Paper Code : 1001CT100115032

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN) — path to success

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2015 - 2016)

JEE (Main + Advanced) : NURTURE COURSE

ANSWER KEY : PAPER-1 **TEST DATE : 03-04-2016**

Test Type : MINOR Test Pattern : JEE-Advanced

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,C	B,C,D	A,D	C,D	A,C	C,D	A,C,D	A,B,D	A,B,C,D	A,B,C,D
SECTION-II	Q.1	A	B	C	D	Q.2	A	B	C	D	
		P,S,T	R,S,T	R,S,T	R,S		P,Q,R,S,T	P,Q,R,S,T	P,Q,R,S,T	P,S	
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	4	4	7	8	4	4	3	7		

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	A,C	B,C	A,B,C	A,D	B,C,D	A	A,B,D	B,D	A,C,D
SECTION-II	Q.1	A	B	C	D	Q.2	A	B	C	D	
		P,R,S,T	Q,S	P,S	P,R,S,T		S	P,S	P,Q,R	P,S,T	
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	5	1	3	5	6	3	4	5		

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A,C	A,B	A,C,D	A,D	A,C	A,C	A,C	A,B,C	B,C	A,C
SECTION-II	Q.1	A	B	C	D	Q.2	A	B	C	D	
		S	P	T	Q		Q	R or P,Q,R,S,T	P	S	
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	6	4	1	3	0	7	6	7		

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ANSWER KEY : PAPER-2 **TEST DATE : 03-04-2016**

Test Type : MINOR Test Pattern : JEE-Advanced

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B	B	B	A	C	A	B	D	B	B,C,D
	Q.	11	12								
	A.	A,D	A,C								
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	5	2	2	5	3	3	5	6		

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	C	C	C	C	A	D	A	B,D	A,C,D
	Q.	11	12								
	A.	A,C	B								
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	8	6	7	3	4	2	3	5		

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	D	A	B	D	D	A	A	Bonus	A,B,D
	Q.	11	12								
	A.	A,B	A,C,D								
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	5	2	1	8	6	4	3	8		

JEE (Main + Advanced) : NURTURE COURSE

Test Type : MINOR Test Pattern : JEE-Advanced

TEST DATE : 03 - 04 - 2015

PAPER-1

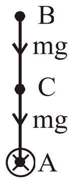
PART-1 : PHYSICS SOLUTION

SECTION-I

1. Ans. (A,C)

Sol. $\frac{1}{2}\left(m\ell^2+\frac{m\ell^2}{4}\right)\omega^2 = mg(2\ell)+mg(\ell)$ and

$\omega = \frac{v}{\ell}$



$\frac{5}{4}m\ell^2\left(\frac{v^2}{\ell^2}\right) = 6mg\ell$

$v = \sqrt{\frac{24g\ell}{5}}$ m/s

Compressive force on AC = 2 mg

Compressive force on BC = mg

ratio $= \frac{2}{1}$

2. Ans. (B,C,D)

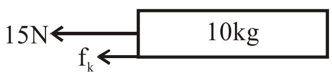
Sol. $f = f_0\frac{C+v}{C-v}$

$\therefore \lambda = \frac{C(C-v)}{f_0(C+v)}$

Beats frequency

$= \Delta f = f_0\frac{C+v}{C-v} - f_0 = f_0\left(\frac{C+v-C+v}{C-v}\right) = \frac{2f_0v}{C-v}$

3. Ans. (A,D)

Sol. 

$f_k = 5$ N

kinetic friction will be in –x direction until its velocity becomes zero so for the time when v = 0

$a = \frac{-20}{10} = -2 m/s^2$

$v = u + at$

$0 = 10 - 2t$

$t = 5$ sec

So $t = 0$ to $t = 5$ sec

$a = -2$ m/s² so $\frac{dv}{dt} = -2$

after $t = 5$

So $a = \frac{-10}{10} = -1$ m/s²

for $t = 0$ to $t = 5$ sec

$s = 10t - \frac{1}{2}\times 2t^2$

$s = 10t - t^2$ at $t = 5$

$s = 25$ m

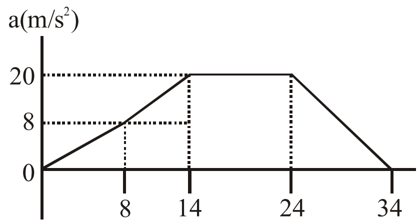
4. Ans. (C,D)

Sol. Since change in velocity is zero, so area under velocity time graph must be zero

$8 + 6 \times 2 = \Delta t \times 2$

$\Rightarrow \Delta t = 10$ sec

$$v = \begin{cases} t \\ 8+2(t-8) \\ 20 \\ 20-2t \end{cases}$$



$$S = \frac{1}{2}\times 8\times 8 + \frac{1}{2}\times 6\times 26 + \frac{1}{2}\times 30\times 20$$

$= 32 + 84 + 300 = 416$ m

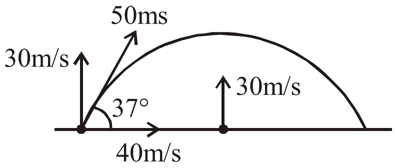
5. Ans. (A,C)

Sol. For isothermal process, PV = constant

For adiabatic process, PV^{γ} = constant

6. Ans. (C,D)

Sol.



Velocity v of both A & B is same in vertical direction.

So A & B will be at same height at any time.

$$t = \frac{120}{40} = 3 \text{ sec.}$$

at t = 3 sec A and B will have same horizontal position also.

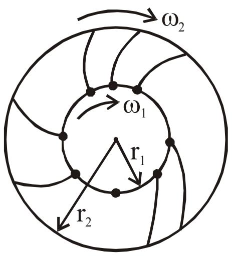
at t = 3 height of both

$$h = 30\times 3 - \frac{1}{2}\times 10\times (3)^2$$

h = 90 – 45 = 45 m.

7. Ans. (A,C,D)

Sol.



As the sand leaves the inner drum through the open holes, it does not exert any force of the drum, angular momentum remains conserved.

At time t,

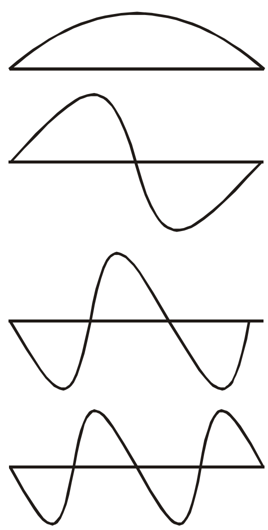
$$m_1 r_1^2 \omega_0 = (m_1 - \lambda t) r_1^2 \omega_0 + (m_2 + \lambda t) r_2^2 \omega_2$$

$$\omega_2 = \frac{\lambda t r_1^2 \omega_0}{(m_2 + \lambda t) r_2^2}$$

The speed of inner drum does not change.

