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ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) path to success

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2016 - 2017)

JEE (Main + Advanced) : ENTHUSIAST COURSE (PHASE : I)

ANSWER KEY **TEST DATE : 30-11-2016**

Test Type : MINOR Test Pattern : JEE-Advanced

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	B	C	D	D	A	A	B	A	A
	Q.	11	12	13	14	15	16				
	A.	B	B	A	D	B	A				
SECTION-IV	Q.	1	2	3	4						
	A.	6	8	5	1						

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	Bonus	B	C	C	C	A	B	D	B	C
	Q.	11	12	13	14	15	16				
	A.	A	A	A	C	A	C				
SECTION-IV	Q.	1	2	3	4						
	A.	4	4	1	4						

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	A	D	B	A	C	A	B	C	D
	Q.	11	12	13	14	15	16				
	A.	D	B	D	A	B	C				
SECTION-IV	Q.	1	2	3	4						
	A.	3	6	7	4						

JEE (Main + Advanced) : ENTHUSIAST COURSE

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PART-1 : PHYSICS   SOLUTION

SECTION-I

1. Ans. (C)

Sol. $\frac{Mdi_2}{dt} - \frac{Ldi}{dt} - i_1R = 0$

$Ma - \frac{Ldi}{dt} - i_1R = 0$

On solving $i_1 = \frac{Ma}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$

2. Ans. (B)

Sol. $\frac{12}{18} = \frac{\ell}{100 - \ell}$

$200 - 2l = 3l$

$l = 40$ cm

$\frac{12}{8} = \frac{\ell'}{100 - \ell'}$

$300 = 5l'$

$l' = 60$ cm

3. Ans. (C)

Sol. $\frac{Q^2}{2 \times 8\pi\varepsilon_0 R} + \frac{(Q)^2}{2 \times 4\pi\varepsilon_0 \times 3R}$

$\frac{Q^2}{8\pi\varepsilon_0 R}\left[\frac{1}{2} + \frac{1}{3}\right]$

4. Ans. (D)

Sol. $\frac{Mv^2}{R} = \frac{GM^2}{4R^2} \Rightarrow M = \frac{4Rv^2}{G}$

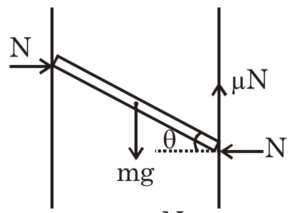
$v = \frac{2\pi R}{T}$

$R = \frac{vT}{2\pi}$

$M = \frac{v^3 T2}{\pi G}$

5. Ans. (D)

Sol.



mg=μN

$mg\frac{L}{2}\cos\theta = N \times L \sin\theta$

$\Rightarrow \frac{\mu}{2} = \tan\theta = \frac{1}{4} = \frac{\sqrt{L^2 - 4}}{2} \Rightarrow L = \frac{\sqrt{17}}{2}$

6. Ans. (A)

Sol. $k_\alpha = \frac{M}{M+4}Q = \frac{222}{226} \times 4.87 = 4.78$ MeV

7. Ans. (A)

Sol. $F = -\frac{e^2}{4\pi \in_0 r^2} + \frac{2\sigma}{r^3} = \frac{mv^2}{r}$

$mv^2 r = \frac{e^2}{4\pi \in_0} - \frac{2\sigma}{r}$

$mvr = \frac{nh}{2\pi}$

$v = \frac{e^2}{2 \in_0 nh} - \frac{4\pi\sigma}{nhr}$

$v < v_0$

$\Rightarrow r > r_0$

8. **Ans. (B)**

9. **Ans. (A)**

Sol. When ball reaches bottom point

$$v = \sqrt{2gh}$$

Now angular momentum will be conserved about any point on the ground surface just before & just after friction starts acting.

$$mvR = mv'R + I\omega$$

$$mvR = mv'R + \frac{2}{5}mR^2 \times \frac{v'}{R}$$

$$\Rightarrow v' = \frac{5}{7}v$$

From energy conservation

$$mgh' = \frac{1}{2}mv'^2 + \frac{1}{2}I\omega^2$$

$$\Rightarrow h' = \frac{5h}{7}$$

10. **Ans. (A)**

Sol. $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\frac{2}{5}mR^2\frac{v^2}{R^2}$$

$$mgh = \frac{7}{10}mv^2$$

$$\Rightarrow v = \sqrt{\frac{10gh}{7}} \quad \Rightarrow \frac{1}{2}mv^2 = mgh' \quad \Rightarrow h' = \frac{5h}{7}$$

11. **Ans. (B)**

Sol. Initially light was corresponding to the transition of hydrogen from n = 2 to n = 1. i.e. E = 10.2eV. To obtain more peaks, the transition of electron in hydrogen atom must start from n = 3.

