Paper Code : 1001CT102116005

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

CLASSROOM CONTACT PROGRAMME
(Academic Session : 2016 - 2017)

JEE (Main + Advanced) : ENTHUSIAST COURSE (PHASE : I)

ANSWER KEY : PAPER-1 TEST DATE : 29-05-2016

Test Type : MINOR Test Pattern : JEE-Advanced

PART-1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	B,C	B,C	A,B	A,B,C	B,D	B,C	A,B	A,B,D	C	A
	Q.	11	12								
	A.	D	B								
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	6	3	4	2	6	9	2	2		

PART-2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	C,D	A,B,C,D	A,B	A,C,D	A,B,C,D	A,B,C,D	A,B,D	D	B
	Q.	11	12								
	A.	A	B								
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	3	6	4	6	0	2	8	5		

PART-3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	B,D	A,D	A	A	A,B,C,D	B,C	A,C or B,C	Bonus	A
	Q.	11	12								
	A.	B	D								
SECTION-IV	Q.	1	2	3	4	5	6	7	8		
	A.	0	8	2	5	6	5	9	0		

Paper Code : 1001CT102116006

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

CLASSROOM CONTACT PROGRAMME
(Academic Session : 2016 - 2017)

JEE (Main + Advanced) : ENTHUSIAST COURSE (PHASE : I)

ANSWER KEY : PAPER-2 TEST DATE : 29-05-2016

Test Type : MINOR Test Pattern : JEE-Main

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	3	4	3	2	1	3	3	3	1	2	2	2	1	3	4	4	4	2	2	1
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	4	1	2	1	3	1	2	2	4	2	4	2	3	2	3	4	3	1	3	2
Que.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Ans.	3	2	2	4	4	1	3	2	4	4	4	3	3	3	3	2	3	4	2	1
Que.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Ans.	1	3	3	4	4	3	4	1	2	4	3	2	4	1	2	4	4	4	4	4
Que.	81	82	83	84	85	86	87	88	89	90										
Ans.	4	2	1	2	3	3	1	4	3	1										

JEE (Main + Advanced) : ENTHUSIAST COURSE

PHASE : I

Test Type : MINOR Test Pattern : JEE-Advanced

TEST DATE : 29 - 05 - 2016

PAPER-1

PART-1 : PHYSICS SOLUTION

SECTION-I

1. Ans. (B,C)

Sol. $\omega = 30\ \pi$

$2\pi f = 30\pi$

$f = 15$ Hz

$k = 20\ \pi$

$$\frac{2\pi}{\lambda} = 20\pi$$

$$\lambda = \frac{1}{10}m = 10cm$$

Wave is travelling along negative x axis with velocity 1.5 m/s

2. Ans. (B,C)

Sol. $$g = \frac{4}{3}\pi GR\rho$$

$$V_e = R\sqrt{\frac{8}{3}\pi G\rho}$$

3. Ans. (A,B)

Sol. Electric field between $a \le r \le 2a$ is E

$$E4\pi r^2 = \frac{q + \int_a^r \frac{C}{r}4\pi r^2 dr}{\varepsilon_0}$$

$$E = \frac{q + C4\pi\frac{[r^2 - a^2]}{2}}{4\pi\varepsilon_0 r^2}$$

$$E = \frac{(q - C2\pi a^2)}{4\pi\varepsilon_0 r^2} + \frac{C}{2\varepsilon_0}$$

If electric field is constant,then

$q = 2\pi Ca^2$

4. Ans. (A,B,C)

Sol. Equilibrium position will shift by $\frac{mg}{k}$

$$\text{Amplitude} = \frac{mg}{k}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

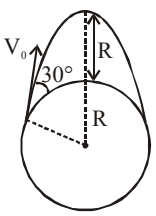
$$v_{max} = \frac{mg}{k}\sqrt{\frac{k}{m}}$$

5. Ans. (B,D)

Sol. Angular momentum remains conserved about centre of earth.

$mv_0\cos 30°R = mv2R$

$$V = \frac{\sqrt{3}V_0}{4}$$



Path will not be parabolic because acceleration due to gravity does not remain constant.

6. Ans. (B,C)

Sol. Charge is given to A,then there will be no charge induced on B. Electric field between A and B will be zero and potential of A and B will be same.

7. Ans. (A,B)

Sol. $P^{3/2}V = C$

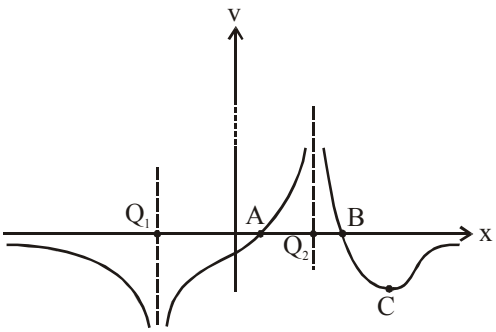
$PV^{2/3} = C$

$$C = C_V + \frac{R}{1-\frac{2}{3}} = C_V + 3R$$

$$\frac{T^{3/2}}{V^{1/2}} \text{ constant}$$

8. Ans. (A,B,D)

Sol.



