ALLEN™ CAREER INSTITUTE KOTA (RAJASTHAN) path to success

Paper Code : 1001CT102116021

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2016 - 2017)

JEE (Main + Advanced) : ENTHUSIAST COURSE (PHASE : I)

ANSWER KEY : PAPER-1 **TEST DATE : 18-09-2016**

Test Type : MINOR Test Pattern : JEE-Advanced

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B,C	A,C	A,C,D	A,D	A,B,C	A	A,C,D	A,D	B,C,D	A,B,C,D
SECTION-IV	Q.	1	2	3	4	5	6	7	8	9	10
	A.	4	2	1 or 7	2	6	5	4	0	8	3

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,B,C,D	A,B,C	B,C	A,D	B,C	B,D	A,B,C,D	A,B,C	A,B,D	B,C
SECTION-IV	Q.	1	2	3	4	5	6	7	8	9	10
	A.	2	5	4	5	8	4	4	6	6	2

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,B,C	A,C	A,B,C,D	A,B,C,D	A,B,C	A,B,D	A,B,C,D	B,C,D	A,C,D	A,C
SECTION-IV	Q.	1	2	3	4	5	6	7	8	9	10
	A.	8	4	2	Bonus	7	4	5	3	2	2

ALLEN™ CAREER INSTITUTE KOTA (RAJASTHAN) path to success

Paper Code : 1001CT102116022

CLASSROOM CONTACT PROGRAMME

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JEE (Main + Advanced) : ENTHUSIAST COURSE (PHASE : I)

ANSWER KEY : PAPER-2 **TEST DATE : 18-09-2016**

Test Type : MINOR Test Pattern : JEE-Main

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	2	3	2	3	2	4	3	3	3	4	4	2	4	1	3	2	2	1	1	3
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	2	2	2	2	2	1	4	1	4	3	4	4	4	3	3	4	1	2	3	2
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	4	3	4	3	3	2	3	4	4	2	2	3	2	2	2	1	3	1	3	3
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	3	1	4	2	4	1	4	3	4	2	1	2	3	1	4	3	1	3	3	3
Que.	81	82	83	84	85	86	87	88	89	90										
Ans.	4	2	4	3	1	2	3	3	1	4										

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PAPER-1

PART-1 : PHYSICS SOLUTION

SECTION-I

1. Ans. (B, C)

Sol. Lenz law

2. Ans. (A, C)

Sol. $T = \frac{2\pi m}{Bq} = \frac{2\pi}{\alpha B_0}$

at $t = \frac{\pi}{\alpha B_0} = \frac{T}{2}$; velocity of particle is $-v_0\hat{i} + v_0\hat{k}$

speed will always remains constant $= v_0\sqrt{2}$

at $t = \frac{2\pi}{\alpha B_0} = T$; displacement is equal to pitch, $\Delta x = V_0 T = \frac{2\pi V_0}{\alpha B_0}$

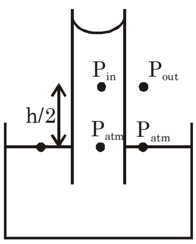
at $t = \frac{2\pi}{\alpha B_0} = T$; distance = speed × T $= \frac{2\sqrt{2}V_0\pi}{\alpha B_0}$

3. Ans. (A,C,D)

Sol. There is no external torque is acting in any case.

4. Ans. (A,D)

Sol.



$P_{out} = P_{atm}$

$P_{in} = P_{atm} - \rho gh/2$

5. Ans. (A,B,C)

Sol. Force $|F| = kx$, zero at $x = 0$

$|a| = \omega^2 x$, maximum at $x = \pm A$

V is maximum at mean position, $x = 0$

PE = KE at $x = \pm\frac{A}{\sqrt{2}}$

6. Ans. (A)

Sol. Work done = Area under P v/s V graph.

7. Ans. (A,C,D)

Sol. $T \propto \frac{1}{\lambda}$ (Wein's law)

emissive power $\propto \frac{T^4}{Area}$.

Rate of heat loss $\propto T^4$

Rate of cooling $\propto \frac{AT^4}{Volume}$

8. Ans. (A,D)

Sol. $P_1 + \rho g h_1 + \frac{1}{2}\rho V_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho V_2^2$

$h_1 = h_2 = 0$ (horizontal pipe)

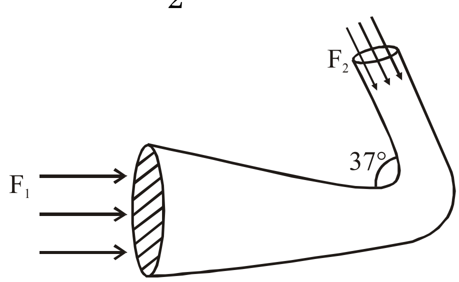
& $A_1V_1 = A_2 V_2$

$4(4) = V_2$

$16 = V_2$

$2.80 \times 10^5 + \frac{1}{2}\rho(4)^2 = P_2 + \frac{1}{2}\rho(16)^2$

$2.80 \times 10^5 + \frac{1}{2}\rho(16 - 256) = P_2$



$2.80\times10^5+\frac{1}{2}900(-240)=P_2$

$172\times10^3\ N/m^2=P_2$

$F_1=P_1A_1=56\times10^3$

$F_2=P_2A_2=8.6\times10^3$

$\vec{F}_1+\vec{F}_2+\vec{F}_{pipe}=\frac{d\vec{P}_{oil}}{dt}$

$\vec{F}_{pipe}=\frac{d\vec{P}_{oil}}{dt}-\left(\vec{F}_1+\vec{F}_2\right)$

$\vec{F}_1=P_1A_1\hat{i}$

$\vec{F}_2=P_2A_2\left(\cos37°\hat{i}-\sin37°\hat{j}\right)$

$\frac{d\vec{P}_{oil}}{dt}=\left(\frac{dm}{dt}\right)\Delta\vec{V}$

$=(\rho A_1V_1)\left(\vec{V}_2-\vec{V}_1\right)$

$\vec{V}_2=16\left(\cos37°\left(-\hat{i}\right)+\sin37°\left(\hat{j}\right)\right)$

$\vec{V}_1=4\hat{i}$

Solving this for F_{pipe} we get, $|F|=76\times10^3$ N

9. Ans. (B,C,D)

Sol. Energy required to eject e^- is more than work function (ϕ) of material.

