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ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2016 - 2017)

JEE (Main + Advanced) : ENTHUSIAST COURSE (PHASE : I)

ANSWER KEY — TEST DATE : 16-10-2016

Test Type : MINOR

Test Pattern : JEE-Advanced

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,B,C	A,B,D or B,D	B	A,B,C,D	C,D	C,D	A,C	B,C,D	B	A
	Q.	11	12	13	14	15	16	17	18		
	A.	C	D	C	B	B	A	D	B		
SECTION-II	Q.1	A	B	C	D	Q.2	A	B	C	D	
		R,S	Q,R,T	R,S	P,S or P,S,T		Q,T	S,T	P,R	Q,T	

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B,C or B,D	A,B,C,D	A	B,C,D	A,B	A,C,D	C,D	A,B,D	B	C
	Q.	11	12	13	14	15	16	17	18		
	A.	B	C	A	C	A	B	A	B		
SECTION-II	Q.1	A	B	C	D	Q.2	A	B	C	D	
		T	P,T	R,T	Q		Q,T	P,T	S	R,T	

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,B,D	B,D	B,D	A,C	B,C	B,C	A	A,D	A	D
	Q.	11	12	13	14	15	16	17	18		
	A.	C	D	B	B	D	B	A	D		
SECTION-II	Q.1	A	B	C	D	Q.2	A	B	C	D	
		R	P	S	Q		S	P	R	Q	

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PART-1 : PHYSICS   SOLUTION

SECTION-I

1. Ans. (A,B,C)

Sol. The charge possessed by the capacitor initially is

$Q_1 = CV_1 = 0.25 \times 10^{-6} \times 10^5$

$= 0.025$ C

At the end of the exposure, the capacitor is left with charge

$Q_2 = CV_2 = 0.25 \times 10^{-6} \times 4 \times 10^4$

$= 0.01$ C

The charge supplied to the X-ray set is thus

$Q_1 - Q_2 = 0.015$ C

This charge is supplied in 0.1 second.

The average current flowing is thus

$$i = \frac{Q_1 - Q_2}{T} = \frac{0.015}{0.1} = 150\text{mA}$$

The energy dissipated in this time is the difference in the energy stored in the capacitor before and after discharge. Thus energy

$$W = \frac{1}{2}CV_1^2 - \frac{1}{2}CV_2^2$$

$$= \frac{1}{2} \times 0.25 \times 10^{-6}(10^{10} - 16 \times 10^8)$$

$= 1050$ J

2. Ans. (A,B,D or B,D)

Sol. $$U_1 = -\frac{GMm}{(2d)} = -\frac{4Gm^2}{2d}$$

$U_1 = -24$ J

From energy conservation

$$0 + 0 - 24 = \frac{1}{2}mv^2 + \left(-\frac{GMm}{d}\right) \times 2 + (-24)$$

$$\frac{2GMm}{d} = \frac{1}{2}mv^2$$

$$\frac{4Gm^2}{d} = \frac{1}{2}mv^2$$

v = 4 m/s

3. Ans. (B)

4. Ans. (A,B,C,D)

Sol Potential difference across two plates must be equal to ε.

So potential difference across any point on plate-1 and any point on plate-2 will be constant

So electric field will be same.

5. Ans. (C,D)

Sol. $$\varepsilon - iR + \frac{Ldi}{df} + \frac{Mdi}{dt} - \frac{Ldi}{dt} - \frac{Mdi}{dt} = 0$$

$\varepsilon - iR = 0$

$$i = \frac{\varepsilon}{R}$$

Time constant $= \dfrac{L_{eff}}{R} = 0$

& Power delivered by battery is constant

$$P = \frac{\varepsilon^2}{R}$$

6. Ans. (C, D)

Sol. If source AC or DC, LC oscillations will take place. So charge on capacitor will not be constant at any time.