E = 12.1 eV

To increase energy by 1.18 times, the potential difference must be increased by 1.18 times.

12. **Ans. (B)**

Sol. Initially

$$1mm = \frac{\lambda D}{d}$$

where $\lambda = \frac{hc}{10.2}$

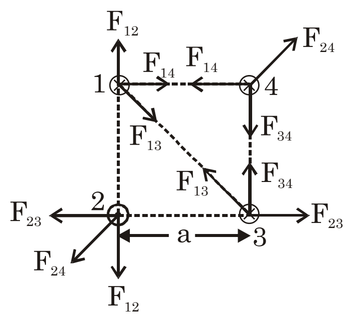
$$\Rightarrow 1mm = \frac{hcD}{(10.2)d}$$

Now $y = \frac{hcD}{(12.1)d}$ or $y = \frac{hcD}{(1.9)d}$

y = 0.843 mm or y = 5.4 mm

13. **Ans. (A)**

Sol. Force per unit length between current carrying wires having current in same direciton is attractive and have magnitude $\frac{\mu_0 i^2}{2\pi r}$ and having current in opposite direction is repulsive and magnitude $\frac{\mu_0 i^2}{2\pi r}$.



$F_{12} = \frac{\mu_0 i^2}{2\pi a}$ $\quad F_{23} = \frac{\mu_0 i^2}{2\pi a}$

$F_{13} = \frac{\mu_0 i^2}{2\sqrt{2}\pi a}$ $\quad F_{24} = \frac{\mu_0 i^2}{2\sqrt{2}\pi a}$

$F_{14} = \frac{\mu_0 i^2}{2\pi a}$ $\quad F_{34} = \frac{\mu_0 i^2}{2\pi a}$

Solve for each wire.

14. **Ans. (D)**

Sol. $\mu_1 \sin 30 = \mu_2 \sin\theta$

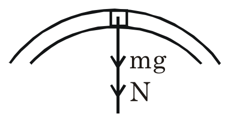
$\theta = 60°$

$$\sin\theta_C = \frac{1}{1.5}$$

$\Rightarrow \theta_C < 45°$

15. **Ans. (B)**

Sol.



$\mu + mg = \frac{mv^2}{r}$

$22 + 1 \times 10 = \frac{1 \times 12}{R}$

$R = \frac{3}{8}$

16. Ans. (A)

Sol. Mass of A = 8 kg

(Since density is same)

$m_1v_1 = m_2v_2$

(since no extrnal force is exists)

$8v_A = 1 \times v_B$

$8v_A = v_B$

$\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = 18 \times 2$

$v_A = 1$ m/s

$v_B = 8$ m/s

$v'_A = 1 \times e$ m/s

$v'_B = 8 \times e$ m/s

SECTION-IV

1. Ans. 6

Sol. We have $f_1 = 50$ cm and $f_2 = 100$ cm let the real distance between A and B be x.

Also let refractive index of liquid be μ. Then

$$\frac{1}{f_1} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{2}{f_1}$$

$$\frac{1}{f'_1} = \left(\frac{3}{2\mu} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \frac{1}{f'_1} = \frac{2}{f_1}\left(\frac{3-2\mu}{2\mu}\right)$$

and $\frac{1}{f'_2} = \frac{2}{f_2}\left(\frac{3-2\mu}{2\mu}\right)$

Now, for A we have $-\left(\frac{1}{200}\right) - \left(\frac{1}{-x}\right) = \frac{2}{50}\left(\frac{3-2\mu}{2\mu}\right)$

$$\Rightarrow \frac{1}{x} = \frac{1}{200} + \frac{2}{50}\left(\frac{3-2\mu}{2\mu}\right) \quad ...(1)$$