$\frac{hC}{\lambda}>\phi$

λ_1 can eject e^- from A & B both

So, its energy $\frac{hC}{\lambda_1}>\phi_1$ & ϕ_2

but, λ_2 can eject e^- only from B.

So, its energy $\frac{hC}{\lambda_2}>\phi_2$

but $\frac{hC}{\lambda_2}<\phi_1$

hence, $\phi_1>\phi_2$

& $\lambda_1<\lambda_2$

threshold wavelength $\lambda_+=\frac{hC}{\phi}$

10. Ans. (A,B,C,D)

Sol. Since the light source is a point source ⇒ intensity at distance a is given as $\frac{W}{4\pi a^2}$

Thus number of photon striking the surface of area S per unit time $=\frac{W\lambda S}{4\pi a^2hc}$

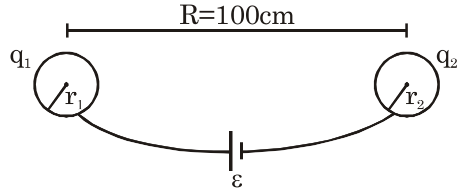
$V_{Stopping}=\frac{1}{e}\left(\frac{hc}{\lambda}-\phi\right)$

Also K.E.$_{max}$ of emitted photo-electron $=\left(\frac{hc}{\lambda}-\phi\right)$

SECTION-IV

1. Ans. 4

Sol.



$F=\frac{kq_1q_2}{R^2}$,

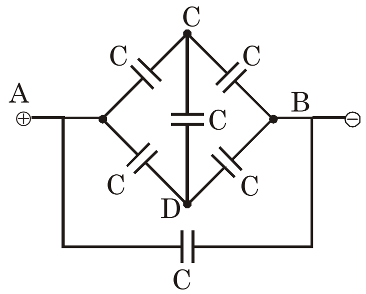
$V_1-V_2=\varepsilon$

$\frac{kq_1}{r_1}-\frac{kq_2}{r_2}=\varepsilon$

and $q_1+q_2=0$

2. Ans. 2

Sol. Redraw :



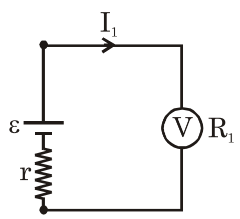
3. Ans. 1 or 7

Sol. $F=-\frac{dU}{dx}$ & $U=\frac{Q^2}{2C}$

Where, $C=\frac{\epsilon_0\ \ell(\ell-x)}{d}+\frac{\epsilon_r\epsilon_0\ \ell(x)}{d}$

4. Ans. 2

Sol. Initially



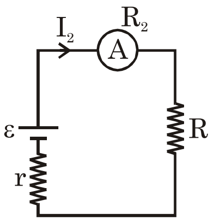
reading $=(\varepsilon-I_1r)$

$=\varepsilon-\frac{\varepsilon r}{r+R_1}$

$2=\left(\frac{\varepsilon R_1}{r+R_1}\right)$

$$2 = \left(\frac{\varepsilon \times 100}{r + 100}\right) \quad .... (1)$$

Finally



reading = I_2 = I

$$1 = \left(\frac{\varepsilon}{r + R + R_2}\right)$$

$$1 = \left(\frac{\varepsilon}{r + 1 + 1}\right) \quad .... (2)$$

Solving equation (1) & (2) we get ε = 2V

5. Ans. 6

Sol. efficiency $\eta = \dfrac{R}{r + R}$

6. Ans. 5

Sol. Total magnetic flux through a super conductor is equal to zero.

$\Rightarrow \phi_{self} + \phi_{ext} = 0$

$|\phi_{self}| = \phi_{ext}$

$Li = B_0 \pi r^2$

$$i = \frac{B_0 \pi r^2}{L}$$

7. Ans. 4

Sol. AC ammeter shows rms current

So, when both currents are flown simultaneously, AC ammeter gives,

$$I_{AC} = \sqrt{6^2 + 8^2} = 10$$

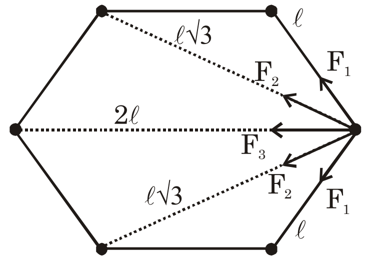
& DC ammeter gives,

$I_{DC} = 6$

difference in readings = 10 – 6 = 4

8. Ans. 0

Sol.



$$F_1 = \frac{Gm^2}{\ell^2}, F_2 \frac{Gm^2}{\left(\ell\sqrt{3}\right)^2}, F_3 = \frac{Gm^2}{(2\ell)^2}$$