7. Ans. (A,C)

Sol. $$13.6z^2\left(\frac{1}{1^2} - \frac{1}{9}\right) = 8 \times 13.6$$

z = 3 ⇒ ion is Lithium

$$E_1 = 13.6\left(\frac{1}{1^2} - \frac{1}{4^2}\right)$$

$$E_2 = 13.6 \times 3^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right)$$

$(KE)_1 = E_1 - \phi$

$(KE)_2 = E_2 - \phi$

$$\lambda_1 = \frac{h}{\sqrt{2m(KE)_1}}$$

$$\lambda_2 = \frac{h}{\sqrt{2m(KE)_2}}$$

Given $\lambda_1 = 3\lambda_2$

On solving $\phi \approx 0.7$ eV

8. Ans. (B,C,D)

Sol. y component of velocity is changing

Mirror must be in x-z plane

Since ray was initially travelling towards +y axis

Normal must be along –ve y i.e. $-\hat{j}$

$\tan\theta = \frac{3}{4}$ angle of incidense = 37°

9. Ans. (B)

Sol. After 4.88 V, dip is observed

Excitation energy of mercury = 7.88 eV

10. Ans. (A)

Sol. $\lambda = \frac{hc}{energy} = \frac{12400}{4.88} \text{Å}$

$\lambda = 2540$ Å

11. Ans. (C)

Sol. In region-2

Potential Energy = 0

Total Energy = KE

for $\ell = 3$

$$\lambda = \frac{2a}{3}$$

$$P = \frac{h}{\lambda} = \frac{3h}{2a}$$

$$E_0 = KE = \frac{P^2}{2m} = \frac{9h^2}{8ma^2}$$

12. Ans. (D)

Sol. For minimum common energy

$$E_0 + \frac{n^2h^2}{8ma^2} = \frac{m^2h^2}{8ma^2}$$

$9 + n^2 = m^2$

$m = 5, n = 4$

$$TE = \frac{25h^2}{8ma^2}$$

13. Ans. (C)

Sol. After head on elastic collsion

$\theta = \pi$

$$\Delta\lambda = \frac{h}{m_0c}(1-(-1))$$

$$\lambda' - \lambda_0 = \Delta\lambda = \frac{2h}{m_0c}$$

$$\lambda' = \lambda_0 + \frac{2h}{m_0c}$$

Since photon's momentum after collision is in –ve x direction

So momentum of electron in +ve x direction must be greater than $\frac{h}{\lambda_0}$

14. Ans. (B)

Sol. $\Delta\lambda = \frac{h}{m_0c}(1-\cos 90) = \frac{h}{m_0c}$

fractional change = $\frac{\Delta\lambda}{\lambda} = \frac{1}{\lambda}\left(\frac{h}{m_0c}\right)$

$= \frac{0.0243}{0.400} = 0.0608$

15. Ans. (B)

16. Ans. (A)

Sol. (P) Source is moving towards observer So wavelength must decreases & frequency must increase.

(Q) Source is moving away from observer. So wavelength must increase & frequency must decrease.

(R) Since wind starts moving So velocity of sound increases but frequency remain constant. So wavelength increase. [$v = f\lambda$]

(S) Since wind starts moving in opposite direction. So velocity of sound decreases but frequency remain constant. So wavelength decreases. [$v = f\lambda$]

17. Ans. (D)

Sol. (P) Magnetic field is constant, so motion of particle is circular

(Q) Magnetic field is change its polarity in time T so motion changes its path so two loops (path) are possible.

(R) Magnetic field is change its polarity in time $\frac{T}{2}$, so path of charge particle is sinusoidal.

18. Ans. (B)

Sol. (P) $x_{F/B} = 5 + \frac{10}{2}(3/2)$

$\frac{25}{2} = 12.5$ m

(Q) $x_{F/B} = 10 + \frac{5\times 2}{(3/2)} = \frac{50}{3}$ m

(R) $|v_{F/B}| = \left|\frac{10\times(3/2)}{2} - 5\right| = \frac{5}{2} = 2.5$ m/s

(S) $|v_{B/F}| = \left|\frac{5\times 2}{(3/2)} - 10\right| = \frac{10}{3}$ m/s

SECTION-II

1. Ans. (A)-(R,S); (B)-(Q,R,T); (C)-(R,S); (D)-(P,S or P,S,T)

Sol. (A) Since wheatstone bridge is formed So current through A will be zero & will not change on reversing or changing the voltage of battery.

(B) Initially current may not be zero or may be zero.
So on increasing V, current may decrease or may not change at all respectively.
On reversing V, current will increase due to potentil difference.

