Paper Code : 1001CT102116013

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2016 - 2017)

JEE (Main + Advanced) : ENTHUSIAST COURSE (PHASE : I)

ANSWER KEY : PAPER-1 **TEST DATE : 24-07-2016**

Test Type : MINOR Test Pattern : JEE-Advanced

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,B,D	A,B,C,D	A,D	A,B,C	A,B,C,D	A,C,D	D	A,B,C	C,D	B,D
	Q.	11	12	13	14						
	A.	A	B	C	D						
SECTION-IV	Q.	1	2	3	4	5	6				
	A.	1	4	5	6	4	8				

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,B,C,D	C,D	A,D	A,B,D	A,B,C,D	A,D	A,C	D	B,D	B,C,D
	Q.	11	12	13	14						
	A.	A	B	B	C						
SECTION-IV	Q.	1	2	3	4	5	6				
	A.	7	9	9	2	3	2				

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,C	A,C	A,B,C	A,B,D	A,C,D	B,D	A,C,D	C,D	B,C	A,B,C
	Q.	11	12	13	14						
	A.	A	C	B	C						
SECTION-IV	Q.	1	2	3	4	5	6				
	A.	7	0	4	3	1	4				

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JEE (Main + Advanced) : ENTHUSIAST COURSE (PHASE : I)

ANSWER KEY : PAPER-2 **TEST DATE : 24-07-2016**

Test Type : MINOR Test Pattern : JEE-Main

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	4	3	3	4	2	3	1	2	3	3	2	4	1	3	2	2	2	1	4	1
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	1	2	4	2	4	4	1	2	2	3	3	4	1 or 2	3	2	2	1	3	2	2
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	1	4	3	4	3	4	4	4	1	3	2	4	3	1	2	4	3	2	1	4
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	2	3	1	1	1	2	3	3	1	1	4	2	1	1	1	2	3	4	1	1
Que.	81	82	83	84	85	86	87	88	89	90										
Ans.	3	1	2	1	2	2	4	1	2	1										

JEE (Main + Advanced) : ENTHUSIAST COURSE
PHASE : I

Test Type : MINOR   Test Pattern : JEE-Advanced

TEST DATE : 24 - 07 - 2016

PAPER-1

PART-1 : PHYSICS   SOLUTION

SECTION-I

1. Ans. (A,B,D)

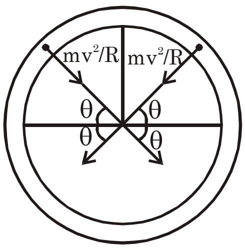
Sol. $\vec{F} = F_0\left(\frac{r}{R_0}\right)^2 \hat{r}$ is a central force, because direction of force is always away from the fix point origin & work done by this force is zero in any close loop $\oint \vec{F}.d\vec{r} = 0$ force is conservative

Work done by the force on bead

$$= \int_0^{2R}\left(\frac{Fr^2}{R_0^2}\hat{r}\right).d\vec{r} = \frac{F}{R_0^2}\left[\frac{r^3}{3}\right]_0^{2R} = \frac{8F_0R_0}{3}$$

2. Ans. (A,B,C,D)

Sol. At any time t.



$$\theta = \frac{vt}{R}$$

Net external force F

$$= \frac{mv^2}{R}\left[\cos\left(\frac{vt}{R}\right)\hat{i} - \sin\left(\frac{vt}{R}\right)\hat{j}\right]$$

$$- \frac{mv^2}{R}\left[\cos\left(\frac{vt}{R}\right)\hat{i} + \sin\left(\frac{vt}{R}\right)\hat{j}\right]$$

$$F_{net} = -\frac{2mv^2}{R}\sin\theta\hat{j}$$

$$a_{COM} = \frac{F_{net}}{2m} = -\frac{v^2}{R}\sin\theta\hat{j}$$

Force on right particle

$$= -\frac{mv^2}{R}\cos\theta\hat{i} - \frac{mv^2}{R}\sin\theta\hat{j}$$

Force on left particle

$$= \frac{mv^2}{R}\cos\theta\hat{i} - \frac{mv^2}{R}\sin\theta\hat{j}$$

3. Ans. (A, D)

Sol. $U = \frac{1}{2}\in_0 E^2$ C value

$$= \frac{1}{2}\in_0\left(\frac{Q_0}{\ell^2\in_0}\right)^2(\ell - x)\,\ell\,d$$

$$\frac{1}{2}\frac{Q_0^2}{\ell^3\in_0}(\ell - x)d$$

$$F = \frac{-dU}{dx} = -\left(\frac{1}{2}\frac{Q_0^2}{\ell^3\in_0}(-1)d\right) = \frac{Q_0^2 d}{2\ell^3\in_0}$$

Energy density

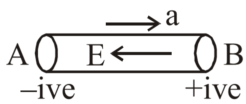
$$= \frac{1}{2}\in_0 E^2 = \frac{1}{2}\in_0\frac{Q_0^2}{(\ell^2\in_0)^2} = \frac{Q_0^2}{2\ell^4\in_0}$$

4. Ans. (A, B, C)

Sol. Properties of field lines.

5. Ans. (A,B,C,D)

Sol. Due to pusedo force, electrons move from B to A.



$\Rightarrow v_B > v_A$

In figure (b) electron moves from A to B

6. Ans. (A,C,D)

Sol. $i = \dfrac{24}{3+9+6} = \dfrac{4}{3}A$

$V_1 = \dfrac{4}{3} \times 9 = 12$

$V_2 = \dfrac{4}{3} \times 6 = 8V$

at $t = \infty$

$V_2 = V_1 = 24$

7. Ans. (D)

Sol. Bacterium act as a dipole & here magnetic field due to earth is non-uniform and bacterium may rise up or fall because in non-uniform field dipole experience net force.