$F_{res} = 2F_1 \cos 60° + 2F_2 \cos 30° + F_3$

$$= \frac{Gm^2}{\ell^2}\left(1 + \frac{1}{\sqrt{3}} + \frac{1}{4}\right)$$

x = 1, y = 3 & z = 4

9. Ans. 8

Sol. By conservation of energy

$\Delta m L_f = MS\Delta T$

$\Delta m \times 80 = m\ (1)\ (10)$

$$\frac{\Delta m}{m} = \frac{1}{8}$$

10. Ans. 3

Sol. $\theta_1 = \theta_0 \sin(\omega t + \delta_1)$

$\theta_2 = \theta_0 \sin(\omega t + \delta_2)$

For the first, θ = 2°,

$\therefore \sin(\omega t + \delta_1) = 1$

For the 2nd, θ = –1°,

$\therefore \sin(\omega t + \delta_2) = -1/2$

$\therefore \omega t + \delta_1 = 90°, \quad \omega t + \delta_2 = -30°$

$\therefore \delta_1 - \delta_2 = 120° = \dfrac{2\pi}{3}$ rad

PART–2 : CHEMISTRY SOLUTION

SECTION-I

1. Ans.(A,B,C,D)
2. Ans.(A,B,C)
3. Ans.(B,C)
4. Ans. (A, D)
5. Ans. (B, C)
6. Ans.(B, D)
7. Ans. (A,B,C,D)
8. Ans. (A,B,C)
9. Ans. (A,B,D)
10. Ans. (B,C)

SECTION-IV

1. Ans. 20 [OMR Ans. 2]
2. Ans. 5
3. Ans. 4
4. Ans. 5.85 [OMR Ans. 5]
5. Ans. 8
6. Ans. 4
7. Ans. 4
8. Ans. 6
9. Ans. 6
10. Ans. 2

PART-3 : MATHEMATICS SOLUTION

SECTION-I

1. Ans. (A,B,C)

$$E=\frac{d^2+4d^2+9d^2}{d^2+d^2r+d^2r^2}=\frac{14}{1+r+r^2}\in\left(\frac{14}{3},14\right)$$

as $r\in(0,1)$

Now, $r^2+r+\left(1-\frac{14}{E}\right)=0\Rightarrow r=-\frac{1}{2}+\sqrt{\frac{56-3E}{4E}}$

$\because$ E is an integer and $5\le E\le 13$ and for $E = 8$, $\frac{56-3E}{4E}$ becomes perfect square

$\therefore\quad E=8, r=\frac{1}{2}$

2. Ans. (A,C)

$\because\ \left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]_{max}=\left|\vec{a}\times\vec{b}\right|=3\sqrt{3}=M$

and $\vec{c}=\frac{\vec{a}\times\vec{b}}{\left|\vec{a}\times\vec{b}\right|}=\frac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$

3. Ans. (A,B,C,D)

Foot of perpendicular from A(1,1,1) to the line BC is (2,3,3), which lies on each of the given plane.

4. Ans. (A,B,C,D)

$\because\ mx^2+(2m-1)x+m-2=0$

$\therefore\ D=(2m-1)^2-4m(m-2)=4m+1=$ odd integer

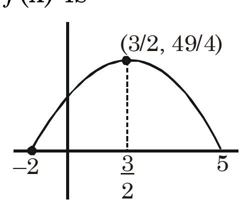
Let $4m+1=(2n+1)^2\Rightarrow 4m=(2n+1)^2-1^2$ $=2n.(2n+2)$

$\Rightarrow m=n(n+1), n\in N$

also $1640=41\times 40$ and $4\lambda^2-10\lambda+6$ $=(2\lambda-2)(2\lambda-3)$

5. Ans. (A,B,C)

Graph of $f(x)$ is



now check options.

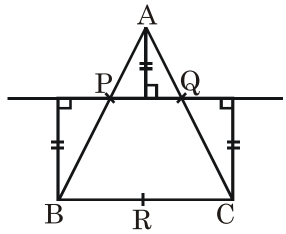
6. Ans. (A,B,D)

$\because$ sum of all coefficient = 0 $\Rightarrow x = 1$ is a root and product of roots

$=\frac{c+2a-3b}{a+2-b-3c}=\frac{1}{4}$

$\therefore$ roots are $1,\frac{1}{4}$.

7. Ans. (A,B,C,D)



Let P,Q,R be the mid points of AB,AC and BC respectively.

Let Π be the plane passing through P and Q and $\perp$ to the plane of ΔABC, then any line ℓ lies on plane Π and parallel to line PQ is equidistant from A,B and C. so infinitely many such lines exist.

8. Ans. (B,C,D)

Very basic, do yourself.

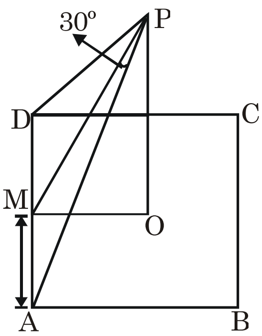
9. Ans. (A,C,D)

Given $a_1 = 2$ and by using given relation, we have $a_2=2^1.3$, $a_3=2^2.4$, $a_4=2^3.5$

$\therefore\ a_{2016}=2^{2015}.2017$

we can also find the general term $a_n=2^{n-1}(n+1)$ by using $a_n=s_n-s_{n-1}$ with no loss of generality.

