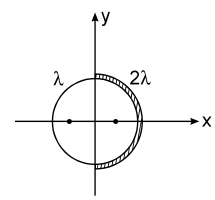
Sol. It can be observed that component of acceleration perpendicular to velocity is

$a_c = 4$ m/s^2

$$\therefore \text{ radius} = \frac{v^2}{a_c} = \frac{(2)^2}{4} = 1 \text{ metre.}$$

7. Two semicircular ………………………

Sol. Let the two half rings be placed in left and right of y-axis with centre as shown in figure.



Then the coordinate of the centre of mass of left and right half rings are $\left(-\frac{2R}{\pi}, 0\right)$ and $\left(\frac{2R}{\pi}, 0\right)$.

$\therefore$ x-coordinates of the centre of mass of complete ring is

$$\frac{m\left(-\frac{2R}{\pi}\right) + 2m\left(\frac{2R}{\pi}\right)}{3m} = \frac{2R}{3\pi}$$

8. Two identical ………………………

Sol. $$\frac{1}{2}\mu v_{rel}^2(1-e^2) = \Delta E$$

$$\frac{1}{2}\left(\frac{m}{2}\right)(2V)^2(1-e^2) = \frac{1}{2}\left(\frac{1}{2}mV^2 + \frac{1}{2}mV^2\right)$$

$$mV^2(1-e^2) = \frac{1}{2}mV^2$$

$$1 - e^2 = \frac{1}{2} \Rightarrow e^2 = \frac{1}{2} \Rightarrow e = \frac{1}{\sqrt{2}}$$

9. Two light vertical ………………………

Sol. AC = x

x_0 = ext^n. in each spring

Torque about C

$K_1 x_0 x = K_2 x_0 (\ell - x)$

$x = \frac{\ell k_2}{k_1 + k_2}$

AC = x

$K_1 x_0 x = K_2 x_0 (\ell - x)$

$x = \frac{\ell k_2}{k_1 + k_2}$

10. A thin and ………………………

Sol. By angular momentum conservation

$$\frac{MR^2}{2}\,\omega = \left(\frac{MR^2}{2} + \frac{MR^2}{8}\right)\omega'$$

$$\omega' = \frac{4}{5}\,\omega$$

11. Equation F = ………………………

Sol. $E = \frac{1}{2}\,kA^2\,e^{-bt/m}$

$t = \frac{\ell n2}{b} m = \frac{\ell n2}{\ell n2} 2$ = 2 sec.

12. Figure shows ………………………

Sol. Level rise because to compensate for the reduction in pressure above A, due to high velocity (from Bernoulli's principle pressure decreases if velocity increases.)

13. A vertical capillary ………………………

Sol. $h_0 = \frac{2S\cos\theta}{r_0 \rho g} = \frac{2 \times 7.5 \times 10^{-2}}{5 \times 10^{-4} \times 10^3 \times 10}$

$= \frac{15 \times 10^{-2}}{5}$ = 3 × 10^{-2}m = 30 mm

$h_0 r_0 = hr$

$\Rightarrow r = \frac{h_0}{h}\, r_0 = \left(\frac{30}{25}\right)(0.5)$ = 0.6 mm

14. A non–uniform ………………………

Sol. $v_y = \sqrt{\frac{T_y}{\mu_y}}$

$T_y = \left\{\int_0^y \mu_0 e^y dy\right\} g$

$T_y = \mu_0(e^y - 1).g$

$v_y = \sqrt{g - \frac{g}{e^y}}$

$v_y^2 = g(1 - e^{-y})$.

15. S_1 & S_2 ………………………

Sol. For minima,

$\Delta x = (2n - 1)\,\frac{\lambda}{2}$

The maximum possible path difference = distance between the sources = 3m.

For no minima

$\frac{\lambda}{2} > 3$

$\lambda > 6$

$\therefore\ f = \frac{V}{\lambda} < \frac{330}{6} = 55$

$\therefore$ If f < 55 Hz, no minimum will occur.

16. An ideal gas ………………………

Sol. If a is isothermal and c is adiabatic processes then for b, ΔU will be negative and Q will be positive so specific heat will be negative.

17. A rod of length ………………………

Sol. The change in length of rod due to increase in temperature in absence of walls is

$\Delta\ell = \ell\,\alpha\,\Delta T$ = 1000 × 10^{-4} × 20 mm

= 2 mm

But the rod can expand upto 1001 mm only.

At that temperature its natural length is = 1002 mm.