(C) Initially if no potential difference was applied i.e. if V = 0 then current was zero. So on increasing V, current will increase.
If the current was non-zero initially, so increasing voltage will have no effect.
On reversing V, current will decrease due to opposite potential difference.

(D) $V_{eq} = \dfrac{\frac{1.5}{2} + \frac{V}{1}}{\frac{1}{2} + \frac{1}{1}}$

$= \dfrac{\left(V + \frac{3}{4}\right)}{\frac{3}{2}} = \frac{2V}{3} + \frac{1}{2}$

$i = \dfrac{\frac{2V}{3} + \frac{1}{2}}{\frac{2}{3} + 4}$

2. Ans. (A)-(Q,T); (B)-(S,T); (C)-(P,R); (D)- (Q,T)

Sol. (A) Since object lies between pole & focus of concave mirror. So image will be enlarged, virtual & errect & on the opposite side of mirror.

(B) Since object is real & mirror is convex. So image will be virtual, erect & diminished.

(C) Image after two reflection will be real, inverted and same size as that of object.

(D) Final image after reflection from convex mirror will be virtual, erect and enlarged

PART–2 : CHEMISTRY — SOLUTION

SECTION-I

1. Ans.(B,C or B,D)

2. Ans.(A,B,C,D)

3. Ans.(A)

4. Ans. (B, C, D)

5. Ans. (A, B)

6. Ans. (A,C,D)

7. Ans. (C,D)

8. Ans. (A,B,D)

9. Ans. (B)

In cathode : $K_{eq} = \frac{K_w}{K_3} = 10^{-4}$

$PO_4^{3-} + H_2O \rightleftharpoons HPO_4^{2-} + OH^-$

$\Rightarrow$ $pOH = \frac{1}{2}pk_b - \frac{1}{2}\log C$

pH = 11.91

10. Ans. (C)

EMF = $E_{cathode} - E_{Anode}$

$E_{H^+/H_2} = E^\circ_{H^+/H_2} - \frac{0.0591}{2}\log\frac{1}{[H^+]^x}$

$E_{H^+/H_2} = -0.0591\,pH$

EMF = $-0.0591\ pH_{cathode} - (-0.0591\ pH_{anode})$

$= (pH_{anode} - pH_{cathod})\ 0.06$

$= (11.91 - 2.09)\ 0.06 = 0.59$

11. Ans. (B)

12. Ans. (C)

13. Ans. (A)

14. Ans. (C)

15. Ans.(A)

16. Ans. (B)

17. Ans. (A)

18. Ans. (B)

SECTION-II

1. Ans. (A)-(T); (B)-(P,T); (C)-(R,T); (D)-(Q)

2. Ans. (A)-(Q,T); (B)-(P,T); (C)-(S); (D)-(R,T)

PART-3 : MATHEMATICS SOLUTION

SECTION-I

1. Ans. (A,B,D)

$f(x) = 4\sin^3 x$, $g(x) = 2[1 - \cos x]^2$, $h(x) = 1 - 2\cos x$

$$I = \int_0^{\pi} f(x)g(x)h(x)dx = \int_0^{\pi} 8\sin^3 x(1-\cos x)^2(1-2\cos x)dx$$

by using king property

$$I = 8\int_0^{\pi} \sin^3 x(1+\cos x)^2(1+2\cos x)dx$$

$$= 8\int_0^{\pi}\left(2\frac{\sin x}{2}\cos\frac{x}{2}\right)^3\left[2\cos^2\frac{x}{2}\right]^2\left[4\cos^2\frac{x}{2}-1\right]dx$$

$$= 256\int_0^{\pi}\left(\sin\frac{x}{2}\right)^3\left(\cos\frac{x}{2}\right)^7\left(4\cos^2\frac{x}{2}-1\right)dx$$

Let $\cos\frac{x}{2} = t$

$$-\sin\frac{x}{2}.\frac{1}{2}dx = dt$$

$$I = -512\int_1^0 (1-t^2)t^7(4t^2-1)dt$$

$$I = 512\int_0^1 (4t^9 - t^7 - 4t^{11} + t^9)dt$$

$$I = 512\left[\frac{2}{5}-\frac{1}{8}-\frac{1}{3}+\frac{1}{10}\right] = \frac{64}{3}$$