10. Ans. (A,C)



Using ΔPMA

$PM=\frac{1}{2}\cot 30^\circ=\frac{\sqrt{3}}{2}$

using ΔPOM,

$OP=\sqrt{\frac{3}{4}-\frac{1}{4}}=\frac{1}{\sqrt{2}}$

$\therefore$ volume of pyramid

$=\frac{1}{3}$. area of base × height $=\frac{1}{3}.1^2.\frac{1}{\sqrt{2}}$

$=\frac{1}{3\sqrt{2}}$

& $\left|\overrightarrow{PP}\right|=2\left|\overrightarrow{OP}\right|=\sqrt{2}$

SECTION-IV

1. Ans. 8

Given parabola is $x^2+4y-6x+k=0$

$\Rightarrow (x-3)^2=-4\left(y+\frac{k-9}{4}\right)$

$\therefore$ equation of directrix is

$$y+\frac{k-9}{4}=1 \Rightarrow y=1-\left(\frac{k-9}{4}\right)=-1$$

$\Rightarrow$ k = 17

$\therefore$ Parabola (1) becomes

$(x-3)^2 = -4(y+2)$

$\therefore$ vertex = (3,–2) and focus = (3,–3)

2. Ans. 4

$$\int_0^\infty \frac{dx}{e^x+e^{-x}}=\int_0^\infty \frac{e^x}{1+e^{2x}}dx=\left(\tan^{-1}e^x\right)_0^\infty=\frac{\pi}{4}$$

3. Ans. 2

Let equation of curve is $(x-a)^2+(y-b)^2 = r^2$

$\because$ r = constant $\Rightarrow$ order = 2

4. Ans. Bonus

Let $\vec{p}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ and $\vec{q}=x\hat{i}+y\hat{j}+z\hat{k}$

$\therefore$ Given condition

$\Rightarrow$ $\vec{p}.\vec{q}=-|\vec{p}||\vec{q}| \Rightarrow \cos\theta=-1 \Rightarrow \theta=\pi$

$\therefore$ vector $\vec{p}$ and $\vec{q}$ are antiparallel

$$\therefore \frac{\alpha}{x}=\frac{\beta}{y}=\frac{\gamma}{z}=k(\text{say})$$

$\Rightarrow \alpha = kx, \beta = ky, \gamma = kz$

$\therefore$ Given condition $\Rightarrow$ k =–1

$\therefore$ Given sum = 1 + 1 + 1 = 3

5. Ans. 7

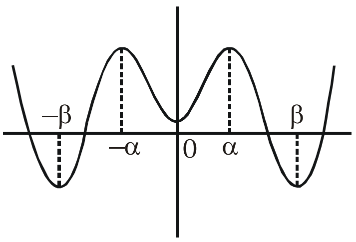
$$\because A=\begin{bmatrix}1 & -1 & -2\\ 1 & 4 & -1\\ 2 & 1 & 9\end{bmatrix}\Rightarrow |A|=58$$

and

$$B=\begin{bmatrix}|A| & 0 & 0\\ 0 & |A| & 0\\ 0 & 0 & |A|\end{bmatrix}\Rightarrow \det B=|A|^3=58^3$$

$$\Rightarrow \left[\frac{3\sqrt{\det B}}{8}\right]=\left[\frac{58}{8}\right]=7$$

6. Ans. 4



$\because f'(\alpha) = f'(\beta) = f'(-\alpha) = f'(-\beta) = f'(0) = 0$

$\therefore$ using rolle's theorem minimum number of points of inflection are four.

7. Ans. 5

$$\because f'(x)=\frac{2}{\sqrt{3}}\left(\frac{1}{1+\left(\frac{2x+1}{\sqrt{3}}\right)^2}\right)\frac{2}{\sqrt{3}}-\frac{(2x+1)}{x^2+x+1}+\left(b^2-5b+3\right)$$

$$\Rightarrow f'(x)=\frac{-2x}{x^2+x+1}+\left(b^2-5b+3\right)$$

$\because f'(x) < 0 \;\forall\; x \in R$

$$\Rightarrow b^2-5b+3 < \frac{2x}{x^2+x+1}$$

$$\because \left.\frac{2x}{x^2+x+1}\right]_{min}=-2$$

$\Rightarrow b^2 - 5b + 3 \le -2 \Rightarrow b^2 - 5b + 5 \le 0$

$$\Rightarrow b\in\left[\frac{5-\sqrt{5}}{2},\frac{5+\sqrt{5}}{2}\right]$$

$$\therefore \frac{\alpha+\beta}{\gamma}=5$$

8. Ans. 3

$$S=\sum_{n=1}^{\infty}\frac{a_n}{2^n}=\frac{1}{2}+\frac{1}{4}+\sum_{n=3}^{\infty}\frac{a_{n-1}+a_{n-2}}{2^n} \quad ...(1)$$

($\because$ from the third term each term is the sum of the previous two terms)

$$(1) \Rightarrow S=\frac{1}{2}+\frac{1}{4}+\sum_{n=3}^{\infty}\frac{a_{n-1}}{2^n}+\sum_{n=3}^{\infty}\frac{a_{n-2}}{2^n}$$

$$=\frac{1}{2}+\frac{1}{4}+\frac{1}{2}\sum_{n=2}^{\infty}\frac{a_n}{2^n}+\frac{1}{4}\sum_{n=1}^{\infty}\frac{a_n}{a^n}$$

$$\Rightarrow S=\frac{3}{4}+\frac{1}{2}\left(S-\frac{1}{2}\right)+\frac{1}{4}S\Rightarrow S=\frac{1}{2}+\frac{3}{4}S\Rightarrow S=2$$

9. Ans. 2

Clearly, point P lies on the plane passing through A,B and $\perp$ to xy-plane.

equation of plane is x = 2y, z = 0

P(α,β) lies on plane $\Rightarrow \alpha = 2\beta$

$$\therefore \left[\frac{\alpha+3\beta}{5}\right]+\left(\frac{\alpha+2}{\beta+1}\right)=\left[\frac{5\beta}{5}\right]+\frac{2\beta+2}{\beta+1}$$

= [β] + 2 = 0 + 2 ($\because$ 0 < β < 1)

we can also find $\alpha=4\sqrt{\frac{3}{5}}-2$ and $\beta=2\sqrt{\frac{3}{5}}-1$ by transformation of axes.