$\therefore$ compression = 1mm

$\therefore$ mechanical stress = $Y\,\dfrac{\Delta \ell}{\ell} = 10^{11} \times \dfrac{1}{1000}$

$= 10^8\ N/m^2$

18. Potential difference ………………………

Sol. $V_C - V_S = \dfrac{3}{2}\dfrac{KQ}{R} - \dfrac{KQ}{R}$

$= \dfrac{KQ}{2R} = \dfrac{1}{8\pi \in_0 R}\left(\dfrac{4}{3}\right)\pi R^3 \rho \qquad = \dfrac{R^2\rho}{6 \in_0}$

19. In the given circuit ………………………

Sol. Current in the circuit is given by

$i = \dfrac{\varepsilon}{7+x}$

Power generated in 5Ω,

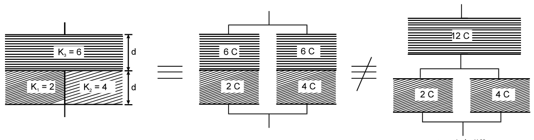
$= \left(\dfrac{\varepsilon}{7+x}\right)^2 \times 5 = 5\left(\dfrac{\varepsilon}{7+x}\right)^2$

Power will be max when 7 + x is minimum

i.e., for x = 0

20. A parallel plate ………………………

Sol.



(right)

as potential difference across dielectrics of dielectric constant 2 and 4 is not same.
(wrong)

The equivalent capacitance $C_{eq} = \dfrac{2C \times 6C}{2C + 6C} + \dfrac{6C \times 4C}{6C + 4C} = 3.9\ C$

21. The figure ………………………

Soln. $\vec{F} = \int I d\vec{\ell} \times \vec{B}$

$|\vec{F}| = I\ell \times |\vec{B}|$

22. In the circuit ………………………

Sol. At the time of maximum current in the circuit.

$\dfrac{di}{dt} = 0$

So potential difference across both capacitors will be equal.

By charge conservation

$3CV' = CV$

$V' = \dfrac{V}{3}$

By energy conservation

$\dfrac{1}{2}CV^2 = \dfrac{1}{2}(C + 2C)\dfrac{V^2}{9} + \dfrac{1}{2}LI^2_{max}$

$I_{max} = \sqrt{\dfrac{2C}{3L}}\ V.$

23. A time varying ………………………

Sol. $q = CV$

$= [6 \times 3 \sin 2t] \times 10^{-6}$

$i = \dfrac{dq}{dt} = \left[6 \times 3 \times 2 \cos 2t\right] \times 10^{-6}$

$V_R = iR = 9 \cos 2t \Rightarrow V_{max} = 9V$

24. The wavefronts ………………………

Sol. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\alpha = \beta = \gamma \Rightarrow \cos\alpha = \dfrac{1}{\sqrt{3}}$

25. If 13.6 eV energy ………………………

Sol. Energy required to remove an electron from nth orbit is

$E_n = \dfrac{13.6}{n^2}$

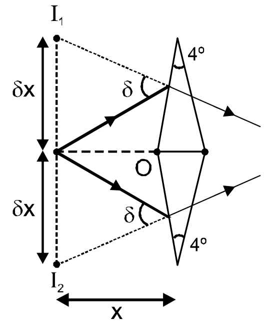
Here n = 2

Therefore $E_2 = \dfrac{13.6}{2^2} = 3.4\ V$

27. A point object ………………………

Sol. $\delta = (n - 1)A$

$= (1.5 - 1) \times 4^\circ \times \dfrac{\pi}{180^\circ}$



Distance between two images = 2δ x

$= 2(1.5 - 1) \times 4° \times \frac{\pi}{180°} \times 60$ cm

$= \frac{4\pi}{3}$ cm

28. Which logic gate ………………………

Sol. $\overline{(\overline{A}+\overline{B})}$ = A.B

29. How many ………………………

Sol. For dark fringe

$\alpha \sin\theta = n\lambda$

$$\frac{n\lambda}{\alpha} \le 1$$

$$n\frac{668 \times 10^{-9}}{4170 \times 10^{-9}} \le 1$$

$n \le 6.24$

30. The output ………………………

Ans. s(t) = 5 cos (1800 πt) + 20 cos (2000 πt) + 5 cos (2200 πt)

$$= 20\left[1 + \frac{1}{2}\cos(200\pi)\right]\cos(2000\pi t)$$

Thus, modulation index is 0.5

PART-B : CHEMISTRY

31. Given are P-V.........

Sol. Slope of PV = constant is $\frac{dP}{dV} = -\frac{P}{V}$

and slope of PV^n = constant is $\frac{dP}{dV} = \frac{-nP}{V}$

Then m > 1 > n

33. Select the correct.........

Sol. $3Br_2 + 6OH^- \xrightarrow{\Delta} 5Br^- + BrO_3^- + 3H_2O$

↑ ↑ ↑ ↑ ↑

p q r s t

Br_2 disproportionate to Br^- and BrO_3^- when treated with hot alkali solution

$$\left(\frac{1}{2}Br_2 + e^- \longrightarrow Br^-\right) \times 5$$

$$\frac{1}{2}Br_2 + 6OH^- \longrightarrow BrO_3^- + 3H_2O + 5e^-$$

$$3Br_2 + 6OH^- \longrightarrow 5Br^- + BrO_3^- + 3H_2O$$

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34. In the given processes.........

