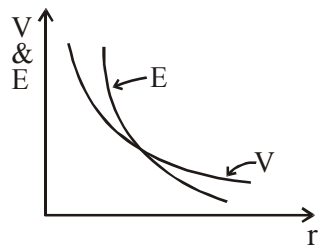


JEE (Main + Advanced) : ENTHUSIAST COURSE**PHASE : I**

Test Type : MINOR

Test Pattern : BOARD PATTERN

PHYSICS**TEST DATE : 16-10-2016****SOLUTION****PART A - PHYSICS**

1.  $E \propto \frac{1}{r^2}$ [½ + ½]
 & $V = \frac{1}{r}$

2. Average power over full cycle of the ac voltage source. [1]
3. Bq or becquerel [½]
 One becquerel activity corresponds to one decay or disintegration per second [½]
4. The numbers are conserved but the total mass is not conserved. The total mass of the free protons/neutrons is more than their total mass within the nucleus.
 The lost mass ($=\Delta m$) gets converted into energy as per the relation $E = (\Delta m)c^2$
 [Also, accept alternative ways of explaining the phenomenon] [2]
5. We have

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \text{ for a point charge}$$

Now, volume charge density

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3} \quad [½]$$

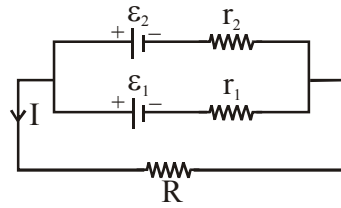
So, charge contained within a sphere of radius r [$0 < r < R$]

$$\text{Thus } Q' = \rho \times \frac{4}{3}\pi r^3 = Q \left(\frac{r^3}{R^3} \right) \quad [1]$$

 \therefore So, electric field

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q'}{r^2} = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^3} r \quad [½]$$

6. as, $r_{eq} < r$, the two cells must have been connected in parallel.
The diagram of the setup is as shown.



[½]

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2} \quad \& \quad \varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} \quad [1]$$

So $I = \frac{\varepsilon_{eq}}{R + r_{eq}}$ [½]

[Also accept if student writes $I = \frac{\varepsilon_1 r_1 + \varepsilon_2 r_2}{R(r_1 + r_2) + r_1 r_2}$]

7. (i) Self inductance of a coil depends on
 (a) It's geometry (area and length of a coil)
 (b) Number of turns
 (c) Medium within the coil [any two] [1]
- (ii) Mutual inductance of a given pair of coils depends on :
 (a) there geometries
 (b) there distance of separation
 (c) Number of turns in each coil
 (d) Nature of medium in the intervening space. [any two] [1]

8. The current leads the voltage in phase.

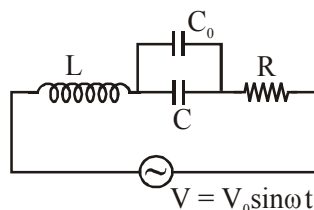
Hence, $X_C > X_L$

For resonance, we must have $X_C = X_L$ [½]

We, therefore, need to decrease $X_C = \frac{1}{\omega C}$. This requires an increase in the value of C. Hence, capacitor

C_0 should be connected in parallel across C. [½]

The diagram of the modified circuit is as shown –



[1]

For resonance, we then have $\frac{1}{\omega(C + C_0)} = \omega L$ [½]

Or $C_0 = \left[\frac{1}{\omega^2 L} - C \right]$ [½]

9. The two required distinct features are :-

- (i) Maximum K.E. of emitted photoelectrons is independent of the intensity of incident light. [½]
- (ii) No photoemission takes place from a given photo emitting surface if the frequency of the incident light is less than a critical or threshold frequency. [½]

We can understand these features on the basis of the photon theory as follows.