$g(x) > h(x) \Rightarrow 2(1-\cos x)^2 > 1 - 2\cos x$

$\Rightarrow 2\cos^2 x + 2 - 2\cos x > 1$

$\Rightarrow \cos^2 x + (1-\cos x)^2 > 0$

always true $\forall\ x \in [-\pi, 2\pi]$

2. Ans. (B,D)

$g_2(x) = -\frac{1}{x-1}$

$g_3(x) = x$

$\therefore$ $g_{3n+1}(x) = g_1(x)$, $g_{3n+2}(x) = g_2(x)$, $g_{3n+3}(x) = g_3(x)$

Given $f(f(g_{51}(2))) = 4$

$\Rightarrow$ $f(f(2)) = 4$ $\Rightarrow$ $f(3) = 4$

By using $f(x+2) + f(x) = f(x+1)$ we get

$f(4) = 1$, $f(6) = -4$, $f(7) = -1$

3. Ans. (B,D)

P can be $\begin{bmatrix} 1 & - & - \\ - & 1 & - \\ - & - & 1 \end{bmatrix}$

6 matrices {6 options for fourth 1}

$\begin{bmatrix} 1 & - & - \\ - & - & 1 \\ - & 1 & - \end{bmatrix}$ 6 matrices

$\begin{bmatrix} - & 1 & - \\ 1 & - & - \\ - & - & 1 \end{bmatrix}$ 6 matrices

$\begin{bmatrix} - & 1 & - \\ - & - & 1 \\ 1 & - & - \end{bmatrix}$ 6 matrices

$\begin{bmatrix} - & - & 1 \\ - & 1 & - \\ 1 & - & - \end{bmatrix}$ 6 matrices

$\begin{bmatrix} - & - & 1 \\ 1 & - & - \\ - & 1 & - \end{bmatrix}$ 6 matrices

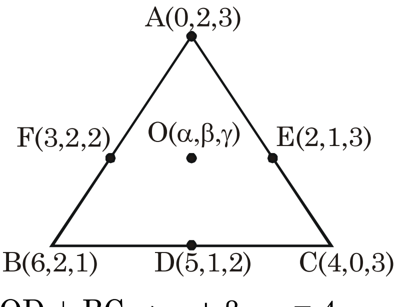
$N = 36 = 2^2.3^2$

Number of divisors is 9

sum of divisions is91

$|\text{adjoint } P| = |P|^2 = (\pm1)^2 = 1$

4. Ans. (A,C)



$OD \perp BC \Rightarrow \alpha + \beta - \gamma = 4$ ...(i)

$OE \perp AC \Rightarrow 2\alpha - \beta = 3$ ...(ii)

$OF \perp AB \Rightarrow 3\alpha - \gamma = 7$ ...(iii)

O,A,B,C are coplaner points.

$$\begin{vmatrix} \alpha & \beta-2 & \gamma-3 \\ 6 & 0 & -2 \\ 4 & -2 & 0 \end{vmatrix} = 0 \Rightarrow \alpha + 2\beta + 3\gamma = 13 \quad ...(iv)$$

$\therefore\ \alpha = \frac{20}{7}, \beta = \frac{19}{7}, \gamma = \frac{11}{7}$

and circumradius

$$r = \sqrt{\alpha^2 + (\beta-2)^2 + (\gamma-3)^2} = \frac{5}{7}\sqrt{21}$$

5. Ans. (B,C)

$C_1 : b^2(x^2 - 2hx + h^2) + a^2(y^2 - 2ky + k^2) = a^2b^2$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Let $P_1(h + a\cos\theta, k + b\sin\theta)$

$$F_1(h - ae_1, k),\ e_1 = \frac{\sqrt{a^2-b^2}}{a}$$

$S_1(h + ae_1, k)$

$\therefore$ centroid of $\Delta P_1F_1S_1$:

$$\left(h + \frac{a}{3}\cos\theta, k + \frac{b}{3}\sin\theta\right)$$

locus of centroid is

$$C_2 : \frac{(x-h)^2}{\left(\frac{a}{3}\right)^2} + \frac{(y-k)^2}{\left(\frac{b}{3}\right)^2} = 1$$