Sol. (I) $Ag_2S + CN^- \longrightarrow [Ag(CN)_2]^-$

(II) $Ni + 4CO \longrightarrow [Ni(CO)_4]$

(III) $AgBr + 2S_2O_3^{2-} \longrightarrow [Ag(S_2O_3)_2]^{3-}$

35. At what temperature ....

Sol. Moles of solute = $\frac{3 \times 10^{23}}{6 \times 10^{23}} = 0.5 = \frac{w_1}{m_1}$

$$\Delta T_f = \frac{1000 \times w_1 \times K_f}{m_1 \times w_2}$$

$$= \frac{1000 \times 0.5 \times 1.86}{250} = 3.72°C \text{ or } 3.72 \text{ K}$$

$\therefore$ FP = 273 – 3.72 = 269.28 K

37. For the octahedral ....

Sol. $Ma_2b_2c_2$

The trans pairs for the possible isomers are :

(aa)(bb)(cc) (optically inactive)

(aa)(bc)(bc) (optically inactive)

(bb)(ac)(ac) (optically inactive)

(cc)(ab)(ab) (optically inactive)

(ab)(ac)(bc) (optically active)

38. To prepare a buffer ....

Sol. Mixing of NH_4OH and $(NH_4)_2SO_4$ gives a basic buffer.

$$pOH = pK_b + \log \frac{[NH_4^+]}{[NH_4OH]};$$

$$pK_{a_{NH_4^+}} = 9.26;\ pK_{b_{NH_4OH}} = 4.74$$

$$5.74 = 4.74 + \log \frac{a}{5};$$

Where a is millimole of $[NH_4]^+$ obtained on mixing,

$\therefore$ a = 50

$\therefore$ Millimole of $(NH_4)_2SO_4$ = 25

or mole of $(NH_4)_2SO_4$ = 0.025 mol

39. The following diagram ....

Sol. from the distribution curve,

U_{mp}(most probable velocity at 150 K) = 500 ms^{-1}

U_{mp}(at T_2 K) = 1000 ms^{-1}

Let, molar mass of the ideal gas = M

$$\sqrt{\frac{2RT}{M}} = U_{mp}$$

At 150 K, $$\sqrt{\frac{2R \times 150}{M}} = 500\ ms^{-1}$$

$$\therefore \frac{2R}{M} \times 150 = (500)^2$$

$$\frac{2R}{M} = (500)^2 / 150$$

At T_2 K $$\sqrt{\frac{2RT_2}{M}} = 1000\ ms^{-1}$$

$$\frac{2RT_2}{M} = (1000)^2$$

$$\frac{(500)^2}{150} \times T_2 = (1000)^2$$

T_2 = 600 K

40. Cl_2 gas is obtained.........

Sol. (I) $HCl + O_2 \xrightarrow{\Delta, CuCl_2} Cl_2 + H_2O$

(II) $2MnO_4^- + 16H^+ + 10Cl^- \xrightarrow{\Delta}$

$2Mn^{2+} + 8H_2O + 5Cl_2$

(III) $NaOH + HCl \longrightarrow NaCl + H_2O$

(IV) $NaCl + MnO_2 + \text{conc. } H_2SO_4 \xrightarrow{\Delta}$

$MnCl_2 + NaHSO_4 + H_2O + Cl_2$

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$MnCl_2 + NaHSO_4 + H_2O + Cl_2$

41. Amongs the colloids ....

Sol. Gel $\rightarrow$ liquid in solid

Milk $\rightarrow$ liquid in liquid

Cloud $\rightarrow$ liquid in gas

42. If the packing fraction ....

Sol. Square sheet = $\frac{\pi}{4}$

Hexagonal sheet = $\frac{\pi}{2\sqrt{3}} = \frac{\sqrt{3}\pi}{6}$

SCC = $\frac{\pi}{6}$

a = 4, b = 6, c = 6

43. Which of the following ....

Sol. Theory based

44. The ore that ....

Sol. $KCl.MgCl_2.6H_2O$ carnalite

$CuFeS_2$ copper pyrite

$CuCO_3.Cu(OH)_2$ malachite

$CaCO_3.MgCO_3$ Dolomite

45. When electric ....

Sol. Theory based

46. The density of ....

Sol. Assuming 1000 gm of water

wt. of solution = 1000 + 2 × 34 = 1068 gm

Vol. of solution = $\frac{1068}{1.068}$ = 1000 ml

Molarity = $\frac{\text{moles of } H_2O_2}{\text{Vol. of solution (in L)}} = \frac{2}{1} = 2$ M

Molarity = $\frac{\text{Volume strength}}{11.2}$

$\therefore$ Volume strength = 2 × 11.2 = 22.4 V

1000 ml of solution contains 68 gm H_2O_2

$\therefore$ 100 ml $\Rightarrow \frac{68}{1000} \times 100 = 6.8$ gm

$\therefore$ % $\left(\frac{w}{v}\right)$ = 6.8% w/v

Mole fraction of water = $\frac{\frac{1000}{18}}{\frac{1000}{18}+2} = \frac{250}{259}$