At point A $V = 0$

$$\frac{k|Q_1|}{r_1} = \frac{k|Q_2|}{r_2}$$

$r_1 > r_2$

$|Q_1| > |Q_2|$

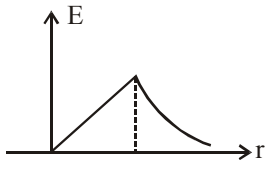
Near Q_1 potential is negative & near Q_2 potential is positive. $Q_1 < 0$, $Q_2 > 0$

At point C , $\frac{dV}{dx} = 0 \Rightarrow E = 0$

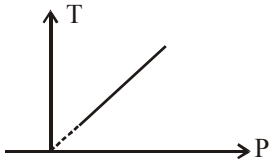
9. Ans. (C)

Sol. (P) $v > v_e$, then speed continuously decreases

(Q)



(R)



(S) $T = 2\pi\sqrt{\frac{\ell}{g}}$

Time period is independent of mass

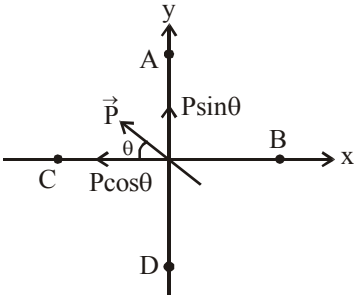
10. Ans. (A)

Sol. $v_A = \frac{kp\sin\theta}{r^2}$

$v_B = \frac{-kp\cos\theta}{r^2}$

$v_C = \frac{kp\cos\theta}{r^2}$

$v_D = \frac{-kp\sin\theta}{r^2}$



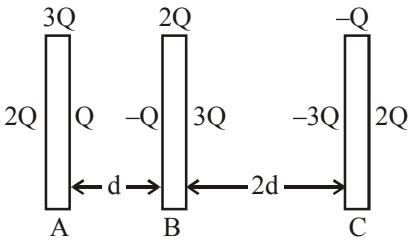
11. Ans. (D)

12. Ans. (B)

SECTION-IV

1. Ans. 6

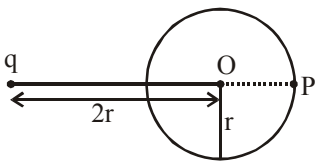
Sol.



$$V = E(2d) = \frac{3Q}{A \times \varepsilon_0} 2d$$

2. Ans. 3

Sol. $V_0 = \frac{kq}{2r} = 9$



$$V_P = \frac{kq}{3r} + V_{ind} = V_0$$

$9 = 6 + V_{ind}$

3. Ans. 4

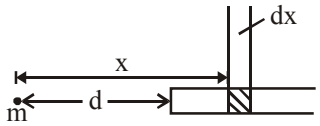
Sol. $\gamma = 5/3$

$2P_0(2V_0)^{\gamma} = P_C V_C^{\gamma}$ ... (i)

$P_0V_0 = P_CV_C$ ... (ii)

4. Ans. 2

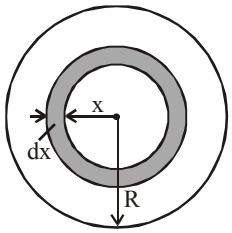
Sol.



$$f = \int_d^{\infty} \frac{Gk}{x}\frac{dxm}{x^2}$$

5. Ans. 6

Sol. $4\pi x^2 \frac{dx}{dt}\rho L = \frac{4\pi kRx\theta}{R-x}$



$$t = \frac{\rho L}{k\theta}\frac{R^2}{6}$$

6. Ans. 9

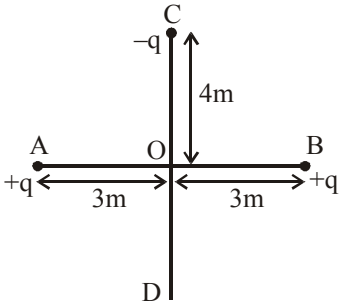
Sol. Let thermal resistance of one rod be R.

$$q_1 = \frac{100\times 3}{R}$$

$$q_2 = \frac{100}{3R}$$

7. Ans. 2

Sol.



$$\frac{-kq^2}{5}\times 2 + 0 = \frac{-2kq^2}{3} + \frac{1}{2}mv^2$$

$$\frac{2kq^2}{3} - \frac{2kq^2}{5} = \frac{1}{2}mv^2$$

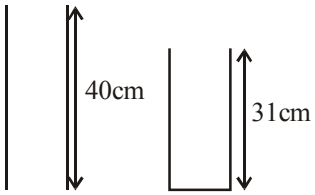
$$\frac{(10-6)kq^2}{15} = \frac{1}{2}mv^2$$

$$q\sqrt{\frac{8}{15\times 4\pi\varepsilon_0 m}} = v$$

$$q\sqrt{\frac{2}{15\pi\varepsilon_0 m}} = v$$

8. Ans. 2

Sol.



$$\frac{2v}{2\ell_1} = \frac{3v}{4\ell_2}$$

$$\frac{2v}{2[40+2e]} = \frac{3v}{4[31+e]}$$

$124 + 4e = 120 + 6e$

$4 = 2e$

$e = 2cm$

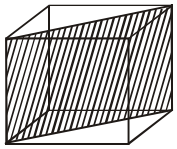
PART–2 : CHEMISTRY — SOLUTION

SECTION - I

1. Ans.(A)

2. Ans.(C, D)

3. Ans.(A,B,C,D)

(D)  There are six planes of this type.