similarly we get $C_n : \frac{(x-h)^2}{\left(\frac{a}{3^{n-1}}\right)^2} + \frac{(y-k)^2}{\left(\frac{b}{3^{n-1}}\right)^2} = 1$

all curves are ellipse with same centre and same eccentricity

$$F_n\left(h - \frac{a}{3^{n-1}}e, k\right)$$

$$S_n\left(h + \frac{a}{3^{n-1}}e, k\right)$$

$$L_n = \frac{2b^2}{3^{n-1}a}$$

$$\sum_{i=1}^{6} x_i = \sum_{i=1}^{6}\left(h - \frac{a}{3^{i-1}}e\right) = 6h - \frac{364}{243}\sqrt{a^2-b^2}$$

$$3^n\sum_{i=1}^{n} L_i = \left(\frac{2b^2}{a}\sum_{i=1}^{n}\frac{1}{3^{i-1}}\right).3^n$$

$$= 4\left[\frac{1-\frac{1}{3^n}}{1-\frac{1}{3}}\right].3^n = \frac{3}{2}\left(3^n - 1\right)L_1$$

we get minimum value 6.

6. Ans. (B,C)

$2\cos2\alpha - 3 = 2\cos(2\alpha + \beta) - 2\cos\beta$

$2(\cos2\alpha + \cos\beta) = 3 + 2\cos(2\alpha + \beta)$

$$2\left(2\cos\left(\alpha+\frac{\beta}{2}\right)\cos\left(\alpha-\frac{\beta}{2}\right)\right) = 3 + 4\cos^2\left(\alpha+\frac{\beta}{2}\right) - 2$$

$$4\cos^2\left(\alpha+\frac{\beta}{2}\right) - 2\left(2\cos\alpha+\frac{\beta}{2}\right)\cos\left(\alpha-\frac{\beta}{2}\right) + 1 = 0$$

$$\left(2\cos\left(\alpha+\frac{\beta}{2}\right) - \cos\left(\alpha-\frac{\beta}{2}\right)\right)^2 + \left(\sin\left(\alpha-\frac{\beta}{2}\right)\right)^2 = 0$$

$\therefore\ \sin\left(\alpha-\frac{\beta}{2}\right) = 0$ and $2\cos\left(\alpha+\frac{\beta}{2}\right) - \cos\left(\alpha-\frac{\beta}{2}\right) = 0$

case : $\cos\left(\alpha-\frac{\beta}{2}\right) = 1, \cos\left(\alpha+\frac{\beta}{2}\right) = \frac{1}{2}$,

$$\alpha - \frac{\beta}{2} \in \left(-\frac{\pi}{2}, 2\pi\right)$$

$$\alpha + \frac{\beta}{2} \in \left(-\frac{\pi}{2}, 3\pi\right)$$

$$\alpha - \frac{\beta}{2} = 0$$

$$\alpha + \frac{\beta}{2} = -\frac{\pi}{3} \text{ or } \frac{\pi}{3} \text{ or } \frac{5\pi}{3} \text{ or } \frac{7\pi}{3}$$

we get $\alpha = \frac{\pi}{6}, \beta = \frac{\pi}{3}$

$\alpha = \frac{5\pi}{6}, \beta = \frac{5\pi}{3}$

Case : $\cos\left(\alpha-\frac{\beta}{2}\right) = 1$ and $\cos\left(\alpha+\frac{\beta}{2}\right) = -\frac{1}{2}$,

$$\alpha - \frac{\beta}{2} \in \left(-\frac{\pi}{2}, 2\pi\right)$$

$$\alpha + \frac{\beta}{2} \in \left(-\frac{\pi}{2}, 3\pi\right)$$

$$\alpha - \frac{\beta}{2} = \pi$$

$$\alpha + \frac{\beta}{2} = \frac{2\pi}{3} \text{ or } \frac{4\pi}{3}$$

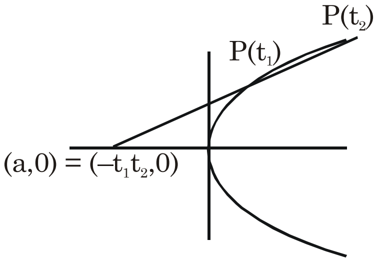
we get $\alpha = \frac{7\pi}{6}, \beta = \frac{\pi}{3}$

we get 3 values of α and 2 values of β.