8. Ans. (A,B,D)

Sol. $K = 0.5\pi = \frac{2\pi}{\lambda} \Rightarrow \lambda = 4\text{cm}$



$\Rightarrow L = \frac{\lambda}{2} = 2\text{cm}$

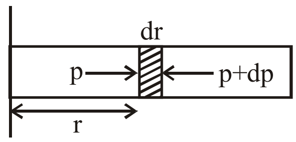
$\Rightarrow L = \lambda = 4$ cm

$\Rightarrow L = \frac{3\lambda}{2} = 6\text{cm}$

$\Rightarrow L = 2\lambda = 8$ cm

9. Ans. (A,B,C,D)

Sol.



$$dP = \frac{(dm)\omega^2 r}{A}$$

A → cross section area (assumed)

$$\int_{P_0}^{P} dP = \int_{r_0}^{r} \frac{\rho (dr)\omega^2 r}{A}$$

$$P - P_0 = \frac{\rho\omega^2}{2}(r^2 - r_0^2)$$

$$P = P_0 + \frac{\rho\omega^2}{2}(r^2 - r_0^2)$$

If an object is there and if it is in equilibrium then horizontal force.

$F = \rho_{ob} v w^2 R_{cm\,ob} = \rho v w^2 R_{cm}$

but if $\rho R_{cm} > \rho_{ob} R_{cmob}$

then in inward direction there will be more force than centripital needed,

So it moves inward when $\rho R_{cm} > \rho_{ob} R_{cm\,ob}$ and vice versa.

10. Ans. (A,B,C,D)

Sol. For option A

$$u_0 = \frac{f}{2} nRT_0$$

$$u_0 = \frac{f}{2} P_A V_0 = \frac{3}{2} P_A V_0$$

for option B

P → constant

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$T_2 = 1.5\ T_0 = \frac{3}{2}T_0$$

for option (C)

$$W = P_A\,(1.5\ V_0 - 0.5\ V_0)$$

$$= \frac{1}{2}P_A V_0$$

for option (D)

$$\Delta Q = \Delta U + W$$

$$\Delta U = \frac{f}{2}nR\Delta T = \frac{f}{2}\left(P_2V_2 - P_1V_1\right)$$

$$\Delta U = \frac{3}{4}P_A V_0$$

$$\Delta Q = \frac{3}{4}P_A V_0 + \frac{1}{2}P_A V_0 = \frac{5}{4}P_A V_0$$

SECTION-II

1. Ans. (A)-(P,S,T); (B)-(R,S,T); (C)-(R,S,T); (D)-(R,S)

2. Ans. (A)-(P,Q,R,S,T); (B)-(P,Q,R,S,T); (C)-(P,Q,R,S,T); (D)-(P,S)

SECTION-IV

1. Ans. 4

Sol. If length of side $= a = 60\left(\sqrt{2}+1\right)$cm

angular momentum conservation,

$$Mav_0 = I\omega$$

$$\omega = \frac{Mav_0}{1} \quad \text{... (1)}$$

$$\frac{1}{2}I\omega^2 \ge mga\left(\sqrt{2}-1\right) \quad \text{.... (2)}$$

$$I = I_{cm} + M\left(a\sqrt{2}\right)^2$$

$$= \frac{8Ma^2}{3} \quad \text{... (3)}$$

Solve these equation,

$$v_0 \ge 4\ \text{m/s}$$

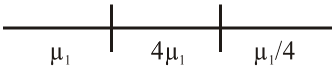
2. Ans. 4

Sol. Frequency of sound wave will be equal to string.

So n = 240 Hz

$$\frac{n}{60} = 4$$

3. Ans. 7

Sol. 

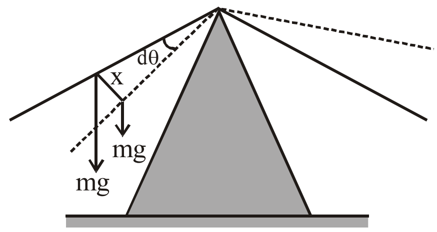
Tension = F

$$v_1 = \sqrt{\frac{F}{\mu_1}};\ v_2 = \frac{1}{2}\sqrt{\frac{F}{\mu_1}};\ v_3 = 2\sqrt{\frac{F}{\mu_1}}$$

So time taken $= \frac{L}{v_1} + \frac{L}{v_2} + \frac{L}{v_3} = \frac{7L}{2}\sqrt{\frac{\mu_1}{F}}$

So $\alpha = 7$

4. Ans. 5

Sol. 

$$x = \frac{L}{2}d\theta$$

so torque = –2 (mg x cos 45°)

$$\text{torque} = -2mg\frac{L}{2}\times\frac{1}{\sqrt{2}}d\theta$$

$$\tau = \frac{-mgLd\theta}{2\sqrt{2}}$$

moment of ineretia $I = \frac{2mL^2}{3}$

$$I\alpha = -\frac{mgLd\theta}{2\sqrt{2}}$$

$$\alpha = -\frac{3gd\theta}{4\sqrt{2}L}$$

$$\text{So } \omega^2 = \frac{3g}{4\sqrt{2}L}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{4\pi}\sqrt{\frac{3g}{\sqrt{2}L}}$$

So $\alpha = 3$; $\beta = 2$

5. Ans. 4

Sol.

$$V = \sqrt{\frac{\gamma\rho}{\rho}} \qquad PV = nRT$$

$$V = \sqrt{\frac{\gamma RT}{M}} \qquad PV = \frac{m}{M}RT$$

$$f_0 = \frac{V}{\lambda} \qquad PM = \rho RT$$

$$\frac{P}{\rho} = \frac{RT}{M}$$

and $\frac{\lambda}{4} = L$

$$\lambda = 4L$$

$$f_0 = \frac{V}{4L}$$

$$4Lf_0 = \sqrt{\frac{\gamma RT}{M}}$$

$$T = \frac{16ML^2f_0^2}{\gamma R}$$

$\beta = 16$

6. Ans. 4

Sol. $V_{30} = V_0 (1 + \gamma \times 30)$

$$\frac{V_{30} - V_0}{V_0} = \gamma \times 30$$

$\gamma = 11.1 \times 10^{-4}$

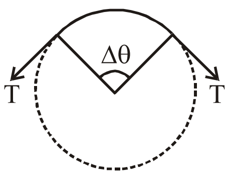
7. Ans. 3

Sol. $2T\sin\frac{\Delta\theta}{2} = dm\omega^2 r$

$$\Rightarrow 2T\frac{\Delta\theta}{2}\rho Ar\Delta\theta\omega^2 r$$

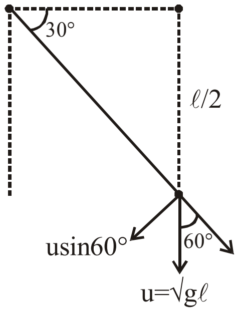
$$\Rightarrow \sigma = \frac{T}{A} = \rho r^2\omega^2$$