10. Ans. 2

Let position vectors of points A,B,C are $\vec{a},\vec{b},\vec{c}$ with respect to point P as origin

$\therefore$ Given condition

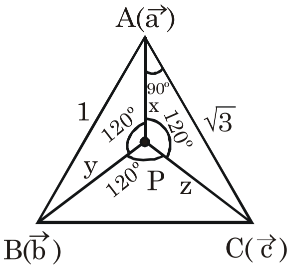
$\Rightarrow \hat{a}+\hat{b}+\hat{c}=\vec{0}$ and $|\hat{a}|=|\hat{b}|\Rightarrow|\hat{c}|=1$

$\Rightarrow \hat{a}+\hat{b}=-\hat{c}$ take mod and square

$|\hat{a}+\hat{b}|^2=|\hat{c}|^2\Rightarrow \cos\angle APB=-\frac{1}{2}\Rightarrow \angle APB=120^\circ$

similarly $\angle APC = \angle BPC = 120^\circ$

let $|\overrightarrow{PA}|=x, |\overrightarrow{PB}|=y$ and $|\overrightarrow{PC}|=z$



apply cosine rule in ΔAPB, ΔBPC, ΔCPA

$x^2 + y^2 + xy = 1^2$ ...(1)

$y^2 + z^2 + yz = 2^2$ ...(2)

$z^2 + x^2 + zx = (\sqrt{3})^2$ ...(3)

and $ar(\Delta ABC) = ar(\Delta APB) + ar(\Delta BPC) + ar(\Delta APC)$

$\Rightarrow xy + yz + zx = 2$ ...(4)

(1) + (2) + (3) $\Rightarrow x^2 + y^2 + z^2 = 3$

(1) + (2) $\Rightarrow y(x + y + z) = 2$

(2) + (3) $\Rightarrow z(x + y + z) = 4$

and $x(x + y + z) = 1$

$\therefore x : y : z = 1 : 2 : 4$

$\Rightarrow x=\frac{\sqrt{7}}{7} y=\frac{2\sqrt{7}}{7}, z=\frac{4\sqrt{7}}{7}$

JEE (Main + Advanced) : ENTHUSIAST COURSE
PHASE : I

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TEST DATE : 18 - 09 - 2016

PAPER-2

SOLUTION

1. **Ans. (2)**

Sol. $P = V_1 i_1 = V_2 i_2$

$10 \times 10^3 = 25 \times V_2$

$V_2 = \frac{10^4}{25}$

Now $V_1 = \frac{n_1}{n_2} \times V_2 = \frac{8}{1} \times \frac{10^4}{25} V$

2. **Ans. (3)**

Sol. Let final temparature be T °C

$20 \times 1 \times (30 - T) =$

$5 \times 1/2 \times 10 + 5 \times 80 + 5 \times 1 \times (T\text{-}0)$

$600 - 20T = 25 + 400 + 5T$

$T = 7°C$

3. **Ans. (2)**

4. **Ans. (3)**

Sol. $\left(P_1 + \frac{4T}{r}\right) \times \frac{4}{3}\pi r^3 = \left(P_2 + \frac{4T}{(r/2)}\right) \cdot \frac{4}{3}\pi\left(\frac{r}{2}\right)^3$

$P_2 = 8P_1 + \frac{24T}{r}$

5. **Ans. (2)**

Sol. $KA\frac{dT}{dx} = \frac{dm}{dt} \times L$

$107.4 \times 0.15 \times \frac{(T_L - 100)}{(1/100)} = \frac{6}{600} \times 2256 \times 10^3$

$T_L = 240°C$

6. **Ans. (4)**

Sol. BC is isothermal

$3P_0 \times V_1 = P_0 \times V_0$

$V_1 = \frac{V_0}{3}$

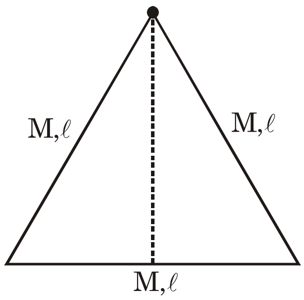
Now AC is adiabatic

$3P_0\left(\frac{V_0}{3}\right)^\gamma = \frac{P_0}{2}(V_0)^\gamma$

$\gamma = \frac{\ell n6}{\ell n3}$

7. **Ans. (3)**

Sol.



$T = 2\pi\sqrt{\frac{I}{(3M)gd}}$

$T = \frac{M\ell^2}{3} + \frac{M\ell^2}{3} + \left[\frac{M\ell^2}{12} + M\left(\frac{\ell\sqrt{3}}{2}\right)^2\right]$

$d = \frac{\ell\sqrt{3}}{2} \times \frac{2}{3} = \frac{\ell}{\sqrt{3}}$

On solving $T = \frac{\pi}{\sqrt{5}}$

8. **Ans. (3)**

Sol. We assume that b is small compared to $\sqrt{\frac{k}{m}}$ and we take $T = 2\pi\sqrt{(m/k)} = 1\,s$. It is given that at t = 4T, the amplitude falls to 3A/4, i.e.

$e^{-bt/2m} = 3/4$

$-2bT/m = \ln(3/4)$

or $b = 0.28$ kg/s.

9. Ans. (3)

Sol. For resonance

$$\omega = \sqrt{\omega_0^2 - b^2} = \sqrt{\frac{100}{1} - 2\times\left(\frac{4}{2}\right)^2}$$

$\omega = \sqrt{92}$

Now initial ω' must be greater than ω because on increasing frequency, amplitude is decreasing

$\Rightarrow$ ω' = 12 rad/s

10. Ans. (4)

Sol. Just after key is pressed.