8. Ans. (A,B,C)

Sol. Energy of dipole interaction

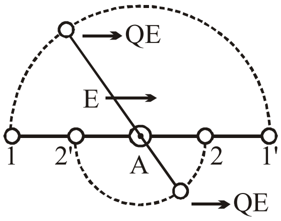
$= \dfrac{\mu_0}{4\pi}\dfrac{2M^2}{d^3} = KT$

$\Rightarrow M = \sqrt{\dfrac{2\pi KTd^3}{\mu_0}}$

9. Ans. (C,D)

10. Ans. (B,D)

Sol.



Position of maximum velocity is 1' and 2' for the balls as it is minimum potential energy position

Potential difference between 1 and 1'

$= V_1 - V_{1'} = E \times \left(\dfrac{3L}{4} \times 2\right)$

Potential difference between 2 and 2'

$= V_2 - V_{2'} = E \times \left(\dfrac{L}{4} \times 2\right)$

Change in electrostatic potential energy

$= Q \times (V_{1'} - V_1) + Q \times (V_{2'} - V_2) = -QEL$

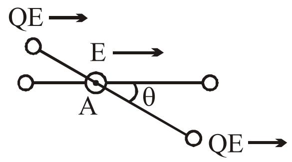
(loss in PE)

$= \text{Gain in KE} = \dfrac{1}{2}mV^2 + \dfrac{1}{2}m(3V)^2$

(where velocity of the ball closer to the axis is V & farther from axis is 3V)

$\Rightarrow V = \sqrt{\dfrac{QEL}{5m}}$

Maximum velocity is $3V = \sqrt{\dfrac{9QEL}{5m}}$



For small angular displacement θ from equilibrium position restoring torque

$= QE\sin\theta\left(\dfrac{3L}{4} - \dfrac{L}{4}\right) = I\alpha$

(since θ is very small $\sin\theta \approx \theta$)

$\Rightarrow \theta = \left(\dfrac{QEL}{2I}\right)(-\theta) \Rightarrow \omega = \sqrt{\left(\dfrac{QEL}{2I}\right)}$ where

I (moment of inertia) $= \left(\dfrac{mL^2}{16} + \dfrac{9mL^2}{16}\right)$

Time Period (T) $= 2\pi/\omega = 2\pi\sqrt{\dfrac{5mL}{4QE}}$

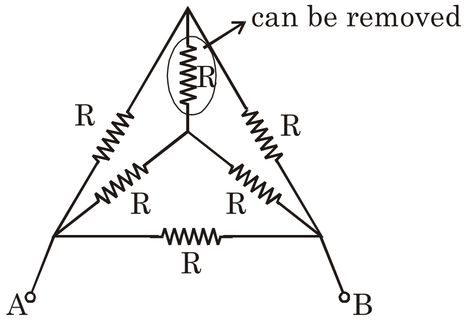
$= \pi\sqrt{\dfrac{5mL}{QE}}$

11. Ans. (A)

12. Ans. (B)

13. Ans. (C)

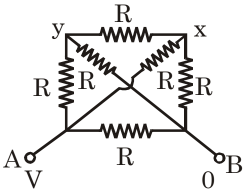
Sol. (Q) Equivalent diagram



$$\Rightarrow \frac{1}{R_{eq}} = \frac{1}{2R} + \frac{1}{2R} + \frac{1}{R}$$

$$\Rightarrow R_{eq} = \frac{R}{2}$$

(S)



Assigning potential to each corner & solving

$$R_{eq} = \frac{R}{2}$$

14. Ans. (D)

Sol. Initially

$$U_1 = \frac{Q^2}{2C}$$

$$u_{V_1} = \frac{Q^2}{2A^2 \in_0}$$

$$F_1 = \frac{Q^2}{2A \in_0}$$

$$V_1 = \frac{Q}{C}$$

Finally

$$C^1 = \in_r C \Rightarrow U_2 = \frac{Q^2}{2 \in_r C}$$

$$u_{V_2} = \frac{Q^2}{2A^2 \in_r \in_0}$$

$$F_2 = \frac{Q^2}{2A \in_0}$$

Since it is force between plates & will not change due to dielectric

$$V_2 = \frac{Q}{\in_r C}$$

SECTION-IV

1. Ans. 1

Sol. F = iLB

$$mv = \int F dt = \int (idt) LB$$

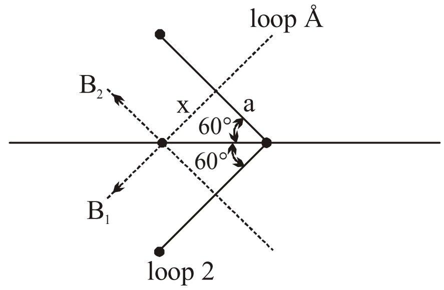
$$m \cdot \sqrt{2gh} = QLB$$

$$m\sqrt{2g\ell(1-\cos\theta)} = QLB$$

$\theta = 37°$

2. Ans. 4

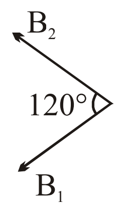
Sol.



$$B_1 = \frac{\mu_0 a^2 i}{2(3a^2 + a^2)^{3/2}} = \frac{\mu_0 i}{16a}$$

$$B_2 = \frac{\mu_0 a^2 i}{2(3a^2 + a^2)^{3/2}} = \frac{\mu_0 i}{16a}$$

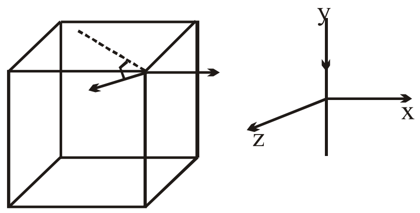
$B_{net} = B_1{}^2 + B_2{}^2 + 2B_1B_2 \cos 120°$



$$B_{net} = \frac{\mu_0 i}{16a}$$

3. Ans. 5

Sol.