Also for B we have

$$-\frac{1}{100} - \left(-\frac{1}{x}\right) = \frac{2}{100}\left(\frac{3-2\mu}{2\mu}\right)$$

so, $\frac{1}{x} = \frac{1}{100} + \frac{2}{100}\left(\frac{3-2\mu}{2\mu}\right)$ ....(2)

from (1) and (2) we get

$$\Rightarrow \frac{2(3-2\mu)}{100(2\mu)} + \frac{1}{100} = \frac{1}{200} + \frac{2(3-2\mu)}{50(2\mu)}$$

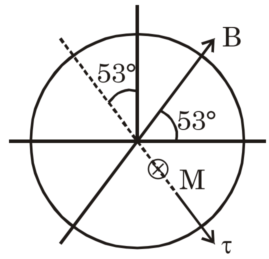
$$\Rightarrow \frac{2(3-2\mu)}{(2\mu)}\left[\frac{1}{50} - \frac{1}{100}\right] = \frac{1}{100} - \frac{1}{200} = \frac{1}{200}$$

$$\Rightarrow \frac{(3-2\mu)}{2\mu} = \frac{1}{2}$$

$$\Rightarrow 6 - 4\mu = \mu \text{ so } \mu = \frac{6}{5} = \frac{12}{10}$$

2. Ans. 8

Sol.



$\tau = 1 \times \pi \times 1^2 \times 5 = 1 \times \frac{1^2}{2} \times \alpha$

$\alpha = 10\pi$

$a = r\alpha$

$= (1 \times \sin 53)\alpha$

$= 8\pi$

3. Ans. 5

Sol. Let potential of a be 3iR

then potential C is 2iR

& potential of b is 0

Let potential of d be V

then (V – 3iR)C + (V – 2iR)C + (V – 0) C = 0

$\Rightarrow V = \frac{5iR}{3}$

$\Rightarrow$ charge of $C_3 = (V - 0)C = \frac{5iR}{3}C$

4. Ans. 1

Sol. $\frac{\Delta a}{a} = \frac{\Delta g}{g} + \frac{\Delta(m_1 + m_2)}{m_1 - m_2} + \frac{\Delta(m_1 + m_2)}{m_1 + m_2}$

$= \frac{0.01}{9.80} + \frac{0.4}{60} + \frac{0.4}{147 \times 5}$

$\frac{15 + 98 + 40}{49 \times 3 \times 4 \times 5} = 1\%$

PART–2 : CHEMISTRY SOLUTION

SECTION-I

1. Ans. (Bonus)

$$\left[n_{N_2}\right]_{1bar} = \frac{P_{N_2}}{K_H}.n_{H_2O} = \frac{0.8}{10^5}\times\frac{5.4\times10^3\times1}{18} = 2.4\times10^{-3}$$

$$\left[n_{N_2}\right]_{10bar} = \frac{8}{K_H}.n_{H_2O} = 2.4\times10^{-2}$$

$\Delta n_{H_2} = 24\times10^{-3} - 2.4\times10^{-3} = 21.6\times10^{-3}$

$= 2.16\times10^{-2}$ mol.

2. Ans.(B)

Sol-1 : $1pH = pK_a + \log\frac{[A^-]}{[HA]} \Rightarrow \frac{[A^-]}{[HA]} = \frac{1}{1}$

Sol-2 : $4.3 = 4 + \log\frac{[A^-]}{[HA]} \Rightarrow \frac{[A^-]}{[HA]} = \frac{2}{1}$

$$[HA]_{final} = \frac{M_1V_1 + M_2V_2}{V_1 + V_2} = \frac{0.1\times1 + 0.1\times2}{3} = 0.1M$$

$$[A^-]_{final} = \frac{0.1\times1 + 0.2\times2}{3} = \frac{0.5}{3}M \;;$$

$$pH = 4 + \log\frac{0.5/3}{0.1} = 4.22$$

3. Ans. (C)

4. Ans. (C)

5. Ans. (C)

6. Ans. (A)

7. Ans. (B)

8. Ans. (D)

9. Ans. (B)

10. Ans. (C)

11. Ans. (A)

12. Ans. (A)

13. Ans. (A)

14. Ans. (C)

15. Ans. (A)

16. Ans. (C)

SECTION - IV

1. Ans. (4)

Cell reaction : $H_2 + Cl_2 \rightarrow 2H^+ + 2Cl^-$

$$\Delta S^0 = nF\left(\frac{\partial E}{\partial T}\right)_P = -2\times96500\times1.0\times10^{-3}$$