$$\Rightarrow \omega = \frac{1}{r}\sqrt{\frac{\sigma}{\rho}} = 3$$



8. Ans. 7

Sol. $\frac{1}{2}mu^2\sin^2 60° = \frac{1}{2}mv^2 + \frac{mg\ell}{2}$



$$\Rightarrow v = \sqrt{\frac{7g\ell}{2}}$$

$$\frac{1}{2}kx^2 = \frac{1}{2}\frac{m \times m}{2m}\frac{7g\ell}{4}$$

$$\Rightarrow x = \sqrt{\frac{70}{8}\left(\frac{m\ell}{K}\right)}$$, m = 0.1 kg and k = 10/8

$\Rightarrow \ell = 7$

PART–2 : CHEMISTRY SOLUTION

SECTION–I

1. Ans. (A)

Sol. (A) Increasing order of wavelength is

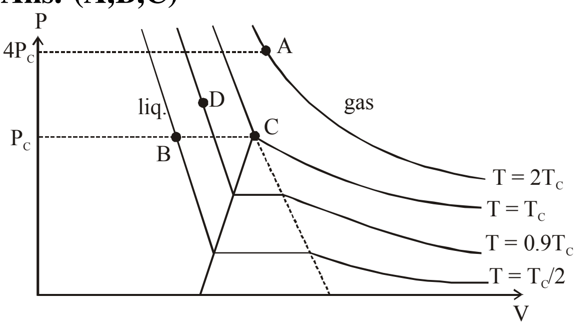
Micro waves > Radio waves > IR waves > visible waves > UV waves

(C) $E_1 > E_2 > E_3 > E_4$

2. Ans. (A,C)

3. Ans. (B,C)

4. Ans. (A,B,C)



5. Ans. (A,D)

6. Ans.(B,C,D)

7. Ans. (A)

8. Ans. (A,B,D)

9. Ans. (B,D)

10. Ans. (A,C,D)

SECTION–II

1. Ans (A)-(P,R,S,T); (B)-(Q,S); (C)-(P,S); (D)-(P,R,S,T)

2. Ans. (A)-(S); (B-(P,S); (C)-(P,Q,R); (D)-(P,S,T)

SECTION–IV

1. Ans. 5

$$Hg\overset{-2}{S} + H\overset{+5}{N}O_3 + 3HCl \rightarrow HgCl_2 + \overset{0}{S} + O\overset{+3}{N}Cl + 2H_2O$$

$nf_{RA} = 2 \quad nf_{AO} = 2$

a + b + c = 1 + 1 + 3 = 5

2. Ans. 1

$$\overset{+3}{Fe_2}(C_2O_4) + K_2\overset{+6}{Cr_2}O_7 \xrightarrow{H^+} Fe^{3+} + CO_2 + Cr^{3+}$$

nf = 6 nf = 6 1 × 6 a × 6

3. Ans. 3

4. Ans. 5

$$[Cu(H_2O)_6]^{2+} + 2NH_3 \longrightarrow [Cu(H_2O)_4(OH)_2] + 2NH_4^+ +$$

171.5gm

34 gm

5. Ans. 6 ; 6. Ans. 3 ; 7. Ans. 4; 8. Ans. 5

PART-3 : MATHEMATICS SOLUTION

SECTION-I

1. Ans. (A,C)

(A) $f(x) = (x^2 - 5x + 9)(x^2 - 5x + 3)$

Let $x^2 - 5x + 9 = t$

$f(t) = t(t - 6) = t^2 - 6t + 9 - 9$

$= (t - 3)^2 - 9 = (x^2 - 5x + 6)^2 - 9$

$f_{min} = -9$

(B) $f(x) = \text{sgn}\left(\cos^{-1}\left(\frac{1}{e^x + e^{-x}}\right)\right) = 1$

(C) $f(x) = \sqrt{1 - \sqrt{2 - \sqrt{3 - x}}}$

$1 - \sqrt{2 - \sqrt{3 - x}} \ge 0$

$1 \ge 2 - \sqrt{3 - x}$

$\sqrt{3 - x} \ge 1$

$3 - x \ge 1$

$x \le 2$ ......(1)

$2 - \sqrt{3 - x} \ge 0$

$2 \ge \sqrt{3 - x}$

$4 \ge 3 - x$

$x \ge -1$ ......(2)

$x \in [-1, 2]$

(D) $f(x) = e^{-x} - e^{|x|} = \begin{cases} e^{-x} - e^{x} & , x \ge 0 \\ 0 & , x < 0 \end{cases}$

Many one.

2. Ans. (A,B)

Coeff. of x in (1+x)(1+2x)(1+3x) .... (1+100x) is (1 + 2 + 3 + ... + 100) = 5050

3. Ans. (A,C,D)

$\sqrt{x^2 - 5|x| + 6} = \sqrt{(|x| - 2)(|x| - 3)}$

$\Rightarrow$ $0 \le |x| \le 2, |x| \ge 3$

and $\sqrt{8 + 2|x| - |x|^2} = \sqrt{-(|x| - 4)(|x| + 2)}$

$\Rightarrow$ $0 \le |x| \le 4$

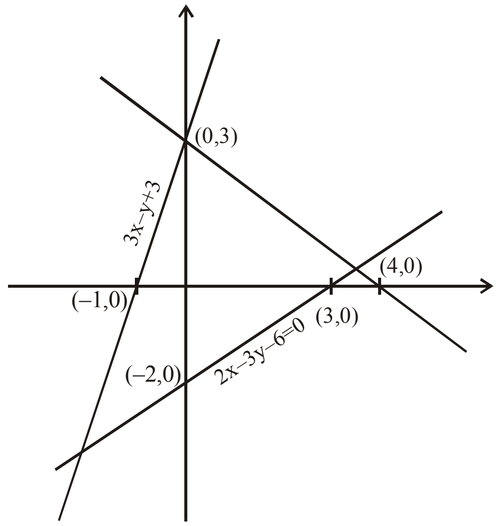
$\Rightarrow$ $|x| \in [0, 2] \cup [3, 4]$