$$B = \frac{4\pi\times10^{-7}}{2\pi}\times\frac{8}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}-\hat{i}+\frac{1}{\sqrt{2}}\hat{k}\right]+1.4\times10^{-6}\,\hat{i}$$

$$= \left(-8\hat{i}+8\hat{k}+14\hat{i}\right)\times10^{-7} = (6\hat{i}+8\hat{k})\times10^{-7}$$

$B = 10 \times 10^{-7}$ T

Ans 10

4. Ans. 6

Sol. In Fig. call I, the ammeter current directed down, I_1 the current up through the left-hand emf, and I_2 the current up through the right-hand emf. Loop and junction equations give $E - I_r - I_1fR_0 = 0$, $E - Ir - I_2(1-f)R_0 = 0$, and $I = I_1 + I_2$. Solving, $I = E/(r + R_0f - R_0f^2)$.

The denominator is smallest when f = 0 or 1, making I a maximum.

Since $f - f^2 = \frac{1}{4}-\left(\frac{1}{2}-f\right)^2$, I is smallest for

$f = \frac{1}{2}$.

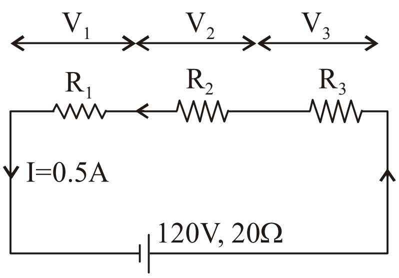
5. Ans. 4

Sol. $V_1 + V_2 + V_3 = 120 - 20\ (0.5)$

$\Rightarrow\ V_1 + V_2 + V_3 = 110$ V .... (1)

Also $V_1 + V_2 = 60 \Rightarrow V_3 = 110 - 60 = 50$V

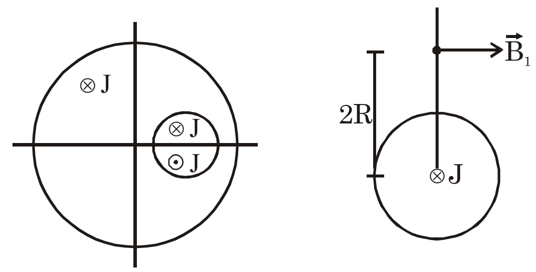
Now $V_2 + V_3 = 90 \Rightarrow V_2 = 40$V



$\therefore\ R_2 = \frac{40}{0.5} = 80\Omega$

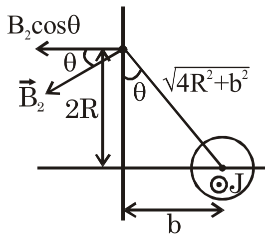
6. Ans. 8

Sol.



B_1 due to total wire at y = 2R is $= \frac{\mu_0 i}{2\pi(2R)}\hat{i}$

$$= \frac{\mu_0\left(J\pi R^2\right)}{2\pi(2R)}\hat{i} = \frac{\mu_0 JR}{4}\hat{i}$$



Horizontal component of magnetic field due to smaller wire $B_{2\,x}$

$$= \frac{-\mu_0 i}{2\pi\left(\sqrt{4R^2+b^2}\right)}\cdot\cos\theta\hat{i}$$

$$= -\frac{\mu_0 J\pi a^2}{2\pi\left(\sqrt{4R^2+b^2}\right)}\cdot\frac{2R}{\sqrt{4R^2+b^2}}\hat{i}$$

$$= -\frac{\mu_0 Ja^2R}{\left(4R^2+b^2\right)}\hat{i}$$

$$\Rightarrow \vec{B}_{net\ x} = \vec{B}_{1x}+\vec{B}_{2x} = \mu_0 JR\left[\frac{1}{4}-\frac{a^2}{4R^2+b^2}\right]$$

$\Rightarrow$ A = 4, B = 4

|A| + |B| = 8

PART–2 : CHEMISTRY SOLUTION

SECTION - I

1. Ans. (A,B,C,D)

2. Ans. (C, D)

3. Ans. (A, D)

4. Ans. (A, B, D)

5. Ans. (A,B,C,D)

6. Ans. (A,D)

7. Ans. (A, C)

8. Ans. (D)

9. Ans. (B,D)

10. Ans. (B,C,D)

11. Ans. (A)

12. Ans. (B)

13. Ans. (B)

14. Ans. (C)

SECTION - IV

1. Ans. (7)

2. Ans. (900) OMR ANS (9)

TV^{x-1} = constant

TV^2 = constant

x = 3

$$w = \frac{nR\Delta T}{x-1} = \frac{1\times2\times900}{2} = 900 \text{ cal.}$$

3. Ans. (1440)

OMR ANS. (9)

Sol. $$\log\frac{k_2}{k_1} = \frac{E_a}{2.303R}\left[\frac{1}{T_1}-\frac{1}{T_2}\right]$$

$$\log\frac{t_1}{t_2} = \frac{E_a}{2.303R}\left[\frac{1}{340}-\frac{1}{350}\right]$$

$$= \frac{23.8}{2\times10^{-3}}\left[\frac{1}{340}-\frac{1}{350}\right]$$

$$\log\frac{t_1}{t_2} = 1 \; ; \; \frac{t_1}{t_2} = 10 = 1$$

$$t_2 = \frac{t_1}{10} = \frac{4\times60\times60}{10} = 1440 \text{ sec.}$$

4. Ans. (2)

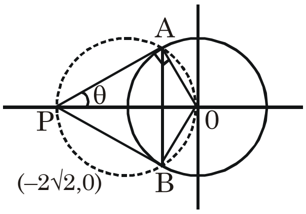
5. Ans. (3)

6. Ans. (2)

PART-3 : MATHEMATICS SOLUTION

SECTION-I

1. Ans. (A,C)



Circumcircle of ΔPAB also passes through the centre of given circle.