- (i) Increase in intensity increases only the number of incident photons but does not increase the energy ($=h\nu$) of these photons. Hence there is no increase in the maximum K.E. of the emitted photoelectrons. [1]
- (ii) No photo emission can take place if the energy of the incident photon ($=h\nu$) is less than the work function of the photo emitting surface. Hence, there is a minimum critical or threshold frequency for given photo emitting surface. [1]

10. Charge stored $Q = CV = 20 \times 10^{-6} \times 100 = 2000 \mu C$ [½]

New capacity after filling with dielectric

$$C = 5 \times 20 \mu F = 100 \mu F \quad [½]$$

Also, energy stored $U = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} CV^2 = \frac{1}{2} QV$ [½]

(i) Energy stored before dielectric

$$= \frac{1}{2} \times \frac{(2000 \times 10^{-6}) \times 2000 \times 10^{-6}}{20 \times 10^{-6}} = 0.1 J \quad [½]$$

(ii) Energy stored after the dielectric introduced (\therefore no change in θ)

so $U' = \frac{1}{2} \times \frac{2000 \times 10^{-6} \times 2000 \times 10^{-6}}{100 \times 10^{-6}} = 0.02 J$ [1]

11. The given circuit can be redrawn as shown -



It is, a wheatstone's bridge.

Using loop law & junction law, we get.

[when $I_g = 0$] [½]

$$I_1 = I_3$$

and $I_2 = I_4$

for loop ABDA, we have

$$-I_1 P + I_2 X = 0 \quad \text{or} \quad I_1 P = I_2 X \quad [½]$$

For the loop BCDB, we have

$$-I_3Q + I_4R = 0 \quad \text{or} \quad I_3Q = I_4R \quad [1/2]$$

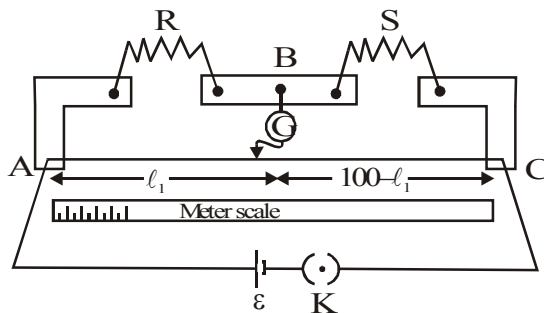
dividing, we get

$$\frac{I_1P}{I_3Q} = \frac{I_2X}{I_4R}$$

$$\text{or} \quad \frac{P}{Q} = \frac{X}{R} \quad [\because I_1 = I_3 \text{ \& } I_2 = I_4] \quad [1/2]$$

(b) A simple device, based on the above condition, is the meter bridge [1/2]

It has a [uniform cross-section] wire of length 1 m stretched taut between two thick metallic clamps. It has two gaps for connecting a resistance box and the unknown resistance. The circuit diagram, for the meter bridge is, as shown -



[1]

We move the jockey, on the wire of the meter bridge till we find a point at which the deflection in (G) is zero.

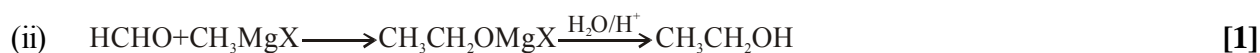
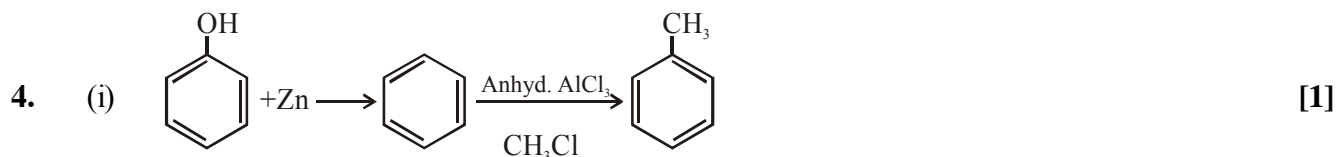
$$\text{So, we have} \quad \frac{R}{l_1} = \frac{S}{l_2}$$

$$\text{or} \quad S = R \left(\frac{l_2}{l_1} \right) \quad [1/2]$$

Knowing R_1 and finding l_1 and $l_2 = 100 - l_1$ we can calculate S. [1/2]

CHEMISTRY
SOLUTION
PART B - CHEMISTRY

1. (i) Inversion of configuration [1]
2. N-methyl-2-methylpropanamine / 2-methyl-N-methylpropanamine [1]
3. (i) zero order [1]
 (ii) $\text{mol L}^{-1} \text{s}^{-1}$



5. Volume of the unit cell = a^3 [2]
- $$= (400 \text{ pm})^3$$
- $$= (4 \times 10^{-8} \text{ cm})^3$$
- $$= 64 \times 10^{-24} \text{ cm}^3$$

Volume of 280 g of the element = mass / density

$$= 280/7 \text{ cm}^3$$

$$= 40 \text{ cm}^3$$

Number of unit cells in this volume = $40 / 64 \times 10^{-24} = 6.25 \times 10^{23}$ unit cells.