4. Ans. (A,D)

$P = (0, \beta)$

$Q = (\alpha, 0)$

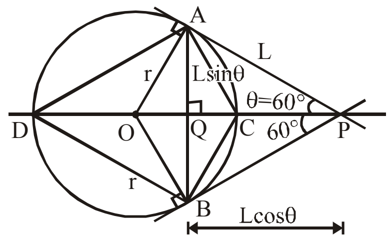
Plot the triangle



Clearly $\alpha \in [-1, 3]$

$\beta \in [-2, 3]$

5. Ans. (A,C)



|Area of ΔABC – Area of ΔABD|

$= \left|\frac{1}{2}AB.CQ - \frac{1}{2}AB.DQ\right| = \frac{1}{2}AB\,|CQ - DQ|$

$= \frac{1}{2}AB\,|(OC - OQ) - (OD + OQ)|$

$= \frac{1}{2}AB\,|(-OD + OC) - 2OQ|$

$= \frac{1}{2}AB\,|OD - OC + 2OQ|$

$= \frac{1}{2}AB\,|2OQ| = AB.OQ$

$= (2L\sin\theta).(\sqrt{r^2 - L^2\sin^2\theta}) = \frac{9\sqrt{3}}{2}$

Circum radius of $\Delta PAB = \frac{OP}{2} = \frac{L\sin\theta}{2} = \sqrt{3}$

6. Ans. (A,C)

For A option :

if $|a - b| \ge 7$, then

a	b		
0	7, 8, 9	→	3 ways
1	8, 9	→	2 ways
2	9	→	1 ways

Number of ways = 2(1 + 2 + 3) = 12 ways.
for C option :
Sum of the digits is 45
So every 10 digit number is always divisible by 3.

7. Ans. (A,C)

a = cos4
b = sin6
$4 \in$ IIIrd quadrant
$6 \in$ IVth quadrant
$\Rightarrow \quad a < 0, b < 0 \quad \Rightarrow \quad ab > 0$

Also $2a^2 - 1 = \cos 8 < 0$ as $8 \in \left(\frac{5\pi}{2}, 3\pi\right)$

$\Rightarrow \quad a^2 < \frac{1}{2}$

8. Ans. (A,B,C)

$f(x) = \sin^6 x + \cos^6 x = 1 - 3\sin^2 x \cos^2 x$

$= 1 - \frac{3}{4}\sin^2 2x$

$f(x) \in 1 - \frac{3}{4}[0,1] = \left[\frac{1}{4}, 1\right]$

$f'(x) = -\frac{3}{4}.2\sin 2x \cos 2x.2 = -\frac{3}{4}\sin 4x$

If $\quad x \in \left[0, \frac{\pi}{4}\right] \quad \Rightarrow \quad f'(x) < 0$

$x \in \left[\frac{3\pi}{4}, \pi\right] \quad \Rightarrow \quad f'(x) > 0$

9. Ans. (B,C)

If A + B + C = π

$\sum \sin 2A = 4\sin A \sin B \sin C$

and (B) corresponds to this identity.

$\sum \sin^2 A = 2 + 2\cos A \cos B \cos C$

and (C) corresponds to this identity.

10. Ans. (A,C)

Let $y = \cos^{-1} x$

$t_{n+1}(x) = \cos((n+1)y) = \cos(y + ny)$

$= \cos y \cos y - \sin y \sin xy$

$= 2\cos y \cos ny - (\cos ny \cos y + \sin ny \sin y)$

$= 2xt_n(x) - \cos(n-1)y$

$= 2xt_n(x) - t_{n-1}(x)$

SECTION – II

1. Ans. (A)→(S); (B)→(P); (C)→(T); (D)→(Q)

(A) $a\sin x + 1 - 2\sin^2 x = 2a - 7$

$\sin x = 2, \frac{a-4}{2}$

$\because \quad \sin x \neq 2$

$\Rightarrow \sin x = \frac{a-4}{2}$

$\Rightarrow a \in [2, 6]$

(B) Value of determinant

$= 2\cos^2 \frac{\theta}{2} + 2 \in [2, 4]$

(C) ${}^{2n+3}C_{2n} - {}^{2n+2}C_{2n-1} = 15(2n+1)$

$\frac{(2n+2)(2n+1)}{2} = 15(2n+1)$

n = 14

(D) $t_n = \frac{1}{1 + 3 + ... + (2n-1) - \frac{1}{4}}$

$= \frac{4}{4n^2 - 1} = \frac{4}{(2n-1)(2n+1)}$

$= 2\left(\frac{1}{2n-1} - \frac{1}{2n+1}\right)$

$S_n = \sum t_n = \frac{4n}{2n+1}$

2. Ans. (A)→(Q); (B)→(R or P,Q,R,S,T); (C)→(P); (D)→(S)

(A) Last digit in powers of 3 are 3,9,7,1,3,9 etc.
$\Rightarrow$ Last digit of $(843)^{843}$ is 7.
Last digit of powers of 2 are 2,4,8,6,2,4 etc.
$\Rightarrow$ Last digit of $(492)^{295}$ is 8
$\Rightarrow$ Last digit of given number is 5.

(B) Let $f(x) = y$

$(y^2 + 2)\left(y - \frac{\cos x}{1 + \sin x}\right) = 0$

$\Rightarrow \quad y = \frac{\cos x}{1 + \sin x}$

(C) $\cos(\pi \sin\theta) = \sin(\pi\cos\theta) = \cos\left(\frac{\pi}{2} - \pi\cos\theta\right)$

$\Rightarrow \quad \frac{\pi}{2} - \pi\cos\theta = 2n\pi \pm \pi\sin\theta$

$\Rightarrow \quad \cos\theta \pm \sin\theta = 2n + \frac{1}{2}, n \in I$

$\cos\left(\frac{\pi}{4} \pm \theta\right) = n\sqrt{2} + \frac{1}{2}\sqrt{2}$

$\Rightarrow \quad \left|\cos\left(\frac{\pi}{4} \pm \theta\right)\right| \le 1$

$\Rightarrow \quad n = -1$ or 0

But $-\sqrt{2} + \frac{1}{2\sqrt{2}} \le -1$ if $n = -1$

$\Rightarrow \quad n = 0$

(D) $(a^2 - b^2)^{2(x + y - 1)} = (a + b)^{x+4} (a - b)^{2y+2}$

$\Rightarrow \quad 2x + 2y - 2 = x + 4$

and $2x + 2y - 2 = 2y + 2$

$\Rightarrow \quad x = y = 2$

SECTION-IV

1. Ans. 6

$f(9) = 9^{2016} - 9.9^{2015} - 2.9^{2014} + 18.9^{2013} + 9^{2011}$

$\quad -10.9^{2010} + 9.9^{2009} + 9 - 3$

$= (9^{2016} - 9^{2016}) - 2(9^{2014} - 9^{2014})$

$\quad +(9^{2009}(9^2 - 10 \times 9 + 9) + 9 - 3$

$= 9^{2009}(9^2 - 9^2) + 6 = 6$

2. Ans. 4

$f(xy) = f(x).f(y) - f(x+y) + 1$

Put y = 1

$f(x) = 2f(x) - f(x + 1) + 1$

$f(x + 1) - f(x) = 1$

$\Rightarrow \quad f(2) - f(1) = 1$

$f(3) - f(2) = 1$

$\vdots$

$f(x+1) - f(x) = 1$

---

add $\Rightarrow \quad f(x + 1) = x + 2$

$f(x) = x + 1$

$1 + \sum_{k=1}^{\infty} \frac{f(k)}{2^k} = 1 + \sum_{k=1}^{\infty} \frac{k+1}{2^k}$