$= -193$ J/K

Ans. = 193 J/K

2. Ans. (4)

3. Ans. (1)

4. Ans. (4)

PART-3 : MATHEMATICS SOLUTION

SECTION-I

1. Ans. (D)

$|z-3| + |z+1| = 8$ represent an ellipse with centre $z = 1 + 0i$ and focus $3 + 0i, -1 + 0i$

$|z-3| + |z+1| \geq 8$ represent ellipse and outer part of ellipse

$$\Rightarrow \quad |z-1|_{min} = \frac{\text{length of minor axis}}{2} = 2\sqrt{3}$$

2. Ans. (A)

Put $e^t + e^{-t} = 2\sec x$, $e^t - e^{-t} = 2\tan x$

$\Rightarrow \quad (e^t - e^{-t})_{dt} = 2\sec x \tan x\, dx$

$dt = \sec x\, dx$

$$\therefore \int_0^{\log(1+\sqrt{2})} \left(e^t + e^{-t}\right)^8 dt = \int_0^{\pi/4} 2^8(\sec x)^9 dx$$

3. Ans. (D)

Let $y = mx - \frac{2}{m}$ is common tangent

perpendicular from centre = radius

$$= \left|\frac{-2}{m\sqrt{1+m^2}}\right| = \sqrt{2}$$

$\therefore \quad m = 1$ or -1

common tangents : $y = x - 2$

$y = -x + 2$

$\therefore \quad T(2, 0)$

$$\text{Area of } \Delta RST = \frac{(S_1)^{3/2}}{2a} = 16$$

4. Ans. (B)

$b + c = 5$, $bc = 3$ or $b + c = 3$, $bc = 5$ (reject)

$\therefore \quad b + c = 5$, $bc = 3$, $a = \sqrt{22}$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = -\frac{1}{2} \Rightarrow A = 120°$$

$$\frac{r}{R} = \frac{\Delta}{S.\left(\frac{abc}{4\Delta}\right)}$$

$$= \frac{8\left(\frac{1}{2}bc\sin A\right)^2}{(a+b+c)(abc)} = \frac{9}{2\left(22 + 5\sqrt{22}\right)}$$

5. Ans. (A)

$$P = \sum_{r=0}^{6} \frac{{}^6C_r}{2^6}.\frac{2^{6-r}}{2^6} = \frac{429}{4096}$$

6. Ans. (C)

$^6C_2[9 + {^2C_1}.{^4C_1}.11 + {^4C_2}\times 14] = 2715$

7. Ans. (A)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = -\frac{3}{5} + \frac{1}{4} + \frac{1}{2} = \frac{3}{20}$

8. Ans. (B)

$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{3}{10}$

Solution for Question 9 and 10

A(2, 3, 4)

B is foot of perpendicular from A to the plane

$\therefore \quad B(\alpha, \beta, 2) \equiv (0, 1, 2)$

Line $L_1 = \frac{x}{1} = \frac{y-1}{1} = \frac{z-2}{1}$

9. Ans. (C)

10. Ans. (D)

11. Ans. (D)

AB is focal chord

$\therefore \frac{y_1}{2}.\frac{y_2}{2} = -1 \;\; \& \;\; x_0 = \frac{x_1 + x_2}{2}, y_0 = \frac{y_1 + y_2}{2}$

$A\left(\frac{y_1^2}{4}, y_1\right)$

$B\left(\frac{y_2^2}{4}, y_2\right) \equiv \left(\frac{y}{y_1^2}, \frac{-4}{y_1}\right)$