Since $z = 4$,

Therefore, total no. of atoms in 280g = $4 \times 6.25 \times 10^{23}$

$$= 2.5 \times 10^{24} \text{ atoms.}$$

6. $\log k = \log A - E_a/2.303RT$ [2]
- $$E_a / 2.303 RT = 1 \times 10^4 \text{ k/ T}$$
- $$E_a = 1.0 \times 10^4 \times 2.303 \times 8.314$$
- $$= 191471.4 \text{ J/mol}$$
- $$t_{1/2} = 0.693/k$$
- $$k = 0.693/200$$
- $$= 0.0034 \text{ min}^{-1} / 3.4 \times 10^{-3} \text{ min}^{-1}$$

7. (i) Sodium Hydrogen Sulphite reaction/ Pentaacetate of glucose does not react with Hydroxylamine/ Schiff's test (any one) [1]
- (ii) Buna-S < Polythene < Nylon 6,6 [1]
8. (i) Chromatography [1]
- (ii) To Separate two sulphide ores [1]
- (iii) It decomposes to CaO which removes impurity (silica) as slag/ Acts as flux. [1]

9.
$$E_{\text{cell}} = E_{\text{cell}}^0 - \frac{0.059}{n} \log \frac{[\text{Cr}^{3+}]^2}{[\text{Fe}^{2+}]^3} \quad [3]$$

$$0.261\text{V} = E_{\text{cell}}^0 - \frac{0.059}{6} \log \frac{[0.01]^2}{[0.01]^3}$$

$$0.261\text{V} = E_{\text{cell}}^0 - \frac{0.059}{6} \log 100$$

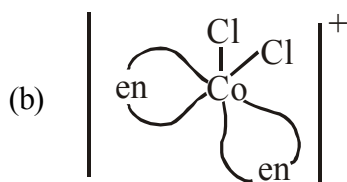
$$E_{\text{cell}}^0 = 0.261 + 0.0197$$

$$= 0.2807\text{V}$$

10. (a) d^2sp^3 , [3]

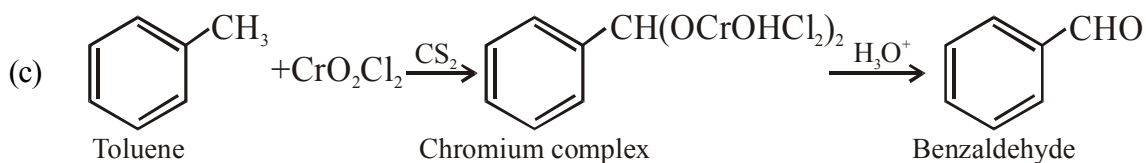
Diamagnetic,

low spin



11. (a) In phenols lone pair of electron on oxygen are delocalized over benzene ring due to resonance but in alcohol lone pair of electron on oxygen are localized & hence available for protonation. [5]

(b) Weaker (O-CH₃) bond and stronger(O-C₆H₅) bond ,due to resonance / carbon in benzene is sp² hybridized due to which partial double bond character.



(d) $\text{C}_6\text{H}_5\text{COCH}_3 < \text{CH}_3\text{CHO} < \text{HCHO}$

(e) Strong electron withdrawing power of Cl. Which stabilized carboxylate anion.