$= 1 + \sum_{k=1}^{\infty} \frac{k+1}{2k} = 1 + \left(\frac{2}{2} + \frac{3}{2^2} + ...\infty\right)$

$= \frac{1}{1 - \frac{1}{2}} + \frac{1/2}{\left(1 - \frac{1}{2}\right)^2} = 2 + 2 = 4$

3. Ans. 1

$2\tan^{-1}\left(\text{cosec}(\tan^{-1} x) - \cot(\tan^{-1} x)\right)$

$= 2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7}$

$\text{LHS} = 2\tan^{-1}\left(\text{cosec}\left(\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) - \cot\left(\cot^{-1}\frac{1}{x}\right)\right)\right)$

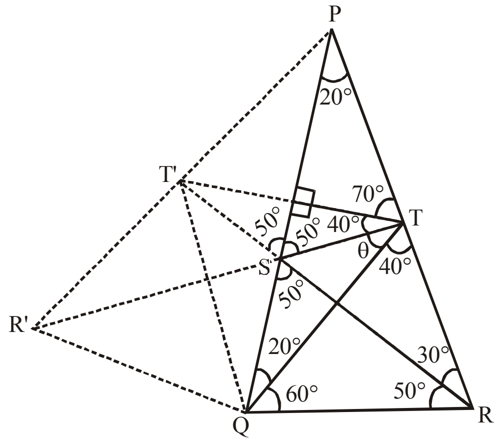
$= 2\tan^{-1}\left(\frac{\sqrt{1+x^2}}{x} - \frac{1}{x}\right)$

$= \tan^{-1}\left(\frac{2\left(\sqrt{1+x^2} - 1\right)x}{x^2 - \left(\sqrt{1+x^2} - 1\right)^2}\right) = \tan^{-1} x$

$\text{RHS} = \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\frac{1}{7} = \tan^{-1} 1$

$\Rightarrow x = 1$

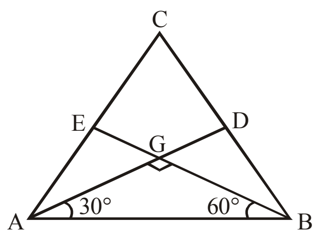
4. Ans. 3



Reflect entire Δ about PQ & create kite PTQT'.

$\Rightarrow \quad \theta = 30°$

Now



$AG = \frac{8}{3}$

InΔ AGB

$$\tan 30^\circ = \frac{BG}{AG}$$

$$\frac{1}{\sqrt{3}} = \frac{BG}{AG} \Rightarrow BG = \frac{AG}{\sqrt{3}}$$

Area of $\Delta ABC = 3.(\Delta AGB) = 3.\frac{1}{2}AG.BG$

$$= \frac{2}{2}.\frac{8}{3}.\frac{8}{3}.\frac{1}{\sqrt{3}} = \frac{32}{3\sqrt{3}}$$

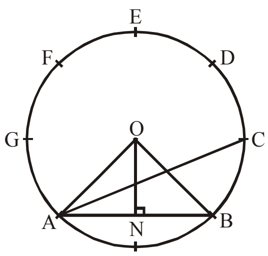
5. Ans. 0

$$\beta = \left(\frac{n+2}{2}\right) - \left\{\left(\frac{1}{n} - \frac{1}{n+1}\right) + 2\left(\frac{1}{n-1} - \frac{1}{n}\right) + 3\left(\frac{1}{n-2} - \frac{1}{n-1}\right) + ... + (n-1)\left(\frac{1}{2} - \frac{1}{3}\right)\right\}$$

$$= \frac{n+2}{2} + \left(\frac{1}{n+1} + \frac{1}{n} + \frac{1}{n-1} + ... + \frac{1}{3} + \frac{1}{2} - \frac{n}{2}\right)$$

$$= \frac{n+2}{2} + \left(\frac{1}{n+1} + \frac{1}{n} + ... + \frac{1}{3} + \frac{1}{2} + 1 - \frac{n+2}{2}\right) = \alpha$$

6. Ans. 7



$$\angle AOB = \frac{2\pi}{7}$$

$$\angle AON = \frac{\pi}{7}$$

$$AB = 2AN = 2\sin\frac{\pi}{7}$$

$$AC = 2\sin\frac{2\pi}{7}$$

$$AD = 2\sin\frac{3\pi}{7}$$

$$AC + AD - AB = 2\left(\sin\frac{3\pi}{7} + \sin\frac{2\pi}{7} - \sin\frac{\pi}{7}\right)$$

$$= 2\left(\sin\frac{2\pi}{7} + \sin\frac{4\pi}{7} + \sin\frac{8\pi}{7}\right) = 2E$$

Let $\frac{\pi}{7} = \theta$

$$E = \sin 2\theta + \sin 4\theta + \sin 8\theta$$

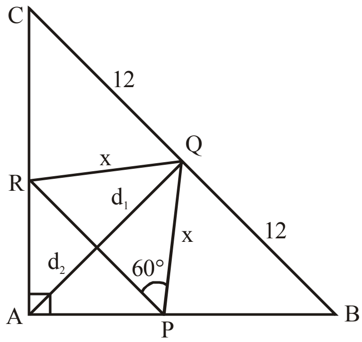
$$E^2 = \sin^2 2\theta + \sin^2 4\theta + \sin^2 8\theta + 2(\sin 2\theta \sin 4\theta + \sin 2\theta \sin 8\theta + \sin 4\theta \sin 8\theta)$$

$$= \frac{1}{2}[(3 - (\cos 4\theta + \cos 8\theta + \cos 16\theta)) + (\cos 2\theta - \cos 6\theta + \cos 6\theta - \cos 10\theta + \cos 4\theta - \cos 12\theta)]$$

$$= \frac{1}{2}\left(3 + \frac{1}{2\sin 4\theta}.\sin 4\theta\right) = \frac{7}{4}$$

$$E = \frac{\sqrt{7}}{2}$$

7. Ans. 6



$$d_1 + d_2 = 12$$

$$\frac{\sqrt{3}}{2}x + \frac{x}{2} = 12$$

$$x = \frac{24}{\sqrt{3}+1}\left(\frac{\sqrt{3}-1}{\sqrt{3}-1}\right) = 12(\sqrt{3}-1)$$