$\therefore$ radius of required circle $=\sqrt{2}$

$\Rightarrow$ Area = 2π.

Also, in ΔPAO,

$$\sin\theta=\frac{2}{2\sqrt{2}}=\frac{1}{2\sqrt{2}}=\frac{1}{\sqrt{2}}\Rightarrow\theta=\frac{\pi}{4}$$

$$\Rightarrow \angle APB=\frac{\pi}{2}$$

2. Ans. (A,C)

Equation of tangent at point P to the given curve is $x + y\sqrt{3} = 4$

$$\frac{x}{4}+\frac{y}{4/\sqrt{3}}=1$$

$$p = 4,\ q=\frac{4}{\sqrt{3}}$$

3. Ans. (A,B,C)

Given curves $r=\dfrac{a}{b\sin\theta+c\cos\theta}$

$\Rightarrow a = b.\ r\sin\theta + c.r\cos\theta$

$\Rightarrow cx + by = a$ ....(1)

& $\dfrac{1}{r}=\dfrac{d}{e\sin\theta+f\cos\theta}$

$\Rightarrow e.\ r\sin\theta + f\ r\cos\theta = d.r^2$

$\Rightarrow x^2+y^2-\dfrac{e}{d}y-\dfrac{f}{d}x=0$ ...(2)

equation (1) represents a straight line and equation (2) represents a circle also number points of intersection of both curves can be 0 or 1 or 2.

4. Ans. (A,B,D)

(A) If $\lim\limits_{x\to\infty} f(x)=\ell(\text{a non}-\text{zero number})$,

then $\lim\limits_{x\to\infty}\left(f(x)+\int_0^x f(t)dt\right)$ does not

exist $\therefore \lim\limits_{x\to\infty} f(x)=0$

(B) Let $\lim\limits_{x\to 0}\dfrac{f(x)}{x^2}=\ell$, Now

$$f'(0)=\lim_{x\to 0}\frac{f(x)}{x}=\lim_{x\to 0}x.\frac{f(x)}{x^2}=\lim_{x\to 0}x.\ell=0$$

(C) Use I.V.T. $\therefore \dfrac{2}{\frac{1}{x_1}+\frac{1}{x_2}}\in[x_1,x_2]$

(D) True.

5. Ans. (A,C,D)

$\because \dfrac{f(1)-f(0)}{1-0}=f'(c)$ where $c \in (0,1)$

$\Rightarrow f(1) = f'(c) \geq f'(1)$ ($\because f'(x)$ is non-increasing)

$\Rightarrow f(1) \geq f'(1)$,

Also, $\dfrac{1}{1+f^2(x)}<1\Rightarrow\int_0^1\dfrac{1}{1+f^2(x)}dx<1$

$$\Rightarrow\int_0^1\frac{1}{1+f^2(x)}dx\le\frac{f(1)}{f'(1)}$$

(D) $\because f'(1) > 0$ and $f'(x)$ is non-increasing

$\Rightarrow f'(x) \geq f'(1)\ \forall\ x \in [0,1]$

but $f'(1) > 0 \Rightarrow f'(x) > 0\ \forall\ x \in [0,1]$

$\Rightarrow f(x) \uparrow$ on $x \in [0,1]$

6. Ans. (B,D)

$$\because\quad [x]-\left[x-\frac{1}{2}\right]=\begin{cases}1 & \text{if}\quad n\le x<n+\frac{1}{2}\\ 0 & \text{if}\quad n+\frac{1}{2}\le x<n+1\end{cases}$$

$$\therefore\quad \int_0^{[x]}\left([x]-\left[x-\frac{1}{2}\right]\right)dx$$

$$=\int_0^{1/2}1dx+\int_{1/2}^{1}0.dx+\int_1^{3/2}1.dx+.....+\int_{[x]-\frac{1}{2}}^{[x]}0.dx=\frac{[x]}{2}$$

$$\therefore\quad f(x)=\frac{1}{2}\int_0^{[x]}[x]dx$$

$$=\frac{1}{2}(0+1+2+3+.....+([x]-1))$$

$$=\frac{([x]-1).[x]}{4}$$

clearly $f(x)$ is discontinuous at x = 0,2

7. Ans. (A,C,D)

$$\because\ f(x)=\begin{cases}\sin 2x & \text{if } 0<x\le\frac{\pi}{6}\\ ax+b & \text{if } \frac{\pi}{6}<x<1\end{cases}$$

$$\Rightarrow f'(x)=\begin{cases}2\cos 2x & \text{if } 0<x<\frac{\pi}{6}\\ a & \text{if } \frac{\pi}{6}<x<1\end{cases}$$

$\because f(x)$ is continuous at $x=\frac{\pi}{6}$

$$\Rightarrow \frac{a\pi}{6}+b=\sin\frac{\pi}{3}\Rightarrow\frac{a\pi}{6}+b=\frac{\sqrt{3}}{2} \quad ...(1)$$

$f'(x)$ is discontinuous at $x=\frac{\pi}{6}$

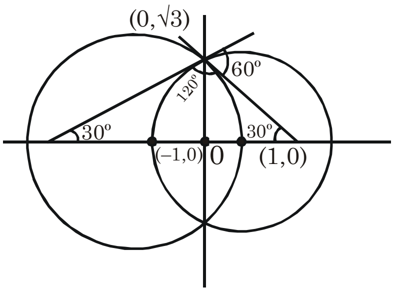
$$\Rightarrow a\ne 2\cos\frac{\pi}{3}\Rightarrow a\ne 1$$

8. Ans. (C,D)

If $f'(x)$ is continuous at $x=\frac{\pi}{6}$

$\Rightarrow a = 1$, then (1) $\Rightarrow$ $b=\frac{\sqrt{3}}{2}-\frac{\pi}{6}=\frac{3\sqrt{3}-\pi}{6}$

9. Ans. (B,C)



$C_1 : (x-1)^2+y^2=4$, $C_2 : (x+1)^2+y^2=4$

point of intersection of C_1 & C_2 is $=\left(0,\sqrt{3}\right)$.