$C\left(\frac{y_3^2}{4}, y_3\right)$

$D\left(\frac{y_4^2}{4}, y_4\right)$

$M_{BC} = \frac{y_3 - y_2}{x_3 - x_2}$

$m_{AE} = \frac{y_1 - 0}{x_1 - 2}$

$\therefore \quad \frac{y_3 - y_2}{x_3 - x_2} = \frac{y_1}{x_1 - 2}$

$\frac{4}{y_3 + y_2} = \frac{4y_1}{y_1^2 - 8}$

$\Rightarrow \quad 4\left(y_1^2 - 8\right) = 4y_1y_3 + 4(-4)$

$\Rightarrow \quad y_3 = \frac{4y_1^2 - 16}{4y_1} = y_1 - \frac{4}{y_1} = y_1 + y_2 = 2y_0$

12. Ans. (B)

Parabola $y^2 = 4x$ & $y_1y_4 = 4$

tangent at D : $yy_4 = 2(x + x_4)$

normal at A : $y - y_1 = \frac{-y_1}{2}(x - x_1)$

$\Rightarrow \quad yy_4 - y_1y_4 = \frac{-y_1y_4}{2}(x - x_1)$

abssissa of intersection point is

$$x = 1 + \frac{x_1 - x_4}{2}$$

$$= 1 + \frac{x_1 - \frac{1}{x_1}}{2} \quad \left\{\begin{matrix} \text{Use}: \frac{1}{x_1} = x_2 \\ 2x_0 = x_1 + x_2 \end{matrix}\right\}$$

$$= 1 + x_0 - x_2$$

13. Ans. (D)

(P) Let $f(x) = ax^2 + bx$

$\int_0^1 (ax^4 + bx^2)dx = 1 \quad \Rightarrow \quad \frac{a}{5} + \frac{b}{3} = 1$

$\Rightarrow \quad 3a + 5b = 15$

$a = 0, b = 3$ or $a = 5, b = 0$

$f(x) = 3x$ or $5x^2$

(Q) If it is minimum at

$x^2 = 2n\pi + \frac{5\pi}{4}, n \in \{0,1\}$

$\therefore$ there are four values of x.

(R) $I = \int_{-2}^{2} \frac{3x^2}{8\left(1 + e^{\sin x}\right)} dx$

Apply King

$I = \int_{-2}^{2} \frac{3x^2 e^{\sin x}}{8(1 + e^{\sin x})} dx$

$2I = \frac{3}{8}\int_{-2}^{2} x^2 dx \quad \Rightarrow \quad I = \frac{[x^3]_0^2}{8} = 1$

(S) $\sin^2 x.\log\left(\frac{1-x}{1+x}\right)$ is odd function

$\therefore \quad \int_{-1/2}^{1/2} \sin^2 x \log\left(\frac{1-x}{1+x}\right) dx = 0$

14. Ans. (A)

(P) $y(x)=\cos\left(\frac{3\pi}{2}-3\sin^{-1}x\right)=\cos(3\cos^{-1}x)$

$= 4x^3 - 3x$

$\frac{x\frac{dy(x)}{dx}-3y(x)}{x}=\frac{x[12x^2-3]-12x^3+9x}{x}=6$

(Q) $\vec{A}=(\vec{A}_2-\vec{A}_1)\times(\vec{A}_4-\vec{A}_3)+(\vec{A}_3-\vec{A}_2)$

$\times(\vec{A}_4-\vec{A}_1)+(\vec{A}_1-\vec{A}_3)\times(\vec{A}_4-\vec{A}_2)$

$=-\vec{A}_2\times\vec{A}_3+\vec{A}_1\times\vec{A}_3-\vec{A}_3\times\vec{A}_1+\vec{A}_2$

$\times\vec{A}_1-\vec{A}_1\times\vec{A}_2+\vec{A}_3\times\vec{A}_2$

$=2[\vec{A}_2\times\vec{A}_1+\vec{A}_3\times\vec{A}_2+\vec{A}_1\times\vec{A}_3]$

$\therefore \quad \frac{|\vec{A}|}{A}=2$

(R) Intersection points are $(1,0)\left(-\frac{13}{5},-\frac{6}{5}\right)$

$\therefore \quad a\sqrt{\frac{8}{5}}=\sqrt{\left(\frac{18}{5}\right)^2+\left(\frac{6}{5}\right)^2}$

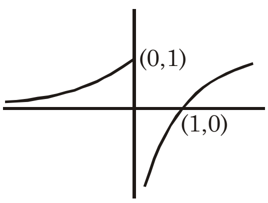
$a^2.\frac{8}{5}=\frac{360}{25}\Rightarrow a=3$