47. The conductivity of ....

Sol. K = $3.2 \times 10^{-5}\ \Omega^{-1}.\text{cm}^{-1}$

$\Lambda = \frac{10^3 K}{C}$

$\Lambda = \frac{3.2 \times 10^{-2}}{0.2}$

$= 16 \times 10^{-2}$

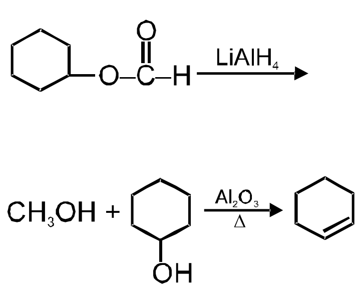
$\alpha = \frac{\Lambda}{\Lambda_\infty}$ $\therefore \Lambda_\infty = \frac{\Lambda}{\alpha} = \frac{16 \times 10^{-2}}{0.02} = 8$

48. Cerium (Z = 58) is ....

Sol. (3) Cerium can also show the oxidation state of +4 in solution as it leads to a noble gas configuration, from $[Xe]^{54}\ 4f^1\ 5d^1\ 6s^2$ to $[Xe]^{54}$, after losing four electrons. It is only Ce^{4+} which exist in solution among the lanthanides.

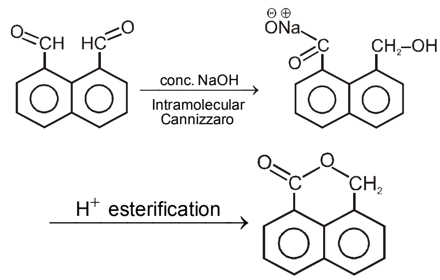
50. A compound M ....

Sol.



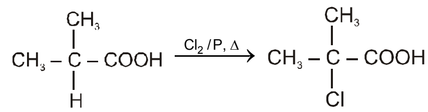
51. Which of the following ....

Sol.



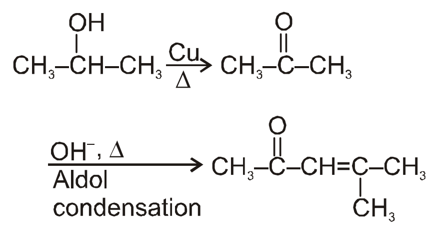
52. In Hell-Volhard-Zelinsky ....

Sol.



53. A(Alcohol) $\xrightarrow[\Delta]{Cu}$ B(Carbonyl compound) ....

Sol.

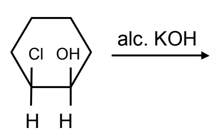


54. In which of the ....

Sol : Due to H-bonding cis isomer is more stable.

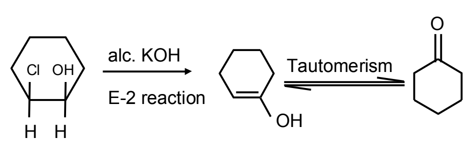
55. Which of the following ....

Sol. Factual.

56. 

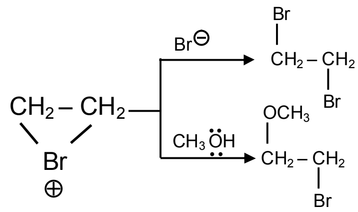
....

Sol.



57. In methanol solution.........

Sol. $CH_2 = CH_2 \xrightarrow[CH_3OH]{Br_2}$



58. Which of the following ....

Sol. Ascorbic acid is one form of vitamin C

PART-C : MATHEMATICS

61. If $\frac{5z_2}{7z_1}$ is a purely………………….

Sol. $\frac{5z_2}{7z_1} = ki \Rightarrow \frac{z_1}{z_2} = \frac{5}{7ki}$

or $\frac{2z_1}{3z_2} = \frac{10}{21ki}$ apply comp. and divi.

$$\frac{2z_1 + 3z_2}{2z_1 - 3z_2} = \frac{10 + 21ki}{10 - 21ki}$$

Take mod of both sides and $\left|\frac{2z_1 + 3z_2}{2z_1 - 3z_2}\right| = 1$

62. In an A.P. $t_7 = 15$………………….

Sol. Both t_2 and t_{12} are equidistant from t_7

$\therefore$ $t_2\, t_7\, t_{12} = (15 - 5d)\; 15\; (15 + 5d)$

$P = 5 \times 5 \times 15\,(3 - d)\,(3 + d) = k\,(9 - d^2)$

Now clearly P will be maximum if d = 0.

63. The value of ………………….

Sol. Condition for A.P. i.e. 2b = a + c

$\Rightarrow$ $2 \log_9 4 = \log_3 (2^{1-x} + 3) + \log_{27} (2^x - 1)^3$

Make base 3.