MATHEMATICS
SOLUTION
SECTION-A

1. We have, $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$

$$\therefore BA = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}_{3 \times 2} \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}_{2 \times 3}$$

$$[b_{ij}] = \begin{bmatrix} 2-12 & -4+6 & 6+15 \\ 4-20 & -8+10 & 12+25 \\ 2-4 & -4+2 & 6+5 \end{bmatrix}_{3 \times 3}$$

$$[b_{ij}] = \begin{bmatrix} -10 & 2 & 21 \\ -16 & 2 & 37 \\ -2 & -2 & 11 \end{bmatrix}_{3 \times 3}$$

[½]

Now $b_{21} = -16$; $b_{32} = -2$
 $\therefore b_{21} + b_{32} = -16 - 2 = -18$

[½]

2. $y = mx$ (i)

$$\frac{dy}{dx} = m$$
(ii)

from equation (i) & (ii)

$$\frac{dy}{dx} = \frac{y}{x}$$

[1]

3. $f: \mathbb{R} \rightarrow \mathbb{R}$

$$f(x) = (3 - x^3)^{1/3}$$

$$f \circ f(x) = f(f(x)) = [3 - \{f(x)\}^3]^{1/3}$$

$$= [3 - \{(3 - x^3)^{1/3}\}^3]^{1/3} = [3 - (3 - x^3)]^{1/3} = x$$

[1]

SECTION-B

4. $y = \tan^{-1} \left[\frac{3x + 2x}{1 - (3x)(2x)} \right] = \tan^{-1} 3x + \tan^{-1} 2x$

[1]

$$\frac{dy}{dx} = \frac{3}{1+9x^2} + \frac{2}{1+4x^2}$$

[1]

5. $y = \sqrt{\frac{1 - \sin 2x}{1 + \sin 2x}} = \sqrt{\frac{(\cos x - \sin x)^2}{(\cos x + \sin x)^2}}$

$$y = \frac{\cos x - \sin x}{\cos x + \sin x} = \frac{1 - \tan x}{1 + \tan x} = \tan\left(\frac{\pi}{4} - x\right)$$

[1]

Diff. w.r.t. x

$$\frac{dy}{dx} = \sec^2\left(\frac{\pi}{4} - x\right) \cdot (-1)$$

$$\Rightarrow \frac{dy}{dx} + \sec^2\left(\frac{\pi}{4} - x\right) = 0$$

[1]

SECTION-C

6. $f(x)$ is continuous at $x = -\frac{\pi}{6}$

$$\therefore f\left(-\frac{\pi}{6}\right) = \text{RHL} = \text{LHL}$$

$$f\left(-\frac{\pi}{6}\right) = \lim_{x \rightarrow -\frac{\pi}{6}} f(x)$$

$$\text{Now } f\left(-\frac{\pi}{6}\right) = k \quad [1/2]$$

$$\text{and } \lim_{x \rightarrow -\frac{\pi}{6}} f(x) = \lim_{x \rightarrow -\frac{\pi}{6}} \frac{\sqrt{3} \sin x + \cos x}{x + \frac{\pi}{6}}$$

$$\Rightarrow \lim_{x \rightarrow -\frac{\pi}{6}} f(x) = \lim_{x \rightarrow -\frac{\pi}{6}} \frac{2\left(\frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x\right)}{x + \frac{\pi}{6}} \quad [1\frac{1}{2}]$$

$$\Rightarrow \lim_{x \rightarrow -\frac{\pi}{6}} f(x) = \lim_{x \rightarrow -\frac{\pi}{6}} \frac{2 \sin\left(x + \frac{\pi}{6}\right)}{x + \frac{\pi}{6}} \quad [1]$$

$$\left(\text{Put } x + \frac{\pi}{6} = h \text{ as } x \rightarrow -\frac{\pi}{6}; h \rightarrow 0\right)$$

$$\Rightarrow \lim_{x \rightarrow -\frac{\pi}{6}} f(x) = \lim_{h \rightarrow 0} \frac{2 \sinh}{h} = 2$$

$$\text{hence } k = 2 \quad [1]$$

7. $I = \int_0^{\pi/2} \frac{\cos^2 x}{1 + 3 \sin^2 x} dx$

$$I = \int_0^{\pi/2} \frac{1}{\sec^2 x + 3 \tan^2 x} dx$$

$$= \int_0^{\pi/2} \frac{1}{(1 + 4 \tan^2 x)} \times \frac{\sec^2 x}{(1 + \tan^2 x)} dx$$