$\Rightarrow$ k = 6

8. Ans. 7

Required number of ways

= Total when all A's seperated

– Total when A's separated and H's are together

$$= \frac{7!}{2!}({}^8C_4) - 6!({}^7C_4)$$

$$= \frac{7!.6!}{4!.3!}(6) = 4^1.5^2.6^3.7^1$$

JEE (Main + Advanced) : NURTURE COURSE

Test Type : MINOR Test Pattern : JEE-Advanced

TEST DATE : 03 - 04 - 2016

PAPER-2

PART-1 : PHYSICS SOLUTION

SECTION-I

1. Ans. (B)

Sol. $2T\sin\theta = mg$

$T = \frac{mg}{2}\text{cosec}\theta$

If $\theta \to 0, T \to \infty$

and $\theta \to \frac{\pi}{2}, T \to \frac{mg}{2}$

2. Ans. (B)

Sol. For bead to be stationary

slope of tangent on the curve $\frac{dy}{dx} = \tan\theta \le \mu$

$\frac{2a}{y} \le \mu$

3. Ans. (B)

Sol. Superposing them gives

$2A\sin\left(kx + \frac{\pi}{6}\right)\cos\left(\omega t + \frac{\pi}{6}\right)$

Nodes are given by $kx + \frac{\pi}{6} = n\pi$

4. Ans. (A)

Sol. Fundamental frequency $f_0 = \frac{v}{2L}$

Beat frequency $= f_1 - f_2$

$= \frac{v}{2\left(\frac{L}{2} - \Delta L\right)} - \frac{v}{2\left(\frac{L}{2} + \Delta L\right)}$

$= \frac{v}{2}\left[\frac{2}{(L - 2\Delta L)} - \frac{2}{(L + 2\Delta L)}\right]$

$= 2f_0 L\left[\frac{4\Delta L}{L^2 - 4\Delta L^2}\right] \approx \frac{8f_0\Delta L}{\ell}$

5. Ans. (C)

Sol. Let the thickness of the wall is t

Weight of the sphere $= \frac{4}{3}\pi\left[R^3 - (R - t)^3\right]\rho$

Buoyancy of the sphere $= \frac{4}{3}\pi R^3\rho$

For flotation $\frac{4}{3}\pi R^3\rho > \frac{4\pi\rho}{3}\left[R^3 - (R - t)^3\right]$

$t \le \frac{R}{3\rho}$

6. Ans. (A)

Sol. $\frac{1}{2}KA^2 + \frac{1}{2}2m\frac{V_0^2}{4} = \frac{1}{2}KA'^2$ (by energy and momentum considerations)

$KA^2 \frac{1}{2}mV_0^2 = KA'^2 \Rightarrow A' = \sqrt{2}A$

$A' = \sqrt{2}A \Rightarrow \varphi = \frac{5\pi}{4}$

$x = A'\sin(\omega' t + \varphi)$

$\omega' = \sqrt{\frac{K}{2m}}$

$x = -\sqrt{2}A\sin\left(\sqrt{\frac{K}{2m}}t + \frac{\pi}{4}\right)$

7. Ans. (B)

Sol. $P_0 A.\ell = PA(\ell - h)$

$\Rightarrow P_0\ell = \left(P_0 + \frac{2T}{r}\right)(\ell - h)$

$\Rightarrow h = \frac{\ell}{\left(1 + \frac{P_0 r}{2T}\right)}$

8. **Ans. (D)**

Sol. Resultant wave amplitude is given by

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_1 - \varphi_2)}$$

Value of cos $(\phi_1 - \phi_2)$ lies, between 1 and –1

Hence $A_1 - A_2 \le A \le A_1 + A_2$

9. **Ans. (B)**

10. **Ans. (B,C,D)**

11. **Ans. (A,D)**

Sol. Case-1: $mv_0 = (m + 3m)v$

and $\frac{1}{2}(4m)v^2 = \frac{1}{2}k(\Delta\ell)^2$

and $T_{max} = k(\Delta\ell)$

Case-2 : $mv_0 = mv_1 + 3mv_2$ and $v_0 = v_2 - v_1$

and $\frac{1}{2}3mv_2^2 = \frac{1}{2}k(\Delta\ell)^2 + \frac{1}{2}3mv_f^2$

After solving $v_2 = \frac{v_0}{2}$; $v_1 = \frac{-v_0}{2}$ and $v_f = \frac{v_0}{\sqrt{6}}$

12. **Ans. (A,C)**

Sol. $$\text{Ratio} = \frac{\frac{3}{2}mv_f^2 + \frac{1}{2}m\left(\frac{v_0^2}{4}\right)}{\frac{1}{2}mv_0^2} = \frac{3}{4}$$

Energy stored in spring

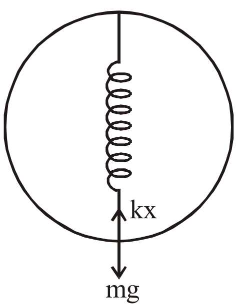
$$= \frac{1}{2}(4m)\left(\frac{v_0}{4}\right)^2 - \frac{1}{2}(4m)\left(\frac{v_0}{8}\right)^2$$

SECTION-IV

1. **Ans. 5**

Sol. $$kr - mg = \frac{mv^2}{r}$$

$\Rightarrow kr^2 - mgr = mv^2$ ... (i)



From conservation of energy

$$3mg\frac{r}{2} = \frac{1}{2}kr^2 + \frac{1}{2}mv^2$$

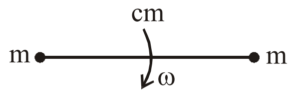
$\Rightarrow mv^2 = 3mgr - kr^2$ ... (ii)

From equation (i) and (ii)

$$k = \frac{2mg}{r} = \frac{100}{\frac{1}{5}} = 500\,N/m$$

2. **Ans. 2**

Sol. $$2mv_0\frac{\ell}{2} = \frac{m\ell^2}{2}\omega$$

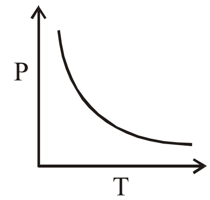


$$\Rightarrow \omega = \frac{2v_0}{\ell}$$

$$T = \frac{m\omega^2\ell}{2} = \frac{2mv_0^2}{\ell}$$

3. **Ans. 2**

Sol. $$\frac{P^2}{\rho} = 2\frac{P_f^2}{\rho}$$



$$P_f = \frac{P}{\sqrt{2}}$$

$$\frac{P^2}{\rho} = C$$

$$P = \frac{\rho RT}{M}$$

$$\frac{\rho^2R^2T^2}{M^2\rho} = C$$

$\rho T^2 = C$

$$\rho T^2 = \frac{\rho}{2}T_f^2$$

$$T_f = \sqrt{2}T$$

$$\frac{P^2RT}{PM} = C$$

$PT = C$

4. **Ans. 5**

Sol. Required velocity of P for completing the cycle at the instant, when OP is along the direction of motion of platform is