$\frac{2}{2} \log 4 = \log (2^{1-x} + 3) + \frac{3}{3} \log (2^x - 1)$

$\therefore$ $4 = \left(\frac{2}{y} + 3\right)(y - 1)$, where $y = 2^x$

$\therefore$ $4y = (3y + 2)(y - 1)$ or $3y^2 - 5y - 2 = 0$

$\therefore$ $y = 2, \; -\frac{1}{3} = 2^x$

Since exponential function cannot be –ve

$\therefore$ $2^x = -\frac{1}{3}$ is rejected.

$\therefore$ $2^x = 2 \Rightarrow x = 1$.

64. It is known………………….

Sol. Since, $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^5} + \ldots\infty = \frac{\pi^2}{8}$

Let $x = \frac{1}{1^2} + \frac{1}{2^2} + \ldots\infty$

$= \left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\infty\right) + \left(\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \ldots\infty\right)$

$= \frac{\pi^2}{8} + \frac{1}{4}\left(\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\infty\right)$

$\Rightarrow x = \frac{\pi^2}{8} + \frac{1}{4}x \Rightarrow \frac{3x}{4} = \frac{\pi^2}{8} \Rightarrow x = \frac{\pi^2}{6}$

65. The number of ………………….

Sol. $630 = 9 \times 70 = 3^2 \times 5 \times 7 \times 2$

or $630 = 2 \,.\, 3^2 \,.\, 5 \,.\, 7$

The number of positive integer solutions

$= ({}^4C_3)^3({}^5C_3) = 640$

66. If the 6^{th} term………………….

Sol. $T_6 = {}^8C_5\left(\frac{1}{x^{8/3}}\right)^3 (x^2 \log_{10} x)^5 = 5600$

or $\frac{8.7.6}{1.2.3} \cdot \frac{1}{x^8} . x^{10} (\log_{10} x)^5 = 5600$

or $(\log_{10} x)^5 = \left(\frac{10}{x}\right)^2$

or $x^2 (\log_{10} x)^5 = 10^2 (\log_{10} 10)^5$

Hence x = 10. (Note $\log_{10} 10 = 1$)

67. Out of 30 ………………….

Sol. $20 = n(A \cap B) + n(B \cap C) + n(C \cap A) - 2n(A \cap B \cap C)$

$\Rightarrow n(A \cap B) + n(B \cap C) + n(C \cap A) = 20 + 2 \times 6 = 32$

Number of computer contain at least one component

$= n(A \cup B \cup C) = 20 + 8 + 25 - 3\;2 + 6 = 27$

68. If $\vec{u} = \vec{a} - \vec{b}$ ....

Sol. u × v = 2 (a × b)

$\therefore$ |u × v| = $2\sqrt{a^2b^2\sin^2\theta}$

$= 2\sqrt{a^2b^2(1-\cos^2\theta)}$

$= 2\sqrt{a^2b^2-(a.b)^2}$

$= 2\sqrt{16-(a.b)^2} \Rightarrow (1)$

69. A variable point..................

Sol. Putting the point in the equations of two parallel lines, we have $\lambda = -\frac{4\sqrt{2}}{5}$ and $\frac{5\sqrt{2}}{6}$.

Since the point lies in between the parallel lines we must have $-\frac{4\sqrt{2}}{3} < \lambda < \frac{5\sqrt{2}}{6}$.

70. The lines a_1x ....

Sol. We know that if the two lines cut the axes in four concyclic points then products of the intercepts on axes of x is equal to product of intercepts on axes of y.

$\therefore \left(-\frac{c_1}{a_1}\right).\left(-\frac{c_2}{a_2}\right) = \left(-\frac{c_1}{b_1}\right).\left(-\frac{c_2}{b_2}\right) \Rightarrow a_1a_2 = b_1b_2$

71. The equation......................

Sol. Using SP = e. PM property for ellipse

SP = e. PM

$$\left(x-\frac{1}{5}\right)^2 + \left(y-\frac{2}{5}\right)^2 = (\lambda-1)^2\left[\frac{3x+4y-1}{\sqrt{3^2+4^2}}\right]^2$$

Above is of the form $SP^2 = e^2. PM^2$ which represents the equation of an ellipse where $0 < e^2 < 1$

$\therefore 0 < (\lambda - 1)^2 < 1$

Above double inequality is equivalent to two inequalities

$0 < \lambda - 1 < 1$ and $-1 < \lambda - 1 < 0$

or $1 < \lambda < 2$ and $0 < \lambda < 1$

$\therefore \lambda \in (0, 2)$

72. The general ....

Sol. $\cos^{25} x = 1 + \sin^{50} x$

But L.H.S. ≤ 1 and R.H.S. ≥ 1

Hence we must have sign of equality

$\therefore x = 2n\pi$

73. If $\sin^{-1}(1 - x) - 2$......................

Sol. $x \in [0, 1]$

LHS $\in \left[0, \frac{\pi}{2}\right]$, RHS $\in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

possible only when $\sin^{-1}(1 - x) = \frac{\pi}{2}$, $\sin^{-1} x = 0$

x = 0

74. $\lim\limits_{x\to\pi/2} \frac{\sin x-(\sin x)^{\sin x}}{1-\sin x+\log\sin x}$ ....