$n = 3, m = 2$

$$\sum_{i=1}^{m}\beta_i = 2\pi$$

7. Ans. (A)



Let $P(t_1^2, 2t_1)$, $t_1 > 0$

$Q(t_2^2, 2t_2)$, $t_2 > 0$

centre of circle $\left(\frac{t_1^2+t_2^2}{2}, t_1+t_2\right)$

$\because$ centre lie on y = x

$\Rightarrow \frac{t_1^2+t_2^2}{2} = t_1+t_2$

$\Rightarrow (t_1-1)^2+(t_2-1)^2 = 2$ ...(1)

slope of line $m = \frac{2}{t_1+t_2}$

$\Rightarrow \frac{1}{m} = \frac{t_1+t_2}{2}$ $\{\because t_1+t_2 \in (2,4)\}$

$\Rightarrow \frac{1}{m} \in (1,2) \therefore M = 0$

$a = -t_1t_2 \Rightarrow a \in (-4,0) \Rightarrow N = 3$

8. Ans. (A,D)

$g(x) = x\cos 5\alpha + \sin 5\alpha$

$g'(x) = \cos 5\alpha$

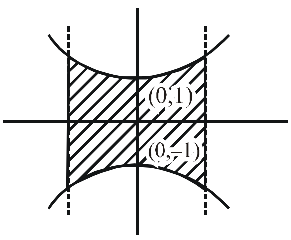
$\therefore\ g(x) = xg'(x) \pm \sqrt{1-(g'(x))^2}$

$(g(x))^2 + x^2(g'(x))^2 - 2xg(x)g'(x) = 1-(g'(x))^2$

$\therefore$ degree 2 and order 1.

$\because\ |g(x)| \le \sqrt{x^2+1}$

$\therefore (g(x))^2 - x^2 \le 1$



area $= 4\int_0^1 \sqrt{1+x^2}\,dx$

$= 4\left[\frac{x}{2}\sqrt{1+x^2} + \frac{1}{2}\ell n\left(x+\sqrt{x^2+1}\right)\right]_0^1$

$= 2\sqrt{2} + \ell n\left(3+2\sqrt{2}\right)$

9. Ans. (A)

$\prod_{i=1}^{9} I_{\left(\frac{i}{10}\right)} = I_{\frac{1}{10}}.I_{\frac{2}{10}}.I_{\frac{3}{10}}.....I_{\frac{9}{10}}$

$= \left(I_{\frac{1}{10}}.I_{\frac{9}{10}}\right)\left(I_{\frac{2}{10}}.I_{\frac{8}{10}}\right)\left(I_{\frac{3}{10}}.I_{\frac{7}{10}}\right)\left(I_{\frac{4}{10}}.I_{\frac{6}{10}}\right)\left(I_{\frac{5}{10}}\right)$

$= \frac{\pi}{\sin\frac{\pi}{10}}.\frac{\pi}{\sin\frac{2\pi}{10}}.\frac{\pi}{\sin\frac{3\pi}{10}}.\frac{\pi}{\sin\frac{4\pi}{10}}.\sqrt{\frac{\pi}{\sin\frac{\pi}{2}}}$

$= \frac{\pi^{9/2}}{\left(\sin\frac{\pi}{10}\cos\frac{\pi}{10}\right)\left(\sin\frac{2\pi}{10}\cos\frac{2\pi}{10}\right)} = \frac{4\pi^{9/2}}{\sin\frac{2\pi}{10}.\sin\frac{4\pi}{10}}$

$= \frac{8\pi^{9/2}}{\cos\frac{2\pi}{10} - \cos\frac{6\pi}{10}} = \frac{16\pi^{9/2}}{\sqrt{5}} = 16\sqrt{\frac{\pi^9}{5}}$

10. Ans. (D)

$AB = \int_0^\infty x^2 e^{-x^4}dx.\int_0^\infty e^{-x^4}dx$

put $x = t^{1/4} \Rightarrow dx = \frac{1}{4}t^{-\frac{3}{4}}dt$

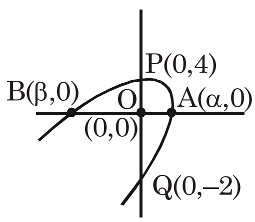
$= \int_0^\infty \frac{t^{1/2}e^{-t}}{4}t^{-3/4}dt \int_0^\infty \frac{e^{-t}t^{-3/4}}{4}dt$