$v_{min} = \sqrt{4 \times 10 \times 0.9} = 6$ m/s

So $T - ma = \frac{mv_{min}^2}{\ell}$

$$T = 10 + \frac{(1)36}{0.9} = 10 + 40 = 50N$$

5. Ans. 3

Sol. Let the sliding acceleration be equal to a, then

rolling acceleration $= \dfrac{a}{\left(1+\dfrac{k^2}{R^2}\right)}$

where k is radius of gravitation.
Let ℓ be length of the incline, then from
$v^2 - u^2 = 2as$

$\therefore \left(\dfrac{5}{4}v_0\right)^2 = 2a\ell$

and $v_0^2 = \dfrac{2a\ell}{1+\dfrac{k^2}{R^2}}$

$\therefore 1+\dfrac{k^2}{R^2} = \dfrac{25}{16}$

$\Rightarrow k = \dfrac{3R}{4}$

6. **Ans. 3**

Sol. Work done $= \dfrac{3}{2}P_0V_0$

$\Delta U = \dfrac{P_2V_2 - P_1V_1}{\gamma - 1}$

$\Delta Q = \dfrac{3}{2}P_0V_0$

$\therefore k = 3$

7. **Ans. 5**

Sol. $F_{net} = F_{pseudo} - F_{Buoyant}$

$= V \times 2\rho \times 10 - v\rho \times 10 = v\rho \times 10$

$a = \dfrac{F_{net}}{n} = \dfrac{v\rho \times 10}{v \times 2\rho} = 5$ m/s

8. Ans. 6

PART–2 : CHEMISTRY SOLUTION

SECTION-I

1. **Ans. (C)**

$10V$ of $H_2O_2 \Rightarrow \dfrac{10}{11.35}M$

$15V$ of $H_2O_2 \Rightarrow = \dfrac{15}{11.35}M$

20 V of $H_2O_2 \Rightarrow = \dfrac{20}{11.35}M$

$M_fV_f = M_1V_1 + M_2V_2 + M_3V_3$

$M_f \times 3000 = \dfrac{10}{11.2}\times 500 + \dfrac{15}{11.2}\times 500 + \dfrac{20}{11.2}\times 500$

V.S = 7.5

2. **Ans. (C)**
3. **Ans. (C)**
4. **Ans. (C)**
5. **Ans. (C)**
6. **Ans. (A)**
7. **Ans. (D)**
8. **Ans. (A)**
9. **Ans. (B, D)**
10. **Ans. (A,C,D)**
11. **Ans. (A,C)**
12. **Ans. (B)**

SECTION-IV

1. **Ans. 8**
2. **Ans. 6**

$\dfrac{dp}{p} = -Kdt$

$\ln p = -Kt$

$\ln \dfrac{p_1}{p_2} = K \times t$

$0.7 = 7 \times t$

$0.1 = t$

$t = 6$

3. **Ans. 25 [OMR Ans. 7]**

$3OH^- + 3H_2O + P_4 \rightarrow PH_3 + 3H_2PO_2^-$

4. **Ans. 3**
5. **Ans. 13 [OMR Ans. 4]**
6. **Ans. 2**
7. **Ans. 3**
8. **Ans. 5**

PART-3 : MATHEMATICS — SOLUTION

SECTION-I

1. Ans. (C)
A = 2, B = 2

2. Ans. (D)
$A = \frac{p+q}{2} = 5$

3. Ans. (A)
$\text{Total ways} = 3! \times \left[{}^4C_3 \times 3 + {}^4C_2 \times 2\right]$
$= 6 \times [12 + 12] = 144$

4. Ans. (B)
$\because \tan x + \tan y + \tan z = \tan x . \tan y \tan z,$
$\tan x . \tan y = 3$
$\tan y . \tan z = 9$
$\therefore \tan x = \sqrt{\frac{3}{5}}, \tan y = \sqrt{15}, \tan z = 3\sqrt{\frac{3}{5}}$
$\tan x = -\sqrt{\frac{3}{5}}, \tan y = -\sqrt{15}, \tan z = -3\sqrt{\frac{3}{5}}$
$\therefore$ x, y, z $\in$ Ist or IIIrd Quad.
x, y, z $\in$ IInd or IVth Quad.

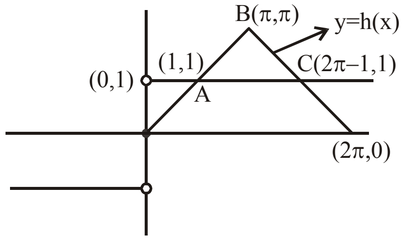
5. Ans. (D)
both points lie on $y = f(x)$
$\therefore 2A - B^2 = A^2 + B \Rightarrow A^2 + B^2 - 2A + B = 0$
$\& (A + B)^2 - 1 = 2AB + 3A + B = 0$
$\Rightarrow A^2 + B^2 - 3A - B - 1 = 0$
on solve we get there are two pairs
(A_1, B_1) and (A_2, B_2)
where $A_1 + A_2 = \frac{8}{5}$
$B_1 + B_2 = -\frac{9}{5}$

6. Ans. (D)
Required ways
$= ({}^8C_1)^2 - {}^8C_1 + ({}^8C_2)^2 - {}^8C_2 + ... + ({}^8C_7)^2 - {}^8C_7$
$= \sum_{r=1}^{7} \left({}^8C_r\right)^2 - \sum_{r=1}^{7} {}^8C_r = 12614$

7. Ans. (A)
$\text{Area} = 5 \sin\frac{\pi}{5}$
$\therefore 10\left(\frac{1}{2}.R^2 \sin\frac{\pi}{5}\right) = 5\sin\frac{\pi}{5}$
$\therefore R = 1$
R is circumradius of polygon.
Use $A_1A_r = \sqrt{2 - 2\cos\left(\frac{(r-1)\pi}{5}\right)}$
$= 2\sin\left(\frac{(r-1)\pi}{10}\right)$
$\therefore (A_1A_2)(A_1A_3) ...... (A_1A_{10})$
$= 2^9 \sin\frac{\pi}{10}.\sin\frac{2\pi}{10}.......\sin\frac{9\pi}{10} = 10$