4. Ans. (A, B)

5. Ans. (A, C, D)

6. Ans. (A, B, C, D)

7. Ans. (A,B,C,D)

8. Ans. (A,B,D)

9. Ans.(D)

10. Ans. (B)

11. Ans. (A)

12. Ans. (B)

SECTION - IV

1. Ans. (3)

$a = 2 (r_+ + r_-)$

$10 = 2 (2 + r_-)$

$r_- = 3Å$

2. Ans. (780) ; OMR ANS (6)

$$\begin{pmatrix}300\\P\end{pmatrix} \xrightarrow[i.e.\ T\propto V^2]{(1)\ P\propto V} \begin{pmatrix}1200\\2P\end{pmatrix} \xrightarrow[T\rightarrow constant]{(2)} \begin{pmatrix}1200\\2K\end{pmatrix}$$

$$\xrightarrow[P\rightarrow constant]{(2)} (300K)$$

$$W_1 = \frac{nR\Delta T}{x-1} = \frac{1\times R\times 900}{-2} = -450\ R$$

$$W_2 = -nRT \ln \frac{P_1}{P_2} = -R \times 200 \ln 2 = -840\ R$$

$$W_3 = -nR\Delta T = -R \times (-900) = 900\ R$$

$$W_{total} = -450\ R - 840\ R + 900\ R$$

$$= -390\ R = -780\ cal.$$

3. Ans. (4)

$$r = \frac{V}{t} \propto \frac{1}{\sqrt{M_w}}$$

$$t \propto \sqrt{M_w}$$

$$\frac{5}{10} = \sqrt{\frac{M_{mix}}{32}}$$

$$M_{mix} = 8$$

$$V.D. = \frac{M_{mix}}{2} = 4$$

4. Ans (6)

5. Ans. (0)

6. Ans. (2)

7. Ans. (8)

8. Ans. (5)

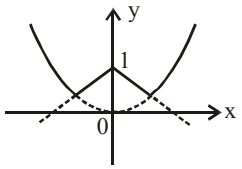
PART-3 : MATHEMATICS SOLUTION

SECTION-I

1. Ans. (D)

$\ell = 1+4+3 = 8$

2. Ans. (B,D)



3. Ans. (A,D)

$$N = {}^{10}C_6.4! = \frac{10!}{6!.4!}.4!$$

$= 10.9.8.7$

4. Ans. (A)

$y = x^{(\sin x)^y}$

$\log y = (\sin x)^y.\log x$

$\log(\log y) = y\log\sin x + \log(\log x)$

$$\frac{1}{\log y}.\frac{1}{y}.y' = y\cot x + \log(\sin x).y' + \frac{1}{\log x}.\frac{1}{x} \quad \ldots\ldots (1)$$

at $x = \frac{\pi}{2}$, $y = \frac{\pi}{2}$

$$(1) \Rightarrow \frac{1}{\log\frac{\pi}{2}}.\frac{2}{\pi}.y' = 0 + 0 + \frac{1}{\log\frac{\pi}{2}}.\frac{2}{\pi}$$

$\therefore \quad y' = 1$

5. Ans. (A)

$\log(\sin^2 x.\cos x) = t$

$\Rightarrow (2\sin x.\cos^2 x - \sin^3 x)dx = dt.\sin^2 x.\cos x$

$\Rightarrow (2\sin x(1-\sin^2 x) - \sin^3 x)dx = dt.\sin^2 x.\cos x$

$$\Rightarrow \quad (2\sin x - 3\sin^3 x)dx = dt\,\frac{\sin x.\sin 2x}{2}$$

$$\Rightarrow \quad \frac{(2-3\sin^2 x)}{\sin 2x}dx = \frac{dt}{2}$$

$$I = \frac{1}{2}\int t\,dt = \frac{t^2}{4} + C$$

$$= \frac{\log^2(\sin^2 x.\cos x)}{4} + C$$

$$= \log^2\left(\frac{1}{2}\sin 2x.\sin x\right) + C$$

6. Ans. (A,B,C,D)

$$\frac{{}^{15}C_5(x^3)^{10}(\sqrt{2}.x^{-2})^5}{{}^{15}C_5(\sqrt{2}x^{-2})^{10}(x^3)^5} = \frac{4\sqrt{2}}{243}$$

$$\Rightarrow \quad \left(\frac{x^3}{\sqrt{2}x^{-2}}\right)^5 = \left(\frac{\sqrt{2}}{3}\right)^5$$

$$\frac{x^5}{\sqrt{2}} = \frac{\sqrt{2}}{3} \qquad \Rightarrow \quad x^5 = \frac{2}{3}$$

$$x = \left(\frac{2}{3}\right)^{\frac{1}{5}}$$

7. Ans. (B,C)

$$\int \log(1+\cos x)dx + \int 2\log\left(\sec\frac{x}{2}\right)dx$$

$$= \int \log\left(2\cos^2\frac{x}{2}\sec^2\frac{x}{2}\right)dx = x\ell n2 + C$$

$= f(x) = x\ell n2$

8. Ans. (A,C or B,C)

By using IVT

9. Ans. (Bonus)

(P) $x^3 - x + 1 = t$

$$\frac{1}{3}\int\frac{dt}{t^2} = -\frac{1}{3t} = -\frac{1}{3}\left(\frac{1}{x^3-x+1}\right) + C$$

(Q) $e^{\lim\limits_{x\to 0}\frac{f(x)}{x^3}} = e^1$

degree of $f(x) = 3$

(R) $$T_{r+1} = {}^{15}C_r(x^2)^{15-r}\left(-\frac{2}{\sqrt{x}}\right)^r$$

$$30 - \left(2r + \frac{r}{2}\right) = 0 \Rightarrow r = 12$$

$\therefore$ n = 13

(S) $$\lim_{n\to\infty}\frac{4-\left[h\log(1+h)\right]^n}{2+\left[h\log(1+h)\right]^n} = 2$$

10. Ans. (A)

$$I = \int\frac{1}{x\sqrt{1-x^4}}dx + \int\frac{x}{\sqrt{1-x^4}}dx$$

$= I_1 \qquad\qquad I_2$

for I_1 put $x^4 = t$ & I_2 put $x^2 = s$

$$\therefore I = \frac{1}{4}\log\left|\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^4}+1}\right| + \frac{1}{2}\sin^{-1}\left(x^2\right) + C$$