Inductor acts as open circuit & capacitor as short circuit.

$i_i = V/R_1$ (max), $i_2 = 0$

11. Ans. (4)

Sol. $R = \frac{mv}{qB} = \frac{\sqrt{2m(KE)}}{qB}$

$(RqB)^2 = 2m\ (KE)$

$$\frac{(RqB)^2}{2m} = (KE)_{max} = \left(\frac{hc}{\lambda} - \phi\right)$$

$\Rightarrow \phi = 3.2$ eV

12. Ans. (2)

Sol. End of cell = E/3

Now when length of wire becomes $\frac{3\ell}{2}$

$$\frac{E}{3} = \frac{E}{(3\ell/2)}\times \ell' \Rightarrow \ell' = \frac{\ell}{2}$$

13. Ans. (4)

Sol. Magnetic field on axis of cirular coil is

$$\frac{\mu_0 i R^2}{2(R^2+x^2)^{3/2}}$$

$$\frac{\mu_0 i_1 R^2}{2(R^2+x^2)^{3/2}} = \frac{\mu_0 i_2 R^2}{2(R^2+x^2)^{3/2}}$$

$x = \sqrt{7}R$

14. Ans. (1)

Sol. V = ωA

Now V' = ω(2A) = 2V

15. Ans. (3)

Sol. $I = \frac{5}{R+1} \Rightarrow R = 4\Omega$

$$2 = \frac{20}{\sqrt{5^2 + (20L)^2}} \Rightarrow L = \frac{\sqrt{3}}{4}H$$

16. Ans. (2)

Sol. $\phi = \frac{NABi}{k}$

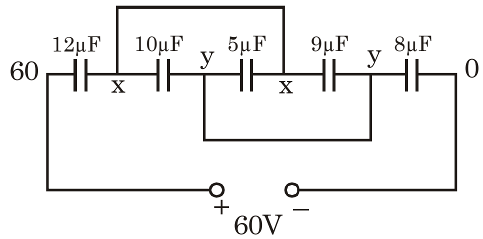
$\phi = \frac{\pi}{6} = 30°$

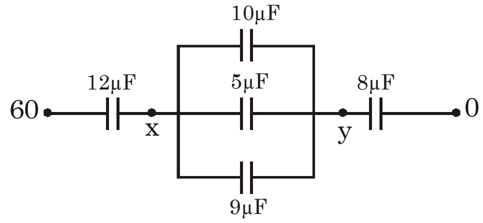
17. Ans. (2)

18. Ans. (1)

19. Ans. (1)

Sol.





$$\frac{1}{C_{eq}} = \frac{1}{12} + \frac{1}{24} + \frac{1}{8}$$

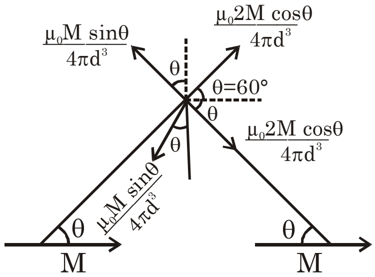
$C_{eq} = 4\mu F$

$$q_{5\mu F} = \frac{240}{24}\times 5 = 50\mu C$$

20. Ans. (3)

21. Ans. (2)

Sol.



$$B_{Net} = 2\left(\frac{2\mu_0}{4\pi}\frac{M}{d^3}\cos\theta\right)\cos\theta - 2\left(\frac{\mu_0}{4\pi}\frac{M\sin\theta}{d^3}\right)\sin\theta$$

$$\frac{\mu_0}{4\pi}\left(\frac{M}{2d^3}\right)$$

22. Ans. (2)

Sol. Let r_2 radius sphere has & ρ & –ρ charge densities.

$$K\left(\frac{4}{3}\pi r_1^3\rho\right)\left(3r_1^2 - \left(\frac{3r_0}{2}\right)^2\right) - \frac{K\left(\frac{4}{3}\pi r_2^3\rho\right)}{\left(\frac{3r_0}{2}\right)} = \frac{101\rho r_0^2}{72\varepsilon_0}$$

23. Ans. (2)

Sol. $\rho_x = \rho_1 + \frac{(\rho_2 - \rho_1)}{L}x$

$dR = \frac{\rho_x dx}{A}$

$R = \left(\frac{\rho_1 + \rho_2}{2}\right)\frac{\ell}{A}$

24. Ans. (2)

25. Ans. (2)

Sol. $|V_0 - V_A| = \frac{B\omega^2}{2}a^2$

$|V_A - V_C| = \frac{B\omega^2}{2}((a+2c+d)^2 - a^2)$

$|V_0 - V_C| = \frac{B\omega^2}{2}(a+2c+d)^2$

$|V_A - V_3| = \frac{B\omega^2}{2}((a+2c)^2 = a^2$

26. Ans. (1)

Sol. Flux = constant

Induced emf = 0

Induced current = 0

27. Ans. (4)

Sol. When $V_{across\ b} = 0 \Rightarrow i = i_{max} \Rightarrow q_c = 0$

28. Ans. (1)

Sol. $E_{coil} = Ee^{-\left(\frac{Rt}{L}\right)} = 6e^{-\left(\frac{10\ell n\sqrt{2}}{5}\right)}$

$\frac{6}{2} = 3V$

29. Ans. (4)

Sol. Current in circuit :

$I = \frac{V}{X} = \frac{250}{(75-25)} = \frac{250}{50} = 5A$

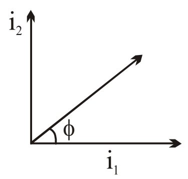
$\therefore V_L = I X_L = 5 \times 25 = 125V$ &

$V_C = I X_C = 5 \times 75 = 375V$

voltage on capacitor is more than that of supply voltage because the phase difference between V_L and V_C is 180° (i.e. out of phase)

30. Ans. (3)

Sol.