$$= \int_0^{\infty} \frac{dt}{(1 + t^2)(1 + 4t^2)} \quad [1]$$

$$\text{Put } t^2 = y$$

$$\frac{1}{(1 + t^2)(1 + 4t^2)} = \frac{1}{(1 + y)(1 + 4y)} = \frac{A}{1 + y} + \frac{B}{1 + 4y}$$

$$\text{at } y = -1 \quad A = -\frac{1}{3}$$

$$y = -\frac{1}{4} \quad B = \frac{4}{3} \quad [1]$$

$$\frac{1}{(1 + y)(1 + 4y)} = -\frac{1}{3} \left(\frac{1}{1 + y} \right) + \frac{4}{3} \left(\frac{1}{1 + 4y} \right)$$

$$\begin{aligned} \text{Put } \tan x &= t \\ \sec^2 x \, dx &= dt \\ x = 0 \quad t &= 0 \\ x = \frac{\pi}{2} \quad t &= \infty \end{aligned}$$

Put $y = t^2$

$$\frac{1}{(1+t^2)(1+4t^2)} = -\frac{1}{3}\left(\frac{1}{1+t^2}\right) + \frac{4}{3}\left(\frac{1}{1+4t^2}\right)$$

$$I = \int_0^{\infty} \frac{1}{(1+t^2)(1+4t^2)} dt = -\frac{1}{3} \int_0^{\infty} \frac{1}{1+t^2} dt + \frac{4}{3} \int_0^{\infty} \frac{1}{1+(2t)^2} dt$$

$$= -\frac{1}{3} (\tan^{-1} t)_0^{\infty} + \frac{4}{3} \times \frac{1}{2} [\tan^{-1} 2t]_0^{\infty} \quad [1]$$

$$= -\frac{1}{3} \left(\frac{\pi}{2}\right) + \frac{2}{3} \times \left(\frac{\pi}{2}\right)$$

$$= -\frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{6} \quad [1]$$

8. Let x and y be the charges of one Eng. and one Hindi page respectively.

$$10x + 3y = 145$$

$$3x + 10y = 180$$

matrix form, $\begin{bmatrix} 10 & 3 \\ 3 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 145 \\ 180 \end{bmatrix}$ [1]

Let $A \quad X = B$

$$X = A^{-1}.B$$

$$A^{-1} = \frac{\text{adj}A}{|A|}$$

$$|A| = 91 \text{ and } \text{adj}A = \begin{bmatrix} 10 & -3 \\ -3 & 10 \end{bmatrix} \quad [1]$$

$$X = \frac{1}{91} \begin{bmatrix} 10 & -3 \\ -3 & 10 \end{bmatrix} \begin{bmatrix} 145 \\ 180 \end{bmatrix}$$

$$X = \frac{1}{91} \begin{bmatrix} 910 \\ 1365 \end{bmatrix}$$

$$X = \begin{bmatrix} 10 \\ 15 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \end{bmatrix} \quad [1]$$

$$x = \text{Rs. } 10, \quad y = \text{Rs. } 15$$

Typist charged from poor student = $2 \times 5 = \text{Rs. } 10$

actual price is $15 \times 5 = \text{Rs. } 75$

The poor boy was charged = $75 - 10 = \text{Rs. } 65$

always helps poor pepule.

[1]

9. We have $\Delta = \begin{vmatrix} yz - x^2 & zx - y^2 & xy - z^2 \\ zx - y^2 & xy - z^2 & yz - x^2 \\ xy - z^2 & yz - x^2 & zx - y^2 \end{vmatrix}$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$, we get

$$= \begin{vmatrix} xy + yz + zx - x^2 - y^2 - z^2 & zx - y^2 & xy - z^2 \\ xy + yz + zx - x^2 - y^2 - z^2 & xy - z^2 & yz - x^2 \\ xy + yz + zx - x^2 - y^2 - z^2 & yz - x^2 & zx - y^2 \end{vmatrix}$$

Taking $(xy + yz + zx - x^2 - y^2 - z^2)$ from C_1 , we get

$$= (xy + yz + zx - x^2 - y^2 - z^2) \begin{vmatrix} 1 & zx - y^2 & xy - z^2 \\ 1 & xy - z^2 & yz - x^2 \\ 1 & yz - x^2 & zx - y^2 \end{vmatrix} \quad [1]$$

Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_1$

$$= (xy + yz + zx - x^2 - y^2 - z^2) \begin{vmatrix} 1 & zx - y^2 & xy - z^2 \\ 0 & xy - z^2 - zx + y^2 & yz - x^2 - xy + z^2 \\ 0 & yz - x^2 - zx + y^2 & zx - y^2 - xy + z^2 \end{vmatrix} \quad [1]$$

$$= (xy + yz + zx - x^2 - y^2 - z^2) \begin{vmatrix} 1 & zx - y^2 & xy - z^2 \\ 0 & x(y - z) + (y^2 - z^2) & y(z - x) + (z^2 - x^2) \\ 0 & z(y - x) + (y^2 - x^2) & x(z - y) + (z^2 - y^2) \end{vmatrix}$$

$$= (xy + yz + zx - x^2 - y^2 - z^2) \begin{vmatrix} 1 & zx - y^2 & xy - z^2 \\ 0 & (y - z)(x + y + z) & (z - x)(x + y + z) \\ 0 & (y - x)(x + y + z) & (z - y)(x - y + z) \end{vmatrix}$$

Taking $(x + y + z)$ common from R_2 and R_3 , we get

$$= (xy + yz + zx - x^2 - y^2 - z^2)(x + y + z)^2 \begin{vmatrix} 1 & zx - y^2 & xy - z^2 \\ 0 & y - z & z - x \\ 0 & y - x & z - y \end{vmatrix} \quad [1]$$

$$= (xy + yz + zx - x^2 - y^2 - z^2)(x + y + z)^2 [1.(yz - y^2 - z^2 + zy - yz + xy + xz - x^2)]$$

$$= (xy + yz + zx - x^2 - y^2 - z^2)^2 (x + y + z)^2$$

Hence $\begin{vmatrix} yz - x^2 & zx - y^2 & xy - z^2 \\ zx - y^2 & xy - z^2 & yz - x^2 \\ xy - z^2 & yz - x^2 & zx - y^2 \end{vmatrix}$ is divisible by $(x + y + z)$. [½]

and quotient is $(xy + yz + zx - x^2 - y^2 - z^2)(x + y + z)$. [½]

SECTION-D

10. Solving the given equations of curves, we have $x^2 + ax = 2ax$

or $x = 0, x = a$, which give $y = 0, y = \pm a$

From figure

$$\text{area ODABO} = \int_0^a (\sqrt{2ax - x^2} - \sqrt{ax}) dx$$

Let $x = 2a \sin^2 \theta$.

Then $dx = 4a \sin \theta \cos \theta d\theta$ and

$$x = 0 \Rightarrow \theta = 0, x = a \Rightarrow \theta = \frac{\pi}{4}$$

$$\begin{aligned} \text{Let } I_1 &= \int_0^a \sqrt{2ax - x^2} dx \\ &= \int_0^{\pi/4} (2a \sin \theta \cos \theta)(4a \sin \theta \cos \theta) d\theta \\ &= a^2 \int_0^{\pi/4} 2 \sin^2 2\theta d\theta = a^2 \int_0^{\pi/4} 2 \left(\frac{1 - \cos 4\theta}{2} \right) d\theta \\ &= a^2 \int_0^{\pi/4} (1 - \cos 4\theta) d\theta = a^2 \left(\theta - \frac{\sin 4\theta}{4} \right) \Big|_0^{\pi/4} = \frac{\pi}{4} a^2 \end{aligned}$$

Further more,

$$I_2 = \int_0^a \sqrt{ax} dx = \sqrt{a} \frac{2}{3} \left(x^{3/2} \right) \Big|_0^a = \frac{2}{3} a^2$$

$$\text{Thus the required area} = I_1 - I_2 = \frac{\pi}{4} a^2 - \frac{2}{3} a^2 = a^2 \left(\frac{\pi}{4} - \frac{2}{3} \right) \text{ sq. units}$$

Aliter :