$\therefore \ell = 1,\ m = 4,\ n = 2,\ k = 2$

11. Ans. (B)

$N = 2^6 3^3 5^2 7^1$

(P) Number of proper even divisors
$= 6.4.3.2 - 1 = 143$

(Q) Number of divisors divisible by 10
$= 6.4.2.2 = 96$

(R) Number of divisors of form $2(2n + 1)$
$= 1.4.3.2 = 24$

(S) Number of divisors divisible by 21
$= 7.3.3 = 63$

12. Ans. (D)

(P) $f(x) = \dfrac{2\sin(-x).\cos 4x}{2\sin 4x.\sin x} = -\cot 4x$

$f'(x) = +4\text{cosec}^2 4x$

$f''(x) = +8\text{cosec}4x.(-\text{cosec}4x.\cot 4x)4$

$= -32\text{cosec}^2 4x.\cos 4x$

$f''(\pi/16) = -64$

(Q) $I = \int \dfrac{e^{-x}(\sin x + 2\cos x)}{\sqrt{e^{2x} - \cos x}}dx$

$= \int \dfrac{e^{-2x}\left(\sin x + 2\cos x\right)}{\sqrt{1 - e^{-2x}.\cos x}}dx$

Put $1 - e^{-2x}.\cos x = t^2$

$\Rightarrow \quad -(e^{-2x}(-\sin x) - 2e^{-2n}\cos x)dx = 2tdt$

$= e^{-2n}(\sin x + 2\cos x)dx = 2tdt$

$$I = \int \frac{2t}{t}dt = 2t + C = 2\sqrt{1 - e^{-2x}}\cos x + C$$

$\Rightarrow \quad a = 2 = b$

(R) $\lim\limits_{x\to 0} \dfrac{(1-e^{2x})(2)\log(1-\{8x^2\})}{(-2x)(-8x^2)}.8 = 16$

SECTION – IV

1. Ans. 0

$$f'(0^+) = \lim_{h\to 0}\frac{f(h) - f(0)}{h}$$

$$= \lim_{h\to 0}\frac{\dfrac{\sin^{-1}\left(h^2 + h^3\right) - h}{h} + 1}{h}$$

$$= \lim_{h\to 0}\frac{\sin^{-1}\left(h^2 + h^3\right)}{h^2}\frac{\left(h^2 + h^3\right)}{\left(h^2 + h^3\right)} = 1$$

$$\lim_{x\to 0^+} f\left(x\right) = \lim_{h\to 0}\frac{\sin^{-1}\left(h^2 + h^3\right) - h}{h}$$

$$= \lim_{h\to 0}\frac{\dfrac{1}{\sqrt{1 - \left(h^2 + h^3\right)^2}}.\left(2h + 3h^2\right) - 1}{1} = -1$$

$$\therefore \lim_{x\to 0^+} f\left(x\right) + f'\left(0^+\right) = 0$$

2. Ans. 8

$$I = \int \frac{\cos^4 x.\cos x}{\sin x}.dx$$

put $\sin x = t$

$$= \int \frac{\left(1 - t^2\right)^2 dt}{t} = \int \frac{\left(1 + t^4 - 2t^2\right)}{t}dt$$

$$= \log|t| + \frac{t^4}{4} - t^2 + C$$

$$= \log|\sin x| + \frac{1}{4}.\sin^4 x - \sin^2 x + C$$

$$\frac{\ell - n}{m} = \frac{2}{\frac{1}{4}} = 8$$

3. Ans. 2

$(1 - x^3)^5(1 + x^2)^4(1 + x^4)^8$

$[{}^5C_0 - {}^5C_1x^3 + {}^5C_2x^6 .....]\ [{}^4C_0 + {}^4C_1x^2 + {}^4C_2x^4 + {}^4C_3x^6 + {}^4C_4x^8]$

$[{}^8C_0 + {}^8C_1x^4 + ...]$

Coefficient of x^6

$= {}^5C_0{}^4C_3{}^8C_0 + {}^5C_0{}^4C_1{}^8C_1 + {}^5C_2{}^4C_0{}^8C_0$

$= 4 + 32 + 10 = 46$

4. Ans. 5

Let $x(1 + x^{-2})^2+1 = t^2$

$x(1 + x^{-4} + 2x^{-2}) + 1 = t^2$

$x + x^{-3} + 2x^{-1} + 1 = t^2$

$(1 - 3x^{-4} - 2x^{-2})dx = 2tdt$

$$\frac{\left(x^4 - 2x^2 - 3\right)}{x^4}dx = 2tdt$$

$$I = \int \frac{2tdt}{t} = 2t + C$$

$$= 2\sqrt{x + \frac{1}{x^3} + \frac{2}{x} + 1} + C$$

$a = 1, b = 1, c = 2, d = 1$

$a + b + c + d = 5$

5. Ans. 6

$g(f(x)) = x$

$$g'\left(f\left(x\right)\right) = \frac{1}{f'\left(x\right)}$$

$$g''\left(f\left(x\right)\right)f'\left(x\right) = -\frac{1}{\left(f'\left(x\right)\right)^2}f''\left(x\right)$$

Put $x = 1$

$$g''\left(5\right) = -\frac{1}{25}$$

$\Rightarrow \quad N = 5$

6. Ans. 5

$$x \to \frac{1}{x^2}$$

$$\frac{\left(x^4 + x^2 + 1\right)^{10}}{x^{20}} = a_0 + \frac{a_1}{x^2} + \frac{a_2}{x^4} + ... + \frac{a_{10}}{x^{20}} + ... \frac{a_{20}}{x^{40}}$$