Sol. Put sin x = t and as $x \to \frac{\pi}{2}$, $t \to 1$

Also $\frac{d}{dx}(x^x) = x^x(1+\log x)$

$\therefore \lim\limits_{t\to1}\frac{t-t^t}{1-t+\log t}$ form $\frac{0}{0}$

$= \lim\limits_{t\to1}\frac{1-t^t(1+\log t)}{-1+\frac{1}{t}}$ form $\frac{0}{0}$

$= \lim\limits_{t\to1}\frac{-\left\{t^t.\frac{1}{t}+t^t(1+\log t)^2\right\}}{-1/t^2}$

$= \frac{1+1}{1} = 2$

75. Let f(x) = $\begin{cases} -\frac{1}{|x|} \text{for} |x| \ge 1 \\ ax^2 - b \text{for} |x| < 1 \end{cases}$ ....

Sol. $f(x) = \begin{cases} -\frac{1}{|x|}, |x| \ge 1 \text{or } x^2-1 \ge 0 \text{or } x \le -1, x \ge 1 ...(i) \\ ax^2 - b, |x| < 1 \text{or } x^2-1 < 0 \text{or } -1 < x < 1 ...(ii) \end{cases}$

$f(x) = \begin{cases} -\frac{1}{|x|}, |x| \ge 1 \text{ या } x^2-1 \ge 0 \text{ या } x \le -1, x \ge 1 ...(i) \\ ax^2 - b, |x| < 1 \text{ या } x^2-1 < 0 \text{ या } -1 < x < 1 ...(ii) \end{cases}$

Also |x| = x when x = +ve ...(iii)

|x| = – x when x = –ve ...(iv)

Hence we redefine the function as under

$$f(x)=\begin{cases} ax^2-b, x<1 & \text{by (ii)} \\ -\dfrac{1}{+x}=-\dfrac{1}{x}, x\ge 1 & \text{by (i)}\end{cases} \quad \text{...(1)}$$

$$f(x)=\begin{cases} -\dfrac{1}{-x}=\dfrac{1}{x}, x\le -1 & \text{by (i)} \\ ax^2-b, x>-1 & \text{by (ii)}\end{cases} \quad \text{...(2)}$$

At x = 1, for continuity, R = L = V

$\therefore$ a – b = – 1

For differentiability, R′ = L′

$\therefore$ 2a = 1

Solving a = $\frac{1}{2}$, b = $\frac{3}{2}$

At x = –1, for continuity R = L = V

$\therefore$ a – b = –1

For differentiability R′ = L′

–1 = –2a

$\therefore$ a = $\frac{1}{2}$, b = $\frac{3}{2}$

76. The minimum ....

Sol. y = $\dfrac{a^2}{\cos^2 x}+\dfrac{b^2}{\sin^2 x}$

= $a^2 \sec^2 x + b^2 \operatorname{cosec}^2 x$

$\dfrac{dy}{dx}$ = $2a^2 \sec x . \sec x \tan x - 2b^2 \operatorname{cosec} x \operatorname{cosec} x \cot x$ = 0

= $2 [a^2 \sec^2 x \tan x - b^2 \operatorname{cosec}^2 x \cot x] = 0$

$\therefore \quad a^2\dfrac{\sin x}{\cos^3 x}-b^2\dfrac{\cos x}{\sin^3 x}=0$

or $\tan^4 x = \dfrac{b^2}{a^2}$ $\therefore \tan^2 x = \dfrac{b}{a}$

$\dfrac{d^2y}{dx^2}=2[a^2(\sec^4 x+2\sec x.\sec x\tan x\tan x)]$ + b^2 $(\operatorname{cosec}^4 x + 2 \operatorname{cosec} x \operatorname{cosec} x \cot x \cot x)]$

Above is clearly +ve when $\tan^2 x = \dfrac{b}{a}$ and hence y is min.

Putting $\tan^2 x = \dfrac{b}{a}$ in the value of y in (i)

y = $a^2 (1 + \tan^2 x) + b^2 (1 + \cot^2 x)$

= $a^2\left(1+\dfrac{b}{a}\right)+b^2\left(1+\dfrac{a}{b}\right)$

= $a^2 + b^2 + 2ab = (a + b)^2$

77. $\displaystyle\int\sqrt{\frac{e^x-1}{e^x+1}}dx$ is ....

Sol. Multiply above a below by $\sqrt{(e^x-1)}$

$\therefore \quad I=\displaystyle\int\frac{e^x(e^x-1)dx}{e^x\sqrt{e^{2x}-1}}$. Now put $e^x = t$

$\therefore \quad I=\displaystyle\int\frac{t-1}{t\sqrt{t^2-1}}dt$

= $I=\displaystyle\int\frac{1}{\sqrt{t^2-1}}dt-\frac{1}{t\sqrt{t^2-1}}dt$

$=\log\left(t+\sqrt{t^2-1}\right)-\sec^{-1}t$

78. $\displaystyle\int_{\log 1/2}^{\log 2}\sin\left(\frac{e^x-1}{e^x+1}\right)dx$ ....