$= \frac{1}{16}\left[\int_0^\infty e^{-t}t^{-1/4}dt \int_0^\infty e^{-t}.t^{-3/4}dt\right]$

$= \frac{1}{6}\left(I_{\frac{3}{4}} I_{\frac{1}{4}}\right) = \frac{1}{16}\frac{\pi}{\sin\left(\frac{\pi}{4}\right)} = \frac{1}{\left(\frac{8\sqrt{2}}{\pi}\right)}$

$\therefore\ k = \frac{8\sqrt{2}}{\pi} \Rightarrow [k] = 3$

Paragraph for Question 11 and 12



11. Ans. (C)

semilatus rectum is harmonic mean of OP and OQ

$\frac{\ell}{2} = \frac{2.4.2}{4+2} = \frac{8}{3} \Rightarrow \ell = \frac{16}{3}$

12. Ans. (D)

by using focus (0,0), point P(0,4) and Q(0,–2) we get directrix

$y = 2\sqrt{2}x - 8$ or $y = -2\sqrt{2}x - 8$

so required parabola is

$$x^2 + y^2 = \left[\frac{y - 2\sqrt{2}x + 8}{3}\right]^2$$

or

$$x^2 + y^2 = \left[\frac{y + 2\sqrt{2}x + 8}{3}\right]^2$$

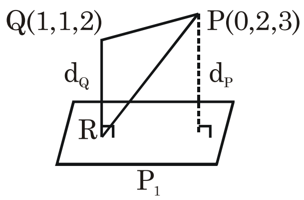
for A,B put y = 0

$\therefore\ 9x^2 = 8x^2 + 64 \pm 32\sqrt{2}x$

$$\Rightarrow x^2 \pm 32\sqrt{2}x - 64 = 0 \begin{cases}\alpha \\ \beta\end{cases}$$

$$|AB| = \sqrt{\left(32\sqrt{2}\right)^2 - 4(-64)} = 48$$

Paragraph for Question 13 and 14



$d_Q < d_P$

$\therefore$ R is foot of perpendicular from point Q on plane P_1

$$\therefore\ R\left(\frac{1}{7}, -\frac{2}{7}, -\frac{4}{7}\right)$$

$$\frac{\alpha + \beta}{\beta + \gamma} = \frac{1}{6}$$

13. Ans. (B)

14. Ans. (B)

15. Ans. (D)

Let y = mx + c is common tangent

$$\therefore\ c = -m^2 = \pm\sqrt{\frac{m^2}{4} + \frac{3}{4}} = \frac{a}{m}$$

$\Rightarrow$ m = –1, a = 1, c = –1

common tangent y = –x – 1

x + y + 1 = 0

ℓ = 1, m = 1

16. Ans.(B)

$$v = \left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]^2 = \begin{vmatrix}1 & 1 & 1 \\ 2 & -3 & 1 \\ 1 & -1 & 2\end{vmatrix}^2 = 49$$

$$\left(\vec{a} \times \vec{b}\right) \times \vec{c} = \left(\vec{a}.\vec{c}\right)\vec{b} - \left(\vec{b}.\vec{c}\right)\vec{a} = -3\hat{i} - 13\hat{j} - 5\hat{k}$$

$$\left(\sqrt{v}\hat{i} - \hat{j} + \hat{k}\right).\left(\hat{i} + \lambda\hat{j} + 2\hat{k}\right) = 0 \Rightarrow \sqrt{v} - \lambda + 2 = 0$$

$\lambda = 9$

17. Ans. (A)

$P(x_1, y_1) \equiv (t_1^2, 2t_1)$

$Q(x_2, y_2) \equiv (t_1^2, 2t_2)$

$R(-15, -2) \equiv (t_1t_2,\ t_1 + t_2)$

$\therefore\ t_1 = -5,\ t_2 = 3$

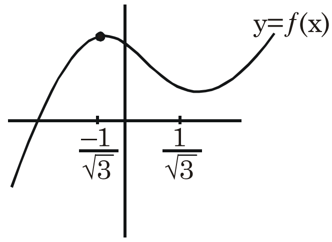
P(25,–10), Q(9, 6)

intersection point of normals at P,Q is T(21,–30)