$\therefore a_0{}^2 + a_1a_2 + ... + a_{10}a_{20}$

= constant term in

$$= \frac{\left(1 + x + x^2\right)^{10}\left(x^2 + x + 1\right)^{10}\left(x^2 - x + 1\right)^{10}}{x^{20}}$$

= coefficient of x^{20} in $(1 + x + x^2)^{20}(1 - x + x^2)^{10}$

$= 20$

$$\therefore \quad \frac{n}{4} = 5$$

7. Ans. 9

$f(x)$ is discontinuous at all integers except at $x = 0$

8. Ans. 0

$$(1 + x)^{10} = \sum_{r=0}^{10} {}^{10}C_r x^r$$

$$10(1 + x)^9 = \sum_{r=0}^{10} {}^{10}C_r . rx^{r-1}$$

Put $x = 1$

$\therefore S_2 = 10.2^9$

$$90(1 + x)^8 = \sum_{r=0}^{10} {}^{10}C_r \left(r^2 - r\right)x^{r-2}$$

Put $x = 1$

$S_2 = 90.2^8$

$$\therefore \frac{S_2}{S_1} = \frac{90.2^8}{20.2^8} = \frac{9}{2}$$

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ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

CLASSROOM CONTACT PROGRAMME

(Academic Session : 2016 - 2017)

JEE (Main + Advanced) : ENTHUSIAST COURSE

PHASE : I

Test Type : MINOR | Test Pattern : JEE-Main

TEST DATE : 29 - 05 - 2016

PAPER-2

SOLUTION

1. Ans. (3)

Sol. E_{max} at $\frac{R}{\sqrt{2}}$

$$\frac{kq\times\frac{R}{\sqrt{2}}}{\left(R+\frac{R^2}{2}\right)^{3/2}}$$

$$\frac{1}{4\pi\varepsilon_0}\frac{2q}{3\sqrt{3}R^2}$$

2. Ans. (4)

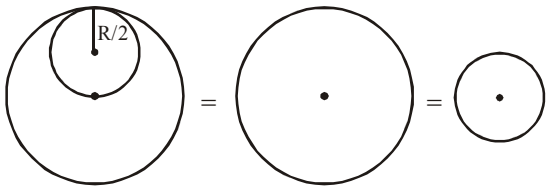
Sol. $v = k\pi\lambda$

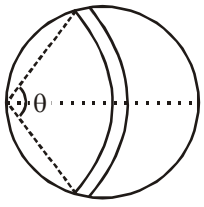
3. Ans. (3)

Sol. $E=\frac{\sigma}{\varepsilon_0}$

4. Ans. (2)

Sol.





$$\frac{\sigma}{2\varepsilon_0}(R-0)-\frac{\sigma R}{2\pi\varepsilon_0}$$

$$=\frac{\sigma R}{2\,\epsilon_0}\left(1-\frac{1}{\pi}\right)=v_0$$

$$dv=\frac{k\sigma 2x\theta dx}{x}$$

$x = 2R\cos\theta$

$dx = -2R\sin\theta\, d\theta$

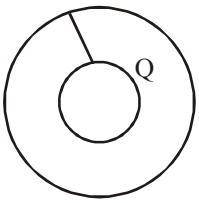
$$v=\frac{\sigma R}{2\pi\,\epsilon_0}$$

5. Ans. (1)

Sol. $V=\int ydx+xdy=-xy+C$

6. Ans. (3)

Sol. Whole charge will flow to outer shell.



7. Ans. (3)

Sol. $-q$ —(a)— $q/2$ ←(x)→ P

$$\left(\frac{kq}{(a+x)^2}\right)-\left(\frac{k\frac{q}{2}}{x^2}\right)=0$$

$$x=\frac{a}{\sqrt{2}-1}$$

$$a+\frac{a}{\sqrt{2}-1}=\frac{\sqrt{2}a}{\sqrt{2}-1}$$

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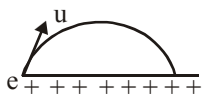
8. Ans. (3)

9. Ans. (1)

Sol.



$$R = \frac{u^2 \sin 2\theta}{a} \Rightarrow R_{max} = u^2/a$$

$$\text{here } a = \frac{e\sigma}{\epsilon_0 m}$$

10. Ans. (2)

Sol. $\vec{\tau} = \vec{P} \times \vec{E}$

$U = -PE\cos\theta$

11. Ans. (2)

Sol. $\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^2$

$R = 10\ r$

$$V = \frac{k(1000q)}{10r}$$

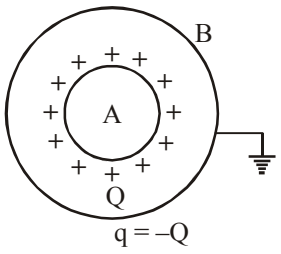
12. Ans. (2)

Sol. $\frac{(100-\theta)}{\ell} kA = \frac{3(\theta-20)5k}{\ell}$

$100 - \theta = 15\ \theta - 300$

13. Ans. (1)

Sol.