$i_3 = \sqrt{i_1^2 + i_2^2}$

& $i_3 < (i_1 + i_2)$

31. Ans.(4)

$\frac{(t_{1/2})_I}{(t_{1/2})_{II}} = \left[\frac{10}{20}\right]^{n-1}$

$\frac{20}{40} = \left(\frac{1}{2}\right)^{n-1}$

1 = n – 1

n = 2

32. Ans.(4)

33. Ans.(4)

34. Ans.(3)

$\overset{+7}{Mn}O_4^- + \overset{+2}{S_2}O_3^{--} \longrightarrow Mn^{+4} + S^{+6}$

$nf = 3 \quad nf = 8$

18 × 3 = a × 8

$a = \frac{54}{8}$

35. Ans.(3)

36. Ans.(4)

$\sqrt{3}a = 4r$

$a = \frac{4}{\sqrt{3}}r$

37. Ans.(1)

38. Ans.(2)

39. Ans.(3)

40. Ans.(2)

41. Ans. (4)

42. Ans. (3)

43. Ans. (4)

44. Ans. (3)

45. Ans. (3)

46. Ans. (2)

47. Ans. (3)

48. Ans. (4)

49. Ans. (4)

50. Ans. (2)

51. Ans. (2)

52. Ans. (3)

53. Ans. (2)

54. Ans. (2)

55. Ans. (2)

56. Ans. (1)

57. Ans. (3)

58. Ans. (1)

59. Ans. (3)

60. Ans. (3)

61. Ans. (3)

3|sinq| + 4|cosq|

for first quadrant y = 3sinθ + 4sinθ

$y_{max} = 5$

62. Ans. (1)

$P_1 = \frac{1}{\sqrt{a^2+b^2+c^2}}$ & $P_2 = \frac{1}{\sqrt{ab+bc+ca}}$

$ab + bc + ca = a^2 + b^2 + c^2$

$\Rightarrow$ a = b = c

Value = 8

63. Ans. (4)

$\frac{[\vec{a}+\vec{b} \;\; \vec{b}+\vec{c} \;\; \vec{c}+\vec{a}]}{[\vec{a}\times\vec{b} \;\; \vec{b}\times\vec{c} \;\; \vec{c}\times\vec{a}]} = \frac{2}{[\vec{a}\vec{b}\vec{c}]}$

64. Ans. (2)

$\int \frac{(2+\sqrt{x})dx}{x^2(1+x^{-1/2}+x^{-1})^2} = \int \frac{(2x^{-2}+x^{-3/2})}{(1+x^{-1/2}+x^{-1})^2}dx$

Let $1 + x^{-1/2} + x^{-1} = t$

$\left(-\frac{1}{2}x^{-3/2} - x^{-2}\right)dx = dt$

$= -2\int \frac{dt}{t^2} = \frac{2}{t} = \frac{2x}{(1+\sqrt{x}+x)} + C$

65. Ans. (4)

Do yourself

66. Ans. (1)

$\vec{a} + 2\vec{b} = -3\vec{c}$

Square $1 + 4 + 4(\vec{a}.\vec{b}) = 9$

$\vec{a}.\vec{b} = 1 \Rightarrow \cos\theta \Rightarrow \theta = 0°$

67. Ans. (4)

$A = \begin{bmatrix} 0 & -15 & -26 \\ 15 & 0 & -65 \\ 26 & 65 & 0 \end{bmatrix}$

A is skew symmetric & $|A| = 0$

68. Ans. (3)

$|4adjA| = 64|adjA| = 64 \times 16 = 1024$

69. Ans. (4)

$\left[(\vec{a}+\vec{b}+\vec{c}) \quad (\vec{a}+\vec{b}) \quad (\vec{a}+\vec{c})\right] = -[\vec{a}\ \vec{b}\ \vec{c}]$

70. Ans. (2)

$\frac{x}{1} = -\frac{2}{y} = -\frac{5}{2}$

$xy = -2$ & $\frac{y}{z} = \frac{2}{5}$

$-\frac{4}{5}$

71. Ans. (1)

(x + y + z – 4) + λ(2x – 3y + z + 5) = 0

Put y = z = 0 $\Rightarrow$ x(1 + 2λ) = (4 – 5λ)

1 + 2λ = 4 – 5λ

$\lambda = \frac{3}{7}$

7(x + y + z – 4) + 3(2x – 3y + z + 5) = 0

13x – 2y + 10z – 13 = 0

72. Ans. (2)

$xdy + ydx = x^2dx$

$d(xy) = x^2dx$

integrate $xy = \frac{x^3}{3} + C$

73. Ans. (3)

$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & -\beta & 3 \\ 1 & -1 & -1 \end{vmatrix} = 0$