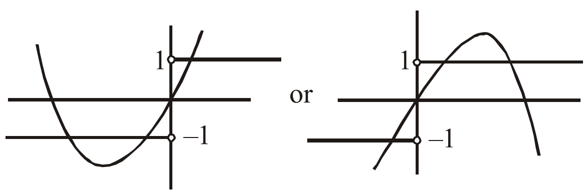
8. Ans. (A)
$\because f(0) = 5 = g(0) \Rightarrow h(0) = 5 \Rightarrow c = 5$
$f(3) = 8 = g(3) \Rightarrow h(3) = 8 \Rightarrow 3a + b = 1$
$\because f(x) \le h(x) \le g(x)$
$\Rightarrow x - 2 \le ax + b \le -x + 4$
$\therefore$ we get (1, –2), (0, 1), (–1, 4) ordered pairs

9. Ans. (Bonus)



Range of h(x) is $(0, \pi]$
$\therefore$ h(x) is into function
Area bounded by $f(x)$ and h(x) = area of ΔABC
$= (\pi - 1)^2$

10. Ans. (A,B,D)
$f(x) = g(x)$ has 4 solutions possible.



By using concept of location of roots, we get integral values of a is N.
y = g(x) = sgn(x) is odd function.

Paragraph for Question 11 to 12

C : $x^2 + y^2 - 8x - 8y + 16 = 0$ ......(i)

Pair of straight lines

$y^2 - 8y + 3 = 0$ ......(ii)

$\therefore \quad y = \frac{8 \pm 2\sqrt{13}}{2} = 4 \pm \sqrt{13}$

On solve (i) & (ii) we get intersection points.

$P(4+\sqrt{3},\ 4+\sqrt{13}),\ Q(4-\sqrt{3},\ 4+\sqrt{13})$

$R(4+\sqrt{3},\ 4-\sqrt{13}),\ S(4-\sqrt{3},\ 4-\sqrt{13})$

PS and QR are diameters of circle

$\therefore$ tangents at P, S are parallel & at Q, R are parallel.

11. Ans. (A,B)

$\therefore \quad \text{Area } A = \frac{2^9}{\sqrt{39}}$

12. Ans. (A,C,D)

By using distance formula

$PT = \sqrt{18 + 2\sqrt{13} + 2\sqrt{3}}$

$QT = \sqrt{18 + 2\sqrt{13} - 2\sqrt{3}}$

$RT = \sqrt{18 - 2\sqrt{13} + 2\sqrt{3}}$

$ST = \sqrt{18 - 2\sqrt{13} - 2\sqrt{3}}$

SECTION-IV

1. Ans. 5

Required value $\frac{\pi}{4} + \frac{\pi}{6} + \frac{\pi}{3} = \frac{3\pi}{4}$

2. Ans. 2

$a = \frac{7\pi}{2},\ b = 3\pi,\ \cos A = \frac{1}{5}$

$\therefore \quad \frac{1}{5} = \frac{9\pi^2 + c^2 - \frac{49\pi^2}{4}}{2.3\pi.c}$

$\therefore \quad c = \frac{5\pi}{2}$

$\sec C = \frac{1}{\cos C} = \frac{2ab}{a^2 + b^2 - c^2} = \frac{21.4}{60} = \frac{7}{5}$

3. Ans. 1

If two circles are orthogonal, then the polar of a point P on first circle w.r.t. second circle passes through the point S which is the other end of the diameter through P.

$\therefore$ PS is diameter of circle A

4. Ans. 8

Let $(1 + a)^{1/6} = t$

$(t^2 - 1)x^2 + (1 - t^3)x + t^2 = t$

$\Rightarrow \quad x^2(t + 1) + [-t^2 - t - 1]x + t = 0$

we get $\quad \alpha(a) = t = (1 + a)^{1/6}$

$\beta(\alpha) = \frac{1}{t+1} = \frac{1}{1+(1+a)^{1/6}}$

$\therefore\ f(\alpha) = (1+a)^{1/6} + \frac{12}{1+(1+a)^{1/6}} + \frac{4[1+(1+a)^{1/6}]^2}{9(1+a)^{1/6}}$

by using A.M. $\geq$ G.M.

we get it's minimum 8

5. Ans. 6

$18b^2 - 4a = 12ab + 3b$ .....(i)

$12ab - 3b = 8a^2$ .....(ii)

On add & subtract given equation

we get $a = \frac{1}{2},\ b = \frac{2}{3}$

$\therefore \quad \text{area} = \frac{1}{2}ab\sin C = \frac{1}{6}\sin C$

Maximum area $= \frac{1}{6}$

6. Ans. 4

Let $\sin A = \tan x$

$\sin B = \tan y$

$\sin C = \tan z$

$\therefore \quad \tan x + \tan y + \tan z = \tan x.\tan y.\tan z$

$\therefore \quad x + y + z = n\pi,\ n \in I.$

$\Rightarrow \quad \tan 2x + \tan 2y + \tan 2z = \tan 2x \tan 2y \tan 2z$

$\frac{\sin A}{\cos^2 A} + \frac{\sin B}{\cos^2 B} + \frac{\sin C}{\cos^2 C} = 4\frac{\sin A.\sin B.\sin C}{\cos^2 A.\cos^2 B.\cos^2 C}$

7. Ans. 3

$2x(2\beta - 6\gamma) - 2y(\alpha + 3\gamma) + 6(2\alpha + 2\beta)$
$+ 2x(4\alpha + 3\beta) - 2y(8\gamma - 6\beta) + 2(-4\gamma - 4\alpha)$
$+ 22\alpha + 144\gamma - 81\beta = 0$

$\therefore \quad \alpha(8x - 2y + 26) + \beta(10x + 12y - 69) + \gamma(-12x - 22y + 136) = 0$

$$\therefore \quad \left.\begin{aligned} 4x - y + 13 &= 0 \\ 10x + 12y - 69 &= 0 \\ \text{and } 6x + 11y - 68 &= 0 \end{aligned}\right\} x = -\frac{3}{2},\ y = 7$$

$\therefore \quad f(x,y) \text{ is point } \left(-\frac{3}{2},\ 7\right)$

$$d = \left|\frac{4\left(\frac{-3}{2}\right) - 3(7) + 12}{5}\right| = 3$$

8. Ans. 8

$a_5 x^{p_5} = {}^{15}C_{10} x^{10}$

$a_{11} x^{p_{11}} = {}^{15}C_{13}.x^{13}$

$a_9 x^{p_9} = {}^{15}C_{12}.x^{12}$

$a_{14} x^{p_{14}} = {}^{15}C_0.x^0$