Sol. $I=\displaystyle\int_{-\log 2}^{\log 2}\sin\left(\frac{e^x-1}{e^x+1}\right)dx$

If f(x) = $\sin\left(\dfrac{e^x-1}{e^x+1}\right)$

f(– x) = $\sin\left(\dfrac{1-e^x}{1+e^x}\right)$ = $-\sin\left(\dfrac{e^x-1}{e^x+1}\right)$ = – f(x)

Hence f(x) is an odd function of x

$\therefore$ I = 0

79. Solution of ....

Sol. Collect the terms of e^{xy} and $e^{x/y}$

$y^2 e^{xy} (x\, dy + y\, dx) = e^{x/y} (y\, dx - x\, dy)$

$e^{xy} d(xy) = e^{x/y} d\left(\dfrac{x}{y}\right)$

Integrating both sides, we have

$e^{xy} = e^{x/y} + c \Rightarrow (1)$

or $xy = \log (e^{x/y} + c) \Rightarrow (3)$

80. If the probabilities………………….

Sol. P(A) = p

P(B) = q

$P(\overline{A}) = 1 - p$

$P(\overline{B}) = 1 - q$

Probability that one person is alive can be written as a sum of two cases : (i) A dies and B lives (ii) B dies and A lives.

(i) A dies and B lives (ii) B dies and A lives.

So required probability = p (1 – q) + q(1 – p)

= p – pq + q – qp = p + q – 2pq

81. Lef f_p (β)………………….

Sol. In the last term write $\frac{\beta}{p}$ as $\frac{p\beta}{p^2}$

$\therefore$ $f_p(\beta) = e^{i\beta/p^2 + 2i\beta/p^2 + ... + pi\beta/p^2}$

$= e^{i\beta/p^2\{1+2+3+...+p\}} = e^{i\beta/p^2\left\{\frac{p(p+1)}{2}\right\}}$

$= e^{\frac{i\beta}{2}\left(1+\frac{1}{p}\right)}$ now put p = n, β = π

$\therefore$ $f_n(\pi) = e^{\frac{i\pi}{2}\left(1+\frac{1}{n}\right)}$

$\lim\limits_{n\to\infty} f_n(\pi) = e^{i\pi/2} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i$

82. Coefficient of ………………….

Sol. Coefficient of variation = $\frac{\text{S.D.}}{\text{A.M.}} \times 100$

$\Rightarrow$ A.M. = $\frac{\text{S.D.}}{\text{Coeff. of variation}} \times 100$

(1) A.M. = $\frac{21}{60} \times 100 = 35$

(2) A.M. = $\frac{16}{70} \times 100 = 22.85$

83. The range of ………………….

Sol. $\Delta \geq 0 \Rightarrow -25m + 150 \geq 0$

$\therefore$ $m \leq 6$

$P = \frac{m+10}{m-5}$ = +ve, as roots are of same sign.

or $\frac{(m+10)(m-5)}{(m-5)^2} > 0$

$\therefore$ m < –10 or m > 5

$\therefore$ m < –10 and 5 < m ≤ 6

84. $\sqrt{x+3-4\sqrt{x-1}} + \sqrt{x+8-6\sqrt{x-1}}$ ......

Sol. Put $\sqrt{x-1} = t$, where t = +ve and x > 1

Equation is

$\sqrt{x-1+4-4\sqrt{x-1}} + \sqrt{x-1+9-6\sqrt{x-1}} = 1$

or $\sqrt{t^2+4-4t} + \sqrt{t^2+9-6t} = 1$

or |t – 2| + |t – 3| = 1 where t is +ve

Consider the three cases t < 2, 2 < t < 3, t > 3

(1) t < 2:

$\therefore$ –(t – 2) – (t – 3) = 1

$\therefore$ –2t = –4

$\therefore$ t = 2

But t < 2. Hence no solution in this case.

(2) 2 < t < 3 : (t – 2) – (t – 3) = 1

or 1 = 1. Hence $2 < \sqrt{x-1} < 3$ satisfies.

or 4 < x – 1 < 9 or 5 < x < 10

(3) t > 3 : t – 2 + t – 3 = 1

$\therefore$ 2t = 6 or t = 3

But t > 3. Hence no solution

Hence by (2) the equation has more than two solutions.