Also, slope of tangents at this point to C_1&C_2 are $\frac{1}{\sqrt{3}}$ and $-\frac{1}{\sqrt{3}}$ respectively

$\Rightarrow$ Angle of intersection $=\frac{\pi}{3}$ or $\frac{2\pi}{3}$

10. Ans. (A,B,C)

11. Ans. (A)

(P) $\because f'(1+x)+f'(1-x)=0$, put $x = 0$, we get $f'(1) = 0$.

(Q) for $x<0$, $f(x)=-2\tan^{-1}x \Rightarrow f'(x)<0$ for $x<0$.

(R) $f(0)=0=f''(0)$ and $f'(-1)=f'(1)=2$

(S) $\int f\left(g(x)\right)dx=\int 1.dx=x+B$ $\therefore$ A = 1

12. Ans. (C)

(P) Using LMVT $\frac{1-2}{0+1}=2c\Rightarrow c=-\frac{1}{2}$

(Q) $\because f(x)=0\ \forall\ x\in(1,2)$

$\Rightarrow f'(x)=0\ \forall\ x\in(1,2)$

$\Rightarrow$ infinitely many critical points.

(R) $f'(x)=2.2^x\ell n2.(2^x-2)$

$\Rightarrow f'(x)>0$ for $x>1$.

(S) Rolle's theorem only applicable on x^2-1

13. Ans. (B)

Let $\ell n|x|=t$, then

$$I=\int\frac{tdt}{\sqrt{1+t}}=\int\frac{(1+t)-1}{\sqrt{1+t}}dt=\int\sqrt{1+t}dt-\int\frac{dt}{\sqrt{1+t}}$$

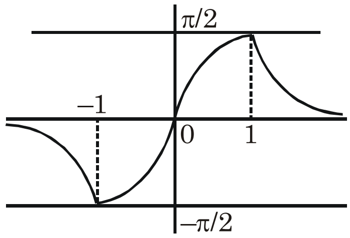
$$=\frac{2}{3}(1+t)^{3/2}-2\sqrt{1+t}+c$$

$$=2\sqrt{1+t}\left(\frac{1+t}{3}-1\right)+c=\frac{2}{3}\sqrt{1+t}\,(t-2)+c$$

$$I=\frac{2}{3}\sqrt{1+\ell n|x|}\left(\ell n|x|-2\right)+C$$

$\Rightarrow f(x)=|x|$, $g(x)=\ell n|x|$

14. Ans. (C)



$$f(x)=\begin{cases}2\tan^{-1}x & \text{for } -1\le x\le 1\\ \pi-2\tan^{-1}x & \text{if } x>1\\ -\pi-2\tan^{-1}x & \text{if } x<-1\end{cases}$$

Now check each part.

SECTION-IV

1. Ans. 7

$$f(x)=\begin{cases} e^{2x^2-3x} & \text{if } x>0 \\ px+q & \text{if } x\le 0 \end{cases}$$

continuous $x = 0 \Rightarrow 1 = q$

differentiability at $x = 0$

$$\Rightarrow\ e^{2x^2-3x}.(4x-3)\Big]_{x=0}=p\Rightarrow p=-3$$

$\therefore\ q - 2p = 1 + 6 = 7.$

2. Ans. 0

Clearly

$$I=2\int_0^{\pi}\frac{e^{|\sin x|}.\cos x}{e^{|\sin x|}+e^{-|\sin x|}}dx \qquad ...(1)$$

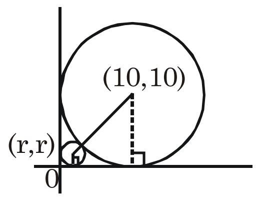
(by queen property)

using king property

$$I=-2\int_0^{\pi}\frac{e^{|\sin x|}\cos x}{e^{|\sin x|}+e^{-|\sin x|}}dx \qquad ...(2)$$

$\therefore$ (1) + (2) $\Rightarrow 2I = 0 \Rightarrow I = 0$

3. Ans. 4



$\because$ circles touches externally

$\Rightarrow C_1C_2 = r_1 + r_2$

$$\Rightarrow \sqrt{(10-r)^2+(10-r)^2}=10+r$$

squaring we get $r^2 - 60r + 100 = 0$

on solving $r = 30 - 20\sqrt{2}$

$$\Rightarrow\ r=10\left(3-2\sqrt{2}\right)$$

$\therefore$ radius of circles are $10, 10\lambda, 10\lambda, 10\lambda^3$....

(where $\lambda = 3 - 2\sqrt{2}$)

$\therefore$ sum of areas of all there circles

$$=100\pi\left(1+\lambda^2+\lambda^4+...\right)=\frac{100\pi}{1-\lambda^2}$$

$$=\frac{100\pi}{1-(9+8-12\sqrt{2})}=\frac{25\pi}{3\sqrt{2}-4}$$

$\Rightarrow m = 5, n = 3$

$$\Rightarrow\ \frac{m+n}{m-n}=4$$

4. Ans. 3

Let

$$f(x)=\left|(3+\sin x)+\frac{2}{3+\sin x}+(a-3)\right|=\left|y+\frac{2}{y}+(a-3)\right|$$

where $y = 3 + \sin x$.