$$\frac{kQ}{b} + \frac{kq}{b} = 0$$

$q = -Q$

14. Ans. (3)

Sol. Between 2 nodes all particle vibrate in same phase

15. Ans. (4)

16. Ans. (4)

17. Ans. (4)

Sol. For charging transfer of electrons is responsible

18. Ans. (2)

Sol. $U = -\frac{GMm}{r}$, kinetic energy $= \frac{GMm}{2r}$

$$U = (-2)\frac{GMm}{2r} = -2 \times \text{kinetic energy}$$

$$= -2 \times \frac{1}{2}mv^2 = -mv^2$$

19. Ans. (2)

Sol. Angle $\angle ADC = 90^\circ$

$\Rightarrow$ distance $AC = DC = \sqrt{2}\,\ell$

$\Rightarrow F_{AB} \perp F_{CD}$

20. Ans. (1)

Sol. Geo-satellite mean whose time period is $T = 24$ hr & from law

$$T^2 \propto R^3 \Rightarrow \frac{T_2^2}{T_1^2} = \frac{R_2^3}{R_1^3} \Rightarrow \frac{T_2^2}{(24)^2} = \frac{(R+2.5R)^3}{(R+6R)^3}$$

$$T_2 = \sqrt{\frac{24\times 24}{8}} = 6\sqrt{2} \text{ hr.}$$

21. Ans. (4)

Sol. $\Delta\phi = (\omega_1 - \omega_2)t = \left(\frac{2\pi}{T} - \frac{2\pi}{5T/4}\right) T = \frac{2\pi}{5}$

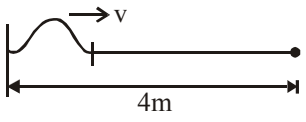
22. Ans. (1)

23. Ans. (2)

Sol. $R = \frac{L}{kA} = \frac{L}{\pi r^2}$ & $\frac{\Delta Q}{\Delta t} \propto \frac{1}{R}$

24. Ans. (1)

Sol. 4 trips means 32 m



$$t = \frac{d}{v} \Rightarrow v = \frac{d}{t} = \frac{32}{0.8} = 40 \text{ m/s}$$

$$v = \sqrt{\frac{T}{\mu}}$$

$\Rightarrow T = \mu v^2$

$$T = \frac{0.2}{4} \times (40)^2 = \frac{2\times 16\times 10}{4}$$

$T = 80$ N

25. Ans. (3)

Sol. $K = .314 = \frac{2\pi}{\lambda}$

$$\frac{\lambda}{2} = \frac{\pi}{.314} = \frac{3.14}{.314} = 10$$

26. Ans. (1)

Sol. From A : $\Delta Q > 0$, $\Delta U = 0$, $\Delta W > 0$

From B : $\Delta Q < 0$, $\Delta U = 0$, $\Delta W < 0$

27. Ans. (2)

Sol. $dQ = -dU$

$$C = -C_V = \frac{-R}{\gamma - 1} = \frac{+R}{\gamma - 1} + \frac{P}{n}\frac{dV}{dT}$$

$$\boxed{-\frac{P}{n}\frac{dV}{dT} = \frac{2R}{\gamma - 1}}$$

$T^5V = \text{const.}$

$$V = \frac{\text{const.}}{T^5}$$

$$\frac{dV}{dT} = -5\frac{\text{const}}{T^6}$$

$PV = nRT$

$P/n = RT/V$

$$+\frac{RT}{\text{const.}}T^5 \times \left(-5\frac{\text{const}}{T^6}\right) = \frac{2R}{\gamma - 1}$$

$$\frac{5}{2} = \frac{1}{\gamma - 1} \Rightarrow \gamma - 1 = 2/5$$

$\gamma = 7/5$

adiabatic compressibility

$$\beta = \frac{1}{\gamma P} = \frac{5}{7P}$$

28. Ans. (2)

Sol. $V_{rms}^2 = <V^2> = \frac{V_1^2 + V_2^2 + V_3^2 + ...}{N}$

$$= \frac{\int V^2 dN}{\int dN} \text{ here } \frac{dN}{dV} = N(V)$$

$$V_{rms}^2 = \frac{1}{N}\int_0^{\infty} N(V)V^2 dV$$

$$= \frac{1}{N}\int_0^{V_0}\left[\frac{3N}{V_0^3}V^2\right]V^2\,dV = \frac{3}{5}V_0^2$$

$$\Rightarrow V_{rms} = \sqrt{\frac{3}{5}}\,V_0$$

29. Ans. (4)

30. Ans. (2)

Sol. $.04 = \frac{1}{2}K \times \left(20 \times 10^{-3}\right)^2$

$$\frac{.04 \times 2}{4 \times 1} \times 10^2 = 200$$

31. Ans. (4)

32. Ans. (2)

33. Ans. (3)

34. Ans. (2)

35. Ans. (3)

36. Ans. (4)

37. Ans. (3)

38. Ans. (1)

39. Ans. (3)

40. Ans. (2)

41. Ans. (3)

42. Ans. (2)

43. Ans. (2)

44. Ans. (4)

45. Ans. (4)

46. Ans. (1)

47. Ans. (3)

48. Ans. (2)

49. Ans. (4)

50. Ans. (4)

51. Ans. (4)

52. Ans. (3)

53. Ans. (3)

54. Ans. (3)

55. Ans. (3)

56. Ans. (2)

57. Ans. (3)

58. Ans. (4)

59. Ans. (2)

60. Ans. (1)

61. Ans. (1)

$f(x) = 1 + x^n$ or $f(x) = 1 - x^n$

$\because \quad f(5) = 126 \quad \Rightarrow \quad f(x) = 1 + x^3$

$$\int (1 + x^3)dx = x + \frac{x^4}{4} + C$$

62. Ans. (3)

A, M, N, N, R, U

$\underline{A}\ \_\ \underline{5!}\ /\ \underline{2!}\ \_ = 60$

$M\ \underline{A/R}\ \underline{4!}\ /\ \underline{2!}\ \_ = 24$

$M\ \underline{N}\ \_\ \_\ \underline{4!}\ \_ = 24$

$M\ U\ \underline{A}\ \underline{3!}\ /\ \underline{2!} = 3$

$M\ U\ N\ \underline{A}\ \underline{2!}\ \_ = 2$

$M\ U\ N\ N\ A\ R$

} 113

63. Ans. (3)

$$\frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)} = \frac{a\cos t}{a\left[-\sin t + \frac{1}{\sin t}\right]}$$