(S) $\tan^{-1}\left[\frac{\frac{1}{4x+1}+\frac{1}{8x+1}}{1-\frac{1}{4x+1}.\frac{1}{8x+1}}\right]=\tan^{-1}\left(\frac{1}{2x^2}\right)$

$\Rightarrow \quad x=0,-\frac{1}{3},\frac{3}{2}$ (x = 0 & $-\frac{1}{3}$ reject)

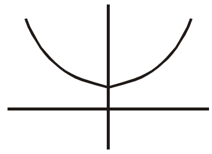
15. Ans. (B)

(P)



neither continuous nor one-one.

(Q) $f_4(x)=\begin{cases}(1-x)^2 & , x<0\\ e^{6x} & , x\ge 0\end{cases}$

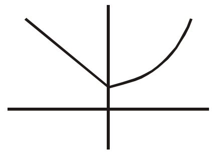


neither onto nor one-one

(R) $f_1 o f_2 = e^{3x^2}$

continuous and one-one

(S)



neither onto nor one-one

16. Ans. (C)

(P) $2\sum_{k=1}^{10} z_{(10,k)}$ = 2(sum of roots of $1^{\frac{1}{10}}$) = 0

(Q) $\sum_{k=1}^{10} z_{(11,k)}$ = (sum of imaginary roots of $1^{\frac{1}{11}}$)

= –1

(R) $\prod_{k=1}^{10} z_{(10,k)}$ = (product of roots of $1^{\frac{1}{10}}$) = –1

(S) $\prod_{k=1}^{10} z_{(11,k)}$ = (product of imaginary roots of $1^{\frac{1}{11}}$) = 1

SECTION-IV

1. Ans. 3

$\lim_{x\to1}\frac{(x+5)(x-1)}{(x-1)(x+1)}=3$

2. Ans. 6

$\frac{dy}{dx}\sqrt{4-x^2}-\frac{x}{\sqrt{4-x^2}}y=7x^6+2x$

$d\left(\sqrt{4-x^2}.y\right)=(7x^6+2x)dx$

$\Rightarrow \quad \sqrt{4-x^2}\,y=x^7+x^2+C$

$\because \quad f(0) = 1 \quad \Rightarrow \quad c = 2$

$\therefore \quad y = \dfrac{x^7 + x^2 + 2}{\sqrt{4 - x^2}}$

$$\int_{-\sqrt{3}}^{\sqrt{3}} \left(\frac{x^7}{\sqrt{4-x^2}} + \frac{x^2+2}{\sqrt{4-x^2}} \right) dx = 2\int_0^{\sqrt{3}} \frac{x^2+2}{\sqrt{4-x^2}} dx$$

put x = 2sinθ

$dx = 2\cos\theta d\theta$

$$= 2\int_0^{\pi/3} \frac{4\sin^2\theta + 2}{2\cos\theta}.2\cos\theta d\theta$$

$$= 2\int_0^{\pi/3} [4 - 2\cos 2\theta] d\theta$$

$$= 2\left[\frac{4\pi}{3} - \frac{\sqrt{3}}{2}\right] = \frac{8\pi}{3} - \sqrt{3}$$

$\therefore \quad a = \dfrac{8}{3}, \ b = -1$

3. Ans. 7

Absolute value of coefficient

$= {}^4C_0.{}^7C_1.{}^{12}C_2.2^{10} + {}^4C_1{}^7C_3{}^{12}C_0.2^{12}$

$+ {}^4C_2.{}^7C_1.{}^{12}C_1.2^{11} + {}^4C_4.{}^7C_1{}^{12}C_0.2^{12}$

$= 2^{10}[1.7.66 + 4.35.4 + 6.7.12.2 + 1.7.4]$

$= 2^{11}.7[147] = 2^{11}.3^1.7^3$

4. Ans. 4

Distance between P_2 & P_3 is $\dfrac{6}{7}$

P_1 & P_3 is 5

P_2 & P_1 is $\dfrac{29}{7}$

$\therefore \quad \text{Probability} = \dfrac{1 + \frac{6}{7}}{5} = \dfrac{13}{35}$