Let $g(y)=y+\frac{2}{y}\Rightarrow g'(y)=1-\frac{2}{y^2}>0\ \forall y\in[2,4]$

$$\therefore\ \text{range of } g(y)=\left[3,\frac{9}{2}\right]$$

$$\therefore\ f(x)=\left|\underbrace{\underbrace{(3+\sin x)+\frac{2}{(3+\sin x)}}_{3\le\ \ \le\frac{9}{2}}+(a-3)}_{a\le\ \ \le a+\frac{3}{2}}\right|$$

$$\therefore\ f(a)=\begin{cases} -a & \text{if } a\le -\frac{3}{4} \\ a+\frac{3}{2} & \text{if } a\ge -\frac{3}{4} \end{cases}$$

(by definition of maximum function)

$\therefore\ f(a)_{min}=\frac{3}{4}=m\ \Rightarrow 4m = 3$

5. Ans. 1

$$\because\ e^{\frac{k\pi}{n^2}}-1=\frac{k\pi}{n^2}+\frac{1}{\underline{|2}}\left(\frac{k\pi}{n^2}\right)^2+........$$

$$\therefore\ \text{Given limit } =\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\frac{k\pi}{n}.\frac{\sin k\pi}{n}$$

(other terms are zero)

$$=\int_0^1 \pi x\sin\pi x dx=1 \text{ (by using by parts)}$$

6. Ans. 4

$I=\int_0^1 f(x)dx$, put $x = \sin t \Rightarrow dx = \cos t dt$

$$\Rightarrow\quad I=\int_0^1 f(\sin t).\cos t dt=\int_0^1 f(\cos t)\sin t dt$$

(by king property)

$$\Rightarrow\ 2I=\int_0^{\pi/2}(f(\cos t)\sin t+f(\sin t)\cos t)dt\le\int_0^{\pi/2}1.dt$$

($\because\ xf(y) + yf(x) \le 1$)

$$\Rightarrow\quad 2I\le\frac{\pi}{2}\Rightarrow I\le\frac{\pi}{4}$$

JEE (Main + Advanced) : ENTHUSIAST COURSE

PHASE : I

Test Type : MINOR   Test Pattern : JEE-Main

TEST DATE : 24 - 07 - 2016

PAPER-2

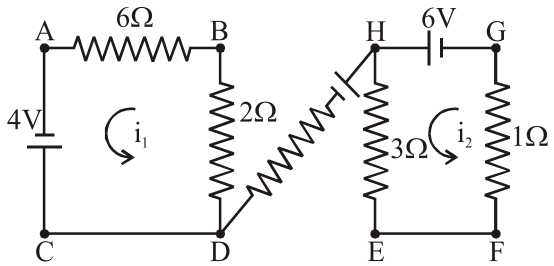
SOLUTION

1. Ans. (4)

Sol. use $\dfrac{B_{axis}}{B_{centre}} = \left(\dfrac{R^2}{R^2 + x^2}\right)^{3/2}$

2. Ans. (3)

Sol.



$i_1 = \dfrac{4}{8} = \dfrac{1}{2}A$

$i_2 = \dfrac{6}{4} = \dfrac{3}{2}A$

$V_H - V_D = 4V$

$V_D - V_B = 1V$

$V_H - V_B = 5v$

$V_H - V_D = 4V$ ....(i)

$V_H - V_G = 6V$ ....(ii)

Equation (ii) – (i)

$V_D - V_G = 2V$

3. Ans. (3)

Sol. $\dfrac{KQ}{r} + \dfrac{KQ}{2r} + \dfrac{KQ}{4r} + ...$

$\dfrac{KQ}{r}\left[1 + \dfrac{1}{2} + \dfrac{1}{4} + ...\right]$

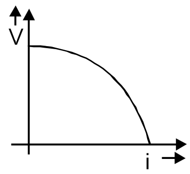
$\dfrac{KQ}{r}\left[\dfrac{1}{1-1/2}\right] = \dfrac{2KQ}{r}$

4. Ans. (4)

Sol. $r = Ki$

$V = E - ir$

$V = E = i^2 K$



5. Ans. (2)

Sol. n–1 capacitance are in series combination.

$C_1 = \dfrac{C}{n-1}$

$C_{eq} = \dfrac{C}{n-1} + C = \dfrac{C + (n-1)C}{n-1} = \dfrac{Cn}{n-1}$

6. Ans. (3)

Sol. $\vec{V} \perp \vec{a}$

$\vec{a} \cdot \vec{V} = 0$

$a_1b_1 + a_2b_2 = 0$

Magnetic force will always perpendicular to velocity ⇒ KE remain constant.

7. Ans. (1)

Sol. $I = neAV_d$ V_d : drift speed

$\dfrac{V}{R} = neAV_d$ I : Current

n : No. of electron per unit volume

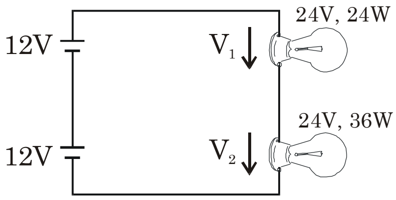
$\dfrac{AV}{\rho L} \times \dfrac{1}{neA} = v_d$ R : Resistance

ρ : Resistivity

$V_d \propto \dfrac{V}{L}$ V : Potential difference

8. Ans. (2)

Sol. Initially when swicth is open



$$R_1 = \frac{(24)^2}{24} = 24$$

$$R_2 = \frac{(24)^2}{36} = 16$$

$V_1 + V_2 = 24$

$V_1 > V_2 \Rightarrow V_1 > 12$

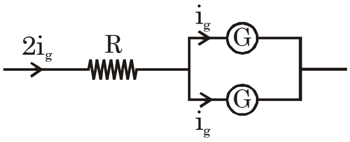
When switch is closed

$V_1 = V_2 = 12.$

Intensity of light bulb A decreases.