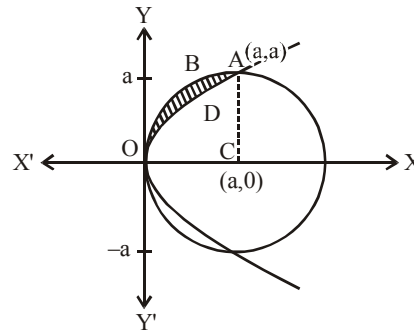
Solving the given equations of curves, we have $x^2 + ax = 2ax$

or $x = 0, x = a$, which give $y = 0, y = \pm a$

From figure

$$\text{area ODAB} = \int_0^a (\sqrt{2ax - x^2} - \sqrt{ax}) dx$$

$$\begin{aligned} \therefore \text{Required area} &= \int_0^a \sqrt{2ax - x^2} dx - \int_0^a \sqrt{ax} dx \\ &= \int_0^a \sqrt{-(x^2 - 2ax + a^2 - a^2)} dx - \sqrt{a} \int_0^a x^{1/2} dx \\ &= \int_0^a \sqrt{-(x-a)^2 + a^2} dx - \sqrt{a} \left[\frac{x^{3/2}}{3/2} \right]_0^a \\ &= \int_0^a \sqrt{a^2 - (x-a)^2} dx - \frac{2\sqrt{a}}{3} [a^{3/2} - 0] \\ &= \left[\frac{1}{2} (x-a) \sqrt{a^2 - (x-a)^2} + \frac{1}{2} a^2 \sin^{-1} \left(\frac{x-a}{a} \right) \right]_0^a - \frac{2a^2}{3} \end{aligned}$$



[2]

[2]

[1]

[1]

[2]

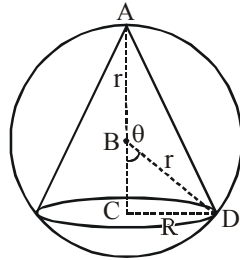
[2]

[1]

$$= \left[0 - \left\{ 0 + \frac{1}{2} a^2 \left(-\frac{\pi}{2} \right) \right\} \right] - \frac{2a^2}{3}$$

$$= \frac{\pi}{4} a^2 - \frac{2a^2}{3} = \left(\frac{\pi}{4} - \frac{2}{3} \right) a^2 \text{ sq. unit} \quad [1]$$

11. Let R and h be the radius and height of the cone respectively.



The volume (V) of the cone is given by

$$V = \frac{1}{3} \pi R^2 h$$

$$r^2 = (h - r)^2 + R^2$$

$$r^2 = h^2 + r^2 - 2hr + R^2$$

$$R^2 = 2hr - h^2 \quad \dots(i) \quad [1]$$

Now Volume of cone $V = \frac{1}{3} \pi R^2 h$

$$V = \frac{\pi h}{3} (2hr - h^2) \quad [\text{Using (i)}]$$

$$V = \frac{\pi}{3} (2h^2 r - h^3)$$

Differentiate both the side w.r.t. h

$$\Rightarrow \frac{dV}{dh} = \frac{\pi}{3} (4hr - 3h^2) = 0$$

$$\Rightarrow h = \frac{4r}{3} \quad [1\frac{1}{2}]$$

and $\frac{d^2V}{dh^2} = \frac{\pi}{3} (4r - 6h)$

$$\frac{d^2V}{dh^2} < 0 \text{ at } h = \frac{4r}{3} \quad [1]$$

V is max at $h = \frac{4r}{3}$

$$\text{max. vol. of cone} = \frac{1}{3} \pi R^2 h$$

$$= \frac{\pi}{3} (2hr - h^2) h \quad [\text{Using (i)}]$$

$$= \frac{\pi}{3} (2h^2 r - h^3)$$

putting the value of $h = \frac{4r}{3}$

$$\Rightarrow \frac{\pi}{3} \left[2r \left(\frac{4r}{3} \right)^2 - \left(\frac{4r}{3} \right)^3 \right] = \frac{4}{3} \pi r^3 \left[\frac{8}{27} \right]$$

$$\Rightarrow \frac{8}{27} \text{ (Vol. of Sphere)} \quad [1\frac{1}{2}]$$