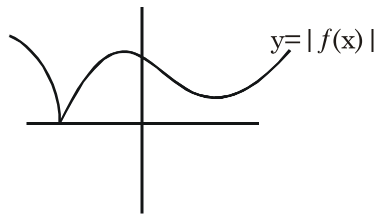
$\therefore$ circumcentre $(x_0, y_0) \equiv (3, -16)$ is midpoint of TR

$$S = \frac{(S_1)^{3/2}}{2a} = 256$$

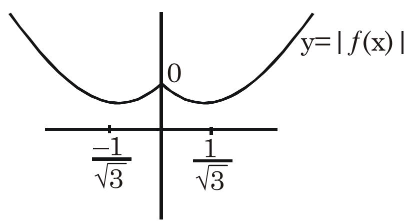
18. Ans. (D)



N = 1



M = 1



m =2

$f''(x) = 6x \quad \therefore\ x_0 = O$ is point of inflection

SECTION – II

1. Ans. **(A)→(R); (B)→(P); (C)→(S); (D)→(Q)**

(A) $f(x)=(x-1)(x+1)\,|(x-1)(x-4)|+|\cos x|$, $x \in (-2\pi, 2\pi)$ it is non differentiable at

$$x=4,\frac{\pi}{2},\frac{-\pi}{2},\frac{3\pi}{2},\frac{-3\pi}{2}$$

(B) $\begin{vmatrix}1 & 2 & 0\\ 2 & k & 1\\ k & 4 & 2\end{vmatrix}=0$

2k – 4 – 2(4 – k) = 0

2k – 4 – 8 + 2k = 0

k = 3

(C) The product of the length's of the perpendicular segments from the foci on any tangent to the ellipse

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a^2<b^2$, is a^2

product is 6

(D) $\int_0^4 \left||x-2|-2\right|dx$

$\int_0^2 |x|\,dx+\int_2^4 |x-4|\,dx$

$\int_0^2 x\,dx+\int_2^4 (4-x)\,dx = 4$

2. **Ans. (A)→(S); (B)→(P); (C)→(R); (D)→(Q)**

(A) these are collinear

$\therefore \begin{vmatrix}60 & 3 & 1\\ 40 & -8 & 1\\ a & 14 & 1\end{vmatrix}=0$

a = 80

(B) $\frac{dy}{dx}=\frac{1}{x}+2bx+a$

$\left(\frac{dy}{dx}\right)_{x=\frac{1}{2}}=0 \Rightarrow 2+b+a=0$

$\left(\frac{dy}{dx}\right)_{x=1}=0 \Rightarrow 1+2b+a=0$

$\therefore$ b = 1

a = – 3

(C) $|\sin^2\alpha + 3\sin\alpha| = 2$

$\therefore \sin^2\alpha + 3\sin\alpha = 2$

$\sin^2\alpha + 3\sin\alpha - 2 = 0$

$\sin\alpha=\frac{-3+\sqrt{17}}{2}, \sin\alpha=\frac{-3-\sqrt{17}}{2}$

or

$\sin^2\alpha + 3\sin\alpha = -2$

$\sin^2\alpha + 3\sin\alpha + 2 = 0$

$\sin\alpha = -1$ or $\sin\alpha = -2$

$\therefore \sin\alpha=\left(-3+\sqrt{17}\right)/2$ or $\sin\alpha = -1$

$\Rightarrow$ 3 values of α.

(D) $\sqrt{|\log\alpha|}=\sqrt{1-\frac{(\log\alpha)^2}{|\log\alpha|}}$

on squaring $|\log\alpha|=\frac{|\log\alpha|-(\log\alpha)^2}{|\log\alpha|}$

put $|\log\alpha| = t$

$t^2 = t - t^2$

t = 0 or $\frac{1}{2}$

$|\log\alpha|=\frac{1}{2}\Rightarrow \alpha=\sqrt{e}$ or $\frac{1}{\sqrt{e}}$