$$= \frac{\cos t \sin t}{(1-\sin^2 t)} = \frac{\sin t}{\cos t} = \tan t$$

64. Ans. (4)

Apply continuity at $x = \frac{\pi}{2}$; $A = \frac{\pi}{2}$

Apply differentiability at $x = \frac{\pi}{2}$

$-B(1) = 1 \Rightarrow B = -1$

65. Ans. (4)

Coefficient of x^{15}

$(1 - x^3)^{15} = -{}^{15}C_5 = p$

$\left(x^2 - \frac{2}{x}\right)^{15}$ coefficient of x^{15}

$= {}^{15}C_r (x^2)^{15-r}\left(-\frac{2}{x}\right)^r$

$q = {}^{15}C_{10}(-2)^{10}$

$\left|\frac{p}{q}\right| = \frac{1}{1024}$

66. Ans. (3)

$f(x) = 1 - \sin x$

$g(x) = x + \cos x$

$h'(g(x)) = \frac{1}{g'(x)}$

$h'(\pi - 1) = \frac{1}{g'(x)} = \frac{1}{1-0} = 1$

67. Ans. (4)

$$\int \frac{(\sin x + \cos x)dx}{3 + 1 - (\sin x - \cos x)^2}$$

$$= \int \frac{(\sin x + \cos x)dx}{(2)^2 - (\sin x - \cos x)^2}$$

Let $\sin x - \cos x = t$

$(\cos x + \sin x)dx = dt$

$$\int \frac{dt}{(2)^2 - t^2} = \frac{1}{4}\ell n\left|\frac{2 + \sin x - \cos x}{2 - \sin x + \cos x}\right| + C$$

68. Ans. (1)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x)f(1 + h/x) - f(x)}{h}$$

$$f'(x) = \frac{f(x)}{x} f'(1)$$

$$\frac{f'(x)}{f(x)} = \frac{4}{x} \Rightarrow f(x) = x^4$$

69. Ans. (2)

$\lim_{x\to 9^+} f(x) = f(9) = \lim_{x\to 9^-} f(x)$

$2a = 1 = b$

70. Ans. (4)

$f(x)$ is not defined at $x = 1, -1$

also not differentiable at $x = 0$

71. Ans. (3)

$$\int \frac{dx}{16 + x^2} = \frac{1}{4}\tan^{-1}\left(\frac{x}{4}\right) + C$$

given $f(4) = \frac{\pi}{16} \quad \Rightarrow \quad f(x) = \frac{1}{4}\tan^{-1}\left(\frac{x}{4}\right)$

$$f(2) + f\left(\frac{4}{3}\right) = \frac{1}{4}\left(\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}\right) = \frac{\pi}{16}$$

72. Ans. (2)

$$\lim_{x\to\infty} x^{1/3}\left((x+1)^{1/3} - (x-1)^{1/3}\right)\left((x+1)^{1/3} + (x-1)^{1/3}\right)$$

$$\lim_{x\to\infty} \frac{x^{1/3}.2\left[(x+1)^{1/3} + (x-1)^{1/3}\right]}{\left[(x+1)^{2/3} + (x-1)^{2/3} + (x^2-1)^{1/3}\right]}$$

73. Ans. (4)

$f(x)$ is continuous at $x = 2$

$\Rightarrow \lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x) = f(2)$

$4p - q = 4$

74. Ans. (1)

$(1 + 2x + 3x^2)^{15} = a_0 + a_1x + a_2x^2 + ...a_{30}x^{30}$,
diff. w.r.t. x
$15(1 + 2x + 3x^2)^{14}(2 + 6x)$
$= a_1 + 2a_2x + ... 30a_{30}x^{24}$
Put x = 1

$$\sum_{k=1}^{30} k.a_k = 120.6^{14} = 20.6^{15}$$

75. Ans. (2)

$$\int f(x)dx = F(x)$$

$$\int 5(x^4 - 1)dx = F(x)$$

$\Rightarrow$ $F(x) = x^5 - 5x + C$
$\because$ $F(1) = 20 \Rightarrow C = 24$
$\therefore$ $F(x) = x^5 - 5x + 24$
$F(2) = 46$

76. Ans. (4)

$5 < n_1 < n_2 < n_3 < n_4$
least value of $n_1 = 6$, $n_2 = 7$, $n_3 = 8$, $n_4 = 9$
$\Rightarrow$ $n_1 + n_2 + n_3 + n_4 = 5$

$$\left.\begin{matrix} 0&0&0&5\\ 0&0&1&4\\ 0&0&2&3\\ 0&1&1&3\\ 0&1&2&2\\ 1&1&1&1 \end{matrix}\right\} 6\,\text{ways}$$

77. Ans. (4)

$$\text{Let } y = \cos\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)$$

$$\text{and } z = \cos^{-1}\left(\sqrt{\frac{1-x}{2}}\right)$$

Put $x = \cos\theta \Rightarrow \theta = \cos^{-1}x \in [0,\pi]$

$$y = \cos\theta \;\&\; z = \frac{\pi}{2} - \frac{\theta}{2}$$

$$\Rightarrow \frac{dy}{dz} = 2\sqrt{1-x^2}$$

78. Ans. (4)

put $\sin x = t$
$\cos x dx = dt$

$$I = \int e^t(2t+2)dt$$

$= 2te^t + C = 2\sin e^{\sin x} + C$

79. Ans. (4)

$$f(x) = \begin{cases} x & x \in \left(n\pi - \frac{\pi}{6}, n\pi + \frac{\pi}{6}\right) \\ \frac{x}{2} & x = n\pi \pm \frac{\pi}{6} \\ 0 & x \in R - \left[n\pi - \frac{\pi}{6}, n\pi + \frac{\pi}{6}\right] \end{cases}$$