α(β + 3) – (–1 – 3) + (–1, + β) = 0

αβ + (α)(3) + 3 + β = 0

α(β + 3) + (β + 3) = 0

α = –1 or β = –3

74. Ans. (1)

$\lim_{x\to 0} \frac{f'(1+\tan x)\sin^2 x - 2xf(1+x^2)}{1} = 3$

75. Ans. (4)

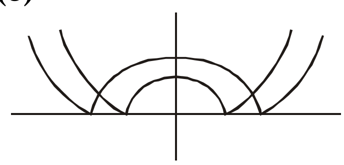
Vector along given line is $\vec{b} = 5\hat{i} + 2\hat{j} + 3\hat{k}$

and $\vec{n} = a\hat{i} + \hat{j} - \hat{k}$

$\vec{n}.\vec{b} = 0 \Rightarrow 5a + 2 - 3 = 0$

$a = \frac{1}{5}$

76. Ans. (3)



Point of intersaction when

$x^2 - 1 = 3 - x^2 \Rightarrow x = \sqrt{2}$

$y = x^2 - 1$ & $y = 3 - x^2$

$\left(\frac{dy}{dx}\right)_1 = 2\sqrt{2}$ & $\left(\frac{dy}{dx}\right)_2 = -2\sqrt{2}$

$\tan\theta = \frac{4\sqrt{2}}{1-8} \Rightarrow \theta = \tan^{-1}\left(\frac{4\sqrt{2}}{7}\right)$

77. Ans (1)

$\vec{n}_1 = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{n}_2 = 2\hat{i} - \hat{j} + \hat{k}$

$\cos\theta = \frac{\vec{n}_1.\vec{n}_2}{|\vec{n}_1||\vec{n}_2|} = \frac{3}{\sqrt{6}\sqrt{6}} = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

78. Ans. (3)

A.M. ≥ G.M.

$$\frac{\left(\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\right)}{3} \geq \left(\frac{a}{b}.\frac{b}{c}.\frac{c}{a}\right)^{\frac{1}{3}}$$

$$\frac{a}{b}+\frac{b}{c}+\frac{c}{a} \geq 3$$

79. Ans. (3)

$$\left(\frac{1}{(a-2b)}+\frac{1}{c}\right)+\left(\frac{1}{a}+\frac{1}{(c-2b)}\right)=0$$

$$\frac{(a+c-2b)}{c(a-2b)}+\frac{(a+c-2b)}{a(c-2b)}=0$$

By solving ac = ab + bc

$$\frac{1}{b}=\frac{1}{a}+\frac{1}{c}$$

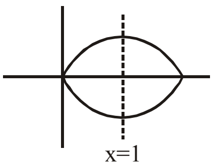
80. Ans. (3)

$$y = x^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+......} = x^1$$

$$\frac{dy}{dx}=1$$

81. Ans. (4)

$$y = x\sqrt{2-x}$$



$$2\int_0^1 x(\sqrt{2-x})dx \qquad \text{Let } 2-x=t^2$$

$$dx = -2$$

$$2\int_{\sqrt{2}}^{1}(2-t^2)t(-2t)dt = 4\int_{\sqrt{2}}^{1}(2t^2-t^4)dt$$

$$4\left(\frac{2t^3}{3}-\frac{t^5}{5}\right)_1^{\sqrt{2}}$$

$$4\left(\frac{4\sqrt{2}}{3}-\frac{4\sqrt{2}}{5}\right)-\left(\frac{2}{3}-\frac{1}{5}\right)$$

$$4\left(\frac{8\sqrt{2}}{15}-\frac{7}{15}\right)$$

$$\frac{4(8\sqrt{2}-7)}{15}$$

82. Ans. (2)

$2x^2 + px + 1 = -1$

$2x^2 + px + 2 = 0$

Use D = 0 $\Rightarrow$ P = 4, P = –4 & Product = –16

83. Ans. (4)

For homogeneous equation $\frac{d^2y}{dx^2}=0$

84. Ans. (3)

$$I = \int_0^{\pi/2}\frac{\sin^2 x dx}{(\sin^2 x+\sqrt{3}\cos^3 x)} \qquad ......(i)$$

$$\sqrt{3}\,I = \int_0^{\pi/2}\frac{\sqrt{3}\cos^3 x dx}{(\sin^2 x+\sqrt{3}\cos^3 x)}$$

(1) + (2)

$$I(1+\sqrt{3})=\frac{\pi}{2}$$

$$I=\frac{\pi}{2(1+\sqrt{3})}=\frac{\pi(\sqrt{3}-1)}{4}$$

85. Ans. (1)

$$\frac{(\alpha^2+\beta^2)-\alpha\beta}{(\alpha+\beta)}=\frac{\frac{b^2}{a^2}-\frac{c}{a}}{-\frac{b}{a}}=\frac{(b^2-ac)}{-ab}=\frac{ac-b^2}{ab}$$

86. Ans. (2)

$x^3 + (\sin\theta)x^2 + (\cos\theta)x + 7 = (x-\alpha)(x-\beta)(x-\gamma)$

put x = 1

$8 + (\sin\theta + \cos\theta) = (1-\alpha)(1-\beta)(1-\gamma)$

Maximum value of = $8+\sqrt{2}$

87. Ans. (3)

$$\frac{1}{2}\begin{vmatrix}0 & 0 & 1\\ 1 & 2 & 1\\ 1 & -2 & 1\end{vmatrix}=\left|\frac{1}{2}(-4)\right|=2$$

88. Ans. (3)

Option C is

$X = A - A^T \qquad .....(1)$

$X^T = A - A^T \qquad .....(2)$

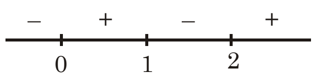
(1) + (2) $\Rightarrow$ $X + X^T = 0$

89. Ans. (1)

$$\frac{dy}{dx}=2x(x-2)^2+2x^2(x-2)$$

$= 2x(x-2)(2(x-1)) < 0$

$x(x-1)(x-2) < 0$



$(-\infty,0)\cup(1,2)$

90. Ans. (4)

$$\left|\frac{16ab}{3}\right|=\frac{256}{3}$$

ab = 16

(1, 16)(2, 8)(4, 4)(8, 2)(16, 1)