85. If the system of ………………….

Sol. For non-trivial solution Δ = 0

$\therefore$ $\begin{vmatrix} 1 & -a & -a \\ b & -1 & b \\ c & c & -1 \end{vmatrix} = 0$

Apply $C_1 - C_2$, $C_2 - C_3$

$\Delta = \begin{vmatrix} 1+a & 0 & -a \\ 1+b & -(1+b) & b \\ 0 & (1+c) & -1 \end{vmatrix} = 0$

(1 + a) {(1 + b) – b (1 + c)} – (1 + b) a (1 + c) = 0

Divide by (1 + a) (1 + b) (1 + c)

$\therefore \quad \frac{1}{1+c} - \frac{b}{1+b} - \frac{a}{1+a} = 0$

Subtract 1 from both sides

$-\frac{c}{1+c} - \frac{b}{1+b} - \frac{a}{1+a} = -1$

$\therefore \quad \sum \frac{a}{1+a} = 1 \Rightarrow (2).$

86. If two different ………………….

Sol. Any tangent to parabola $y^2 = 4x$, (a = 1)

$y = mx + \frac{1}{m}$

Now any normal to parabola $x^2 = 4ay$ is

$x = My - 2aM - aM^3$

or $y = \frac{1}{M}x + 2a + aM^2$

If (1) and (3) are same, then

$m = \frac{1}{M}$ and $2a + aM^2 = \frac{1}{m} = M$

or $aM^2 - M + 2a = 0$

It will have two real roots if $1 - 8a^2 > 0$

or $a^2 < \frac{1}{8} \quad \therefore |a| < \frac{1}{2\sqrt{2}}$

87. If p, q, r are ………………….

Sol. $(\sim T \vee F) \wedge \sim T \Rightarrow T$

$\therefore (F \vee F) \wedge F \Rightarrow T$

$\therefore \; F \wedge F \Rightarrow T \quad \therefore \; F \Rightarrow T$

88. The curve ………………….

Sol. $\frac{dy}{dx} = -\frac{f_x}{f_y} = -\frac{n}{a}\left(\frac{x}{a}\right)^{n-1} + \frac{n}{b}\left(\frac{y}{b}\right)^{n-1}$

$\therefore$ (a, b) at $\frac{dy}{dx} = -\frac{b}{a}$

It is independent of n.

$\therefore$ Tangent is $y - b = -\frac{b}{a}(x-a)$

or $bx + ay = 2ab$

or $\frac{x}{a} + \frac{y}{b} = 2$ for all values of n.

89. $\int_0^{\pi^2/4} \frac{\sin\sqrt{x}}{\sqrt{x}} dx$ is………………….

Sol. Put $\sqrt{x} = t$ and adjust the limits.

90. If $\left(1 - \frac{r_1}{r_2}\right)\left(1 - \frac{r_1}{r_3}\right) = 2$, then………………

Sol. L.H.S. = $\left(1 - \frac{s-b}{s-a}\right)\left(1 - \frac{s-c}{s-a}\right) = 2$

or $(b - a)(c - a) = 2(s - a)^2$

or $2(bc - ac - ab + a^2) = (b + c - a)^2$

$\therefore \; a^2 = b^2 + c^2$

$\therefore \; \angle A = 90°$

JEE PREPARATORY TEST (JPT-1)
(JEE MAIN PATTERN)
TARGET : JEE (MAIN+ADVANCED) 2019

DATE : 10-03-2019 **COURSE : (JP, JF, JR, JCC, EP, EF, ER)**

ANSWER KEY

SET-1

PART-A (PHYSICS)

1.	(1)	**2.**	(3)	**3**	(3)	**4.**	(1)	**5.**	(1)	**6.**	(1)	**7.**	(2)
8.	(1)	**9.**	(4)	**10.**	(3)	**11.**	(2)	**12.**	(1)	**13.**	(2)	**14.**	(1)
15.	(1)	**16.**	(2)	**17.**	(2)	**18.**	(1)	**19.**	(4)	**20.**	(1)	**21.**	(1)
22.	(2)	**23.**	(3)	**24.**	(4)	**25.**	(3)	**26.**	(1)	**27.**	(3)	**28.**	(3)
29.	(3)	**30.**	(1)										

PART-B (CHEMISTRY)

31.	(4)	**32.**	(3)	**33.**	(3)	**34.**	(2)	**35.**	(2)	**36.**	(2)	**37.**	(1)
38.	(2)	**39.**	(4)	**40.**	(3)	**41.**	(1)	**42.**	(1)	**43.**	(3)	**44.**	(4)
45.	(1)	**46.**	(2)	**47.**	(2)	**48.**	(3)	**49.**	(2)	**50.**	(2)	**51.**	(2)
52.	(2)	**53.**	(2)	**54.**	(2)	**55.**	(1)	**56.**	(4)	**57.**	(3)	**58.**	(2)
59.	(1)	**60.**	(2)										

PART-C (MATHEMATICS)

61.	(4)	**62.**	(3)	**63.**	(4)	**64.**	(3)	**65.**	(4)	**66.**	(3)	**67.**	(3)
68.	(1)	**69.**	(2)	**70.**	(1)	**71.**	(2)	**72.**	(2)	**73.**	(1)	**74.**	(2)
75.	(2)	**76.**	(3)	**77.**	(1)	**78.**	(4)	**79.**	(3)	**80.**	(2)	**81.**	(1)
82.	(1)	**83.**	(3)	**84.**	(4)	**85.**	(2)	**86.**	(2)	**87.**	(1)	**88.**	(4)
89.	(1)	**90.**	(3)										