80. Ans. (4)

Let degree of $f(x)$ be n
$n - 1 = n - 2 + n - 3$
$n = 4$
Let $f(x)$ be $= ax^4 + bx^3 + cx^2 + dx + e$
$(32ax^3 + 12bx^2 .....)$
$= (108ax^2 + 18bx + 2c).(24ax + 6b)$
coefficient of x^3
$32a = 108a.24a(a \neq 0)$

$$a = \frac{1}{81}$$

81. Ans. (4)

Let $x = \cos^2\theta$
$dx = -2\sin\theta\cos\theta\, d\theta$

$$\therefore \quad I = -2\int \tan\frac{\theta}{2}\tan\theta d\theta$$

$$= -4\int \frac{\sin^2\theta/2}{\cos\theta}d\theta = -2\int\frac{1-\cos\theta}{\cos\theta}d\theta$$

$$= -2\ell n\,|\sec\theta + \tan\theta| + 2\theta + C$$

$$= -2\ell n\left|\frac{1+\sqrt{1-x}}{\sqrt{x}}\right| + 2\cos^{-1}\sqrt{x} + C$$

82. Ans. (2)

${}^nZ_m = {}^nC_m$
${}^{12}C_4 - {}^{11}C_3 = {}^{11}C_3.2$
$\Rightarrow$ $2.{}^{11}Z_3$

83. Ans. (1)

$$b = \frac{4}{1+x^2} \Rightarrow x^2 = \frac{4}{b} - 1 > 0 \Rightarrow b \in (0,4)$$

84. Ans. (2)

$R(7^{2016}, 4)$ $\equiv$ Remainder
when 7^{2016} is divided by 4.
$(4 + 3)^{2016} \Rightarrow 3^{2016} = (4 - 1)^{2016} = 1$

85. Ans. (3)

$\int (7\tan^6 x - 4\tan^3 x)\sec^2 x\,dx$

put $\tan x = t$

$\sec^2 x\,dx = dt$

$\int (7t^6 - 4t^3)dt = t^7 - t^4 + C$

$= \tan^7 x - \tan^4 x + C$

$g(0) = 0 \Rightarrow C = 0$

$\Rightarrow \quad g\left(\frac{\pi}{4}\right) = 0$

86. Ans. (3)

$\lim\limits_{x\to 0}\left(\sin^2 \frac{\pi}{2-3x}\right)^{\sec^2\left(\frac{\pi}{2-3x}\right)}$

Let $\frac{\pi}{2-3x} = \alpha$

$\lim\limits_{\alpha\to\frac{\pi}{2}}(\sin^2\alpha)^{\sec^2\alpha} = 1^\infty$

$e^{\lim\limits_{\alpha\to\frac{\pi}{2}}\frac{\sin^2\alpha - 1}{\cos^2\alpha}} = e^{-1}$

87. Ans. (1)

$\int \frac{2x^2+5}{x^4+5x^2+4} = \int \frac{(x^2+4)+(x^2+1)}{(x^2+4)(x^2+1)}dx$

$= \int \frac{dx}{1+x^2} + \int \frac{dx}{x^2+4}$

$= \tan^{-1} x + \frac{1}{2}\tan^{-1}\frac{x}{2} + C$

88. Ans. (4)

$\int \frac{6}{x^2\left(x^3+3x\right)^{\frac{1}{3}}}dx$

$= \int \frac{6x^{-3}}{\left(1^2+\frac{3}{x^2}\right)^{1/3}}dx \qquad 1+\frac{3}{x^2} = t$

$6x^{-3}dx = -dt$

$= -\int \frac{dt}{t^{1/3}}$

$= -\frac{3}{2}t^{2/3} + C = -\frac{3}{2}\left(1+\frac{3}{x^2}\right)^{2/3} + C$

89. Ans. (3)

$\sin x + \sin y = 1$

$\cos x + \cos y.\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{\cos x}{\cos y}$

$\sin y = 1 - \sin x$

$\sqrt{1-\cos^2 y} = 1 - \sin x$

$1 - \cos^2 y = (1-\sin x)^2$

$1 - \cos^2 y = 1 + \sin^2 x - 2\sin x$

$\cos y = \sqrt{2\sin x - \sin^2 x}$

$\lim\limits_{x\to 0^+} x^\alpha.\frac{dy}{dx} = -\lim\limits_{x\to 0^+}\frac{x^\alpha \cos x}{\sqrt{2\sin x - \sin x}}$

$-\lim\limits_{x\to 0}\frac{x^\alpha}{\sqrt{\sin x}}\frac{\cos x}{\sqrt{2-\sin x}}$

$\Rightarrow \alpha = \frac{1}{2}$

90. Ans. (1)

Let $x^2 = t \Rightarrow 2x\,dx = dt$

$\frac{1}{2}\int \frac{dt}{t^4+1} = \frac{1}{4}\int \frac{(t^2+1)-(t^2-1)}{t^4+1}dt$

$= \frac{1}{4}\int \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}}dt - \frac{1}{2}\int \frac{x^5-x}{x^8+1}dx$

$= \frac{1}{4}\int \frac{1+\frac{1}{t^2}dt}{\left(t-\frac{1}{t}\right)^2+2} - \frac{1}{2}\int \frac{x^5-x}{x^8+1}dx$

$= \frac{1}{4\sqrt{2}}\tan^{-1}\left(\frac{x^4-1}{\sqrt{2}x^2}\right) - \frac{1}{2}\int\left(\frac{x^5-x}{x^8+1}\right)dx + C$