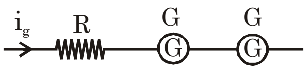
9. Ans. (3)

Sol.



$V_T = 2i_gR = i_g \times R_g$ [$R_g = R$]

$= 3i_gR$



$V_T = 3i_gR$

10. Ans. (3)

11. Ans. (2)

Sol. Use $\vec{F} = q(\vec{v} \times \vec{B}) = (-e)(\vec{v} \times \vec{B})$

12. Ans. (4)

Sol. Initially swithc is closed.

$$U_1 = \frac{1}{2}CV^2$$

When switch is open

$$U_2 = \frac{1}{2} \times \frac{C}{2} V^2 = \frac{CV^2}{4}$$

$$\frac{U_1}{U_2} = 2$$

13. Ans. (1)

Sol. Potential energy of an object at both surface are same by energy conservation.

Kinetic energy will also same.

14. Ans. (3)

Sol. If effect of other satellites will consider then the gravitational potential energy will increase.

15. Ans. (2)

Sol. $V_R = \varepsilon . e^{-t/RC}$

$V_C = \varepsilon[1 - e^{-t/RC}]$

At t = 100 ms

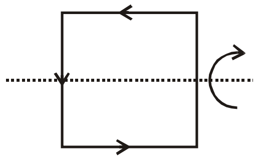
$V_R = V_C$

$\Rightarrow e^{-t/RC} = 1/2$

$$RC = \frac{100}{\ell n(2)} \approx 145.45 \text{ ms}$$

16. Ans. (2)

Sol. $T = \mu \times B = I \propto$

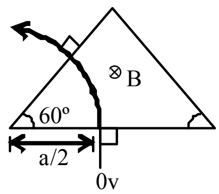


$$IL^2 \times B = \frac{2}{3} ML^2 \times \propto$$

$$\propto = \frac{3IB}{2m}$$

17. Ans. (2)

Sol. The charged particle moves in a circle of radius $\frac{a}{2}$ $qvB = \frac{mv^2}{a/2}$ or $B = \frac{2mv}{qa}$



18. Ans. (1)

Sol. $iR_1 = \frac{q}{C_1}$ ....(i)

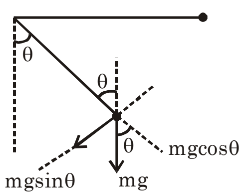
$iR_2 = \frac{q}{C_2}$ ....(ii)

$$\frac{R_1}{R_2} = \frac{C_2}{C_1}$$

19. Ans. (4)

20. Ans. (1)

Sol. $a_t = g\sin\theta = a_C = \frac{V^2}{L}$



$mg\,L\cos\theta = \frac{1}{2} \times mv^2$

$V^2 = 2gL\cos(\theta)$

$$a_C = \frac{2gL\cos(\theta)}{L} = 2g\cos(\theta)$$

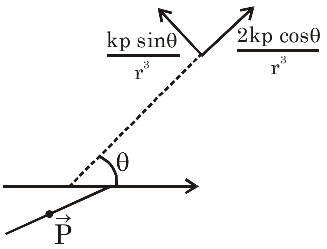
$2g\cos(\theta) = g\sin(\theta)$

$\tan(\theta) = 2$

21. Ans. (1)

22. Ans. (2)

Sol.



23. Ans. (4)

24. Ans. (2)

Sol. $5 = (2t^3 + 2)m$

$$I = \int_0^1 F dt$$

$V = 6t^2$

$a = 12t$

$F = ma = 24t$

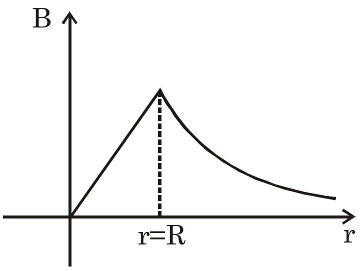
$$\Rightarrow \int_0^1 24 dt = \frac{24t^2}{2} = 12 \text{ N-S}$$

25. Ans. (4)

$$\text{Current sensitivity} = \frac{NAB}{C}$$

26. Ans. (4)

Sol.



Magnetic field due to solid pipe.

27. Ans. (1)

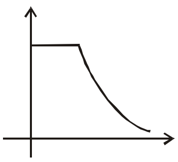
Sol. $V = IR$

$\frac{V}{L}$: Potential gradient

$$\frac{I \times \rho L}{A \times L} = \frac{0.1 \times 8 \times 10^{-7}}{4 \times 10^{-7}} = 0.2 V/m$$

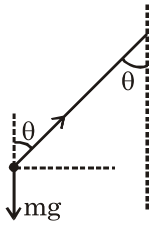
28. Ans. (2)

Sol. Electric field inside the spherical shell = 0



29. Ans. (2)

Sol. $T\sin(\theta) = \frac{m_2 V^2}{R}$



$T\cos(\theta) = m_2 g$

$$\tan(\theta) = \frac{V^2}{Rg} = \frac{10}{1 \times 10} = 1$$

$\tan(\theta) = 1 \Rightarrow \theta = 45°$

$T = (m_2 g) \times \sqrt{2}$

$(m_2 g)\sqrt{2} = m_1 V_1^2 \times R$

$$\frac{1}{\sqrt{2}} \times g \times \sqrt{2} = \frac{1}{10} \times 1 \times V_1^2$$

$V_1^2 = 100$

$V_1 = 10$ m/s

30. Ans. (3)

Sol. For maximum elongation charge on the blocks must be equal to $\frac{Q}{2}$ on each block.

$$\therefore \frac{1}{4\pi\varepsilon_0} \frac{\left(\frac{Q}{2}\right)^2}{\left(\ell_0 + x^2\right)} = kx$$

$$Q = 2\left(\ell_0 + x\right)\sqrt{4\pi\varepsilon_0 kx}$$

31. Ans.(3)

32. Ans.(4)

33. Ans.(1 or 2)

34. Ans.(3)

35. Ans.(2)

$\Delta H_{ionisation} + 2\ (-57) = -109$

36. Ans.(2)

37. Ans.(1)

38. Ans.(3)

39. Ans.(2)

40. Ans.(2)

41. Ans. (1)

42. Ans.(4)

43. Ans.(3)

44. Ans.(4)

45. Ans.(3)

46. Ans.(4)

47. Ans.(4)

48. Ans.(4)

49. Ans.(1)

50. Ans.(3)

51. Ans. (2)

52. Ans. (4)

53. Ans. (3)

54. Ans. (1)

55. Ans. (2)

56. Ans. (4)

57. Ans. (3)

58. Ans. (2)

59. Ans. (1)

60. Ans. (4)

61. Ans. (2)

$$\lim_{x\to\infty}\frac{x\left(x\sin\frac{1}{x}-1\right)}{1+|x|}=\frac{1-1}{1+0}=0$$

62. Ans. (3)

$\because f(1-x)+f(1+x)=2$

Put $x = 1 - x$

$f(x)+f(2-x)=2$

$$\int_0^2 f(x)dx=\int_0^1 f(x)dx+\int_1^2 f(x)dx$$

63. Ans. (1)

equation of chord of contact

$5x - 5y = 5$

$\Rightarrow (x - y = 1)$

orthocentre always lies on the perpendicular to $(x - y = 1)$ passing through $(0,0)$

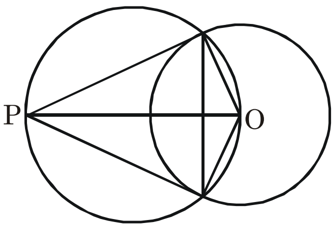
64. Ans. (1)

$\sqrt{x+y}=y$

65. Ans. (1)

$a = b$ & $a + b = 4$

$\Rightarrow a = 2, b = 2$



(PO will be a diameter)

66. Ans. (2)

$a = b = c = 1$, $f(x)$ is continuous

67. Ans. (3)

Odd function

68. Ans. (3)

$$y=\cos^{-1}\left(\frac{3}{5}\cos x-\frac{4}{5}\sin x\right)$$

$=\cos^{-1}\cos(x+\alpha)$

$=(x+\alpha)\ \left(\cos\alpha=\frac{3}{5}\right)$

69. Ans. (1)

70. Ans. (1)

Continuity $0 = a + m\pi$

$a = (-m\pi)$

Differentiable

$(-1 = m) \Rightarrow (a = \pi)$

71. Ans. (4)

$$I_8+I_6=\int_0^{\pi/4}\tan^6\theta.\sec^2\theta d\theta$$

$\Rightarrow$ Put $\tan\theta = t$

72. Ans. (2)

Use expansion.

73. Ans. (1)

74. Ans. (1)

$y = e^{2x} + x^2$

at $x = 0, y = 1$

$$\frac{dy}{dx}=2e^{2x}+2x$$

$$\frac{dy}{dx}(0.1)=2$$

equation of tangent is

$y - 1 = 2(x - 0) = 2x$

$2x - y + 1 = 0$

$$d=\frac{1}{\sqrt{5}}$$

75. Ans. (1)

$f'(x) \le 0$

then $x \in R$, $D \le 0$

76. Ans. (2)

77. Ans. (3)

78. Ans. (4)

$$\int\frac{(x-1)}{x\sqrt{x^2-1}}dx=\int\frac{1}{\sqrt{x^2-1}}dx-\int\frac{1}{x\sqrt{x^2-1}}dx$$

79. Ans. (1)

$$\left[\frac{t^3}{3}\right]_0^{f(x)} = x\cos\pi x$$

$$\Rightarrow \quad \frac{f^3(x)}{3} = x\cos\pi x \qquad .....(i)$$

Differentiate

$f^2(x) = -\pi x\sin\pi x + \cos\pi x \qquad .....(ii)$

Solve (i) & (ii)

80. Ans. (1)

$e^x = t$

81. Ans. (3)

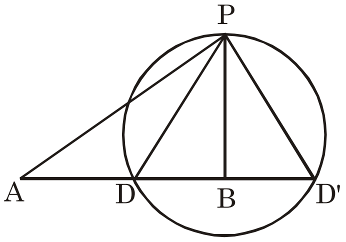
Take I_2 & put $x^3 = t$

82. Ans. (1)

23. Ans. (2)

Multiply by sinx in N^r & D^r

83. Ans. (1)



$$\because \quad \frac{PA}{PB} = \lambda$$

D & D' be the diameter and point.

84. Ans. (2)

$f'(x) = m - e^{-x} \le 0 \ \forall\ x \in R$

$\Rightarrow \quad m \le 0$

85. Ans. (2)

$$\left|y\frac{dy}{dx}\right| = k \ \Rightarrow \ ydy = \pm kdx$$

$$\Rightarrow \quad \frac{y^2}{2} = \pm kx + C$$

86. Ans. (4)

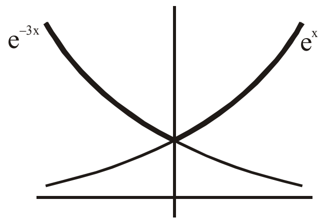
$$\int_{-3}^{2} \text{sgn}(x)dx = \int_{-3}^{0} -dx + \int_{0}^{2} dx = -1$$

87. Ans. (1)

Line passes through centre of circle $\forall$ a,b $\in$ R

$\Rightarrow$ Line is normal to circle.

88. Ans. (2)



30. Ans. (1)

$$f'(x) = x(2\sin x + x\cos x) > 0 \ \forall\ x \in \left(0, \frac{\pi}{